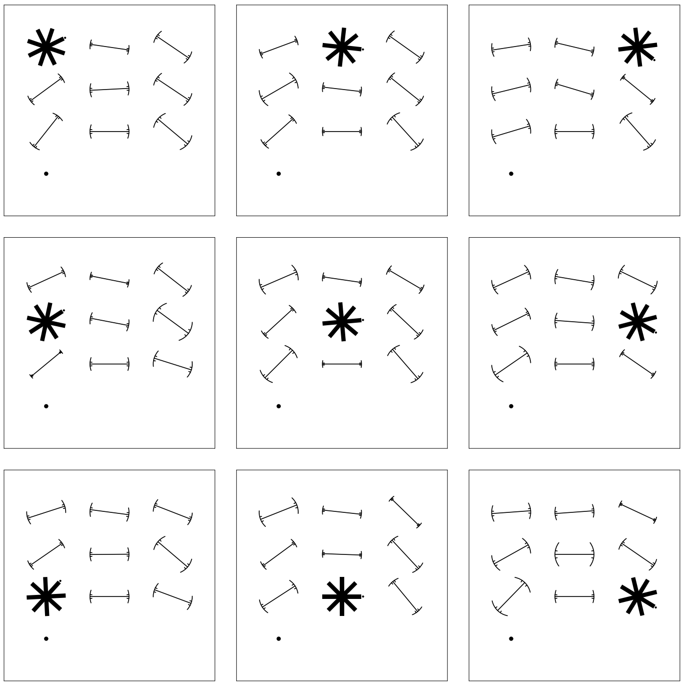
Michiel Meeuwissen

Analysis Done on the Results of Experiments in which Blindfolded Subjects Had to Set Angles in the Plane of a Table



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Abstract

The results of several kinds of measurements, previously done, were described. They deal with haptic perception on a horizontal plane in front of the subject. Three blindfolded subjects had to set directions as a function of place in this plane. They had to set a rod parallel to another rod somewhere else in the plane, had to point to a certain point in the plane or had to set two rods in-line. It was evident that the subjects made certain errors systematically, and it was assumed that these errors could be described with a real function of the plane coordinates. The error made is then the difference of this function on the two involved positions (The one on which the direction should be set, and the one which gives the direction to be set). This function was described in terms of a model of the arm, and could be characterized by a certain factor, which could be found by comparing the measurements to this model. Logically, one wouldn't expect the function to be the same for all three types of described measurements, because sometimes a direction had to be set and perceived, and sometimes only to be set. It was shown that they indeed didn't. The factors also are different for different subjects.

Further, the from literature already known 'oblique effect' was searched for in the measurements. There are two, one in the standard-deviation (preciseness) for which no evidence was found, and one in the absolute error (systematic), which was found. It was shown that this last effect can for an important part be attributed to an 'in-line' effect.

Keywords

Haptic perception, psychophysics, oblique effect

Resumo

La rezultoj de kelkaj tipoj de mezurado estis priskribitaj. Ili temas pri palpa percepto sur horizontala surfaco fronte de la subjekto. Tri blindigitaj subjektoj devis meti direktojn kiel funkcio de loko en ĉi tiu surfaco. Ili devis paraleligi stangon al alia aliloke sur la surfaco, devis montrigi stangon al iu loko en la surfaco aŭ devis sursamliniigi du stangojn. Estis klare ke la subjektoj eraris laŭsisteme, kaj estis supozite ke ĉi tiuj eraroj priskribeblas de reala funkcio de la surfacaj koordinatoj. La farita eraro sekve estas la diferenco de ĉi tiu funkcio je la du koncernataj lokoj (Tiu kie la direkto metendas, kaj tiu kiu donas la metendan direkton). Ĉi tiu funkcio estis priskribita per modelo de la brako, kaj karakterizeblas de iu faktoro, kiu troveblas per komparado la mezuradrezultojn kun ĉi tiu modelo. Logike, oni ne atendus ĉi tiun faktoron esti sama por ĉiuj tri tipoj de priskribitaj mezuradoj, ĉar foje oni devis kaj percepti kaj meti direkton, kaj foje devis nur meti. Montriĝis ke ili efektive ne samas. La faktoroj ankaŭ diferencis por malsamaj subjektoj.

Aldone, la elliteratura 'oblikva efekto' estis serĉata en la mezuraĵoj. Ekzistas du da tiu, unu en la norma devio (precizeco), por kiu neniu pruvo trovitis, kaj unu en la absoluta eraro (sistemeco) kiu ja estis trovita. Estis montrate ke ĉi tiu lasta efekto povas esti grandparte kaŭzita de 'sursamlinia efekto'.

Contents

Chapter 1

Introduction

Mathematical descriptions of the so called ‘visual’ space are already for many years known in the perception sciences. One can describe and measure the structure of visual space in contrast to the real space which is euclidian (in, of course, a classical approach). Luneburg [11] cannot be left unmentioned here, because as one of the first he declared visual space noneuclidian as one of the first.

Vision, however, is not the only sense by which we directly percieve our three dimensional environment. The other is our haptic perception. This includes in fact every sensitivity of the individual to the world adjacent to his body by the use of his body (Gibson[4] ch. 6). This includes the sense of pressure, pain, warm, cold and the sensitivity of the joints. We are interested in the perception of space itself, so not in the more ‘local’ pressure, pain, warm and cold but only in the perception of the joint-angles (joints have receptors that discharge at a given rate for a given angle of the joint), or the superposition of them. One also could argue that the length of the muscles can be important for sensing the state of one’s limbs but it was proven not to be (Rose en Mountcastle, 1959 cited in [4]).

Already a century ago (1898) Goldscheider experimented with the bending of a single joint such as shoulder or wrist. He already showed that posture of a joint as well as the change of it could be registered. To sense blindfolded your environment you have to do it with the information your joints supply you to ‘calculate’ where your hand is. In the past, several related measurements were already done, e.g. the several experiments of Gibson in which subjects had to set things haptically vertical or horizontal. Haptic perception is very related to vision, because everything you can feel in this way, you also can see, and blindfolded this sense will take over much of the function of seeing. It is therefore interesting to question whether also our haptic space is not simply euclidian, but possibly related to visual space or deformed in another way.

As a start to explore the properties of haptic space the perception of a subject of the direction of rods on the plane of a table were explored in this research. Blindfolded subjects were asked to directionalize one or two rods such that they point to each other, to a given point, or such that one becomes parallel to the other.

This report is a reflection of the description and interpretation of the results of these measurements.

Chapter 2

Theory

2.1 Vision science

The visual world is much better known than the haptic one, and something will be said about it first. It was shown that visual space is probably not Euclidean, which is however a pretty strange thing.

It is however true, and this is quickly understood if one becomes aware of the fact that 'infinity' doesn't exist in visual space. An object in infinity, such as the sky or the moon, one sees much closer. This is however no real proof, because these things have no cues at all to judge them anywhere else. Luneburg [11] however had more indication of the non-euclidian properties of visual space and recent measurements also indicate a very complexly distorted visual space (Koenderink[7]). We now only are interested in the nearby structure of it, which has according to Koenderink [7] a positive Gaußian curvature.

2.2 Haptic

Before studying haptic space one can have several hypotheses about it. In every case, one assumes that humans are sufficiently able to handle things in the real three dimensional Euclidean world, even blindfolded, so that haptic space must at least be similar to Euclidean space. One could argue that an eventual deformation of the haptic space will be due to the non-euclidian properties of visual space. Visual perception is, after all, for humans the most important for orienting them in reality, and one is often locating his hand by vision. This mechanism may, one way or another, somehow have calibrated ones haptic sense when blindfolded. Also the opposite can be mentioned; the haptic sense may have influence on the visual space. This would also cause the haptic and the visual world to be similar.

Secondly is it possible that haptic space is influenced by the construction of the arm. Everybody of course knows, be it unconsciously, this and the brains will translate the position of the arm in a direction of the hand and try to correct for it.

With these two different assumptions one can construct (at least) two models to obtain certain deviations of Cartesian space.

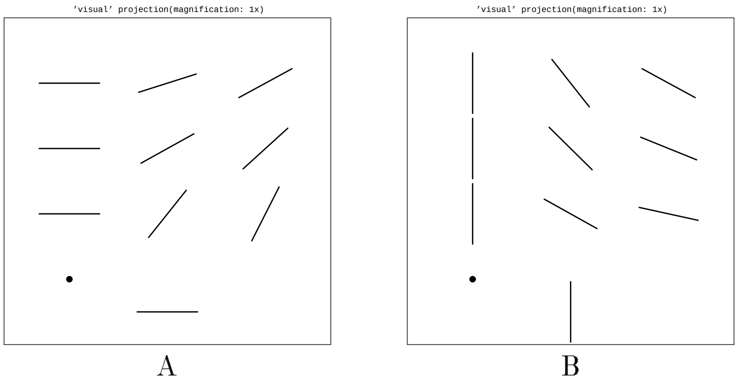


Figure 2.1: Two times the perspectively deformed angles on 9 different positions. The used position of the eye was how it would be for the author. The 10th direction to be seen, the lower middle one, is an unprojected direction.

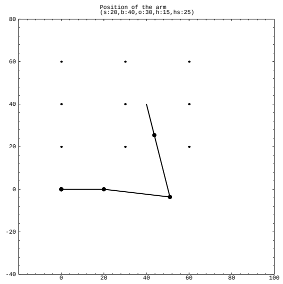
The first model is just saying that the deviations are the same as in visual space and in the second one assumes that the correction of the brain is not completely, say imperfectly. Both these models will be given closer attention in the following subsections.

2.2.1 A visual model

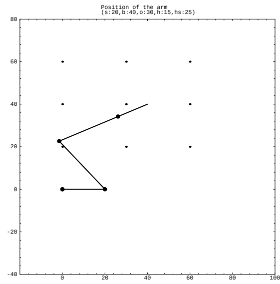
We could compare the haptic space with the visual and predict that the haptic space is similar to visual space. The simplest we can do is look at how directions are deformed due to simple projection on the retina. The world around us, we see in fact projected on a sphere around us. Projected on a sphere the plane table surface looks a lot different than it is. As an example 'vertical' and 'horizontal' directions are projected on the sphere and the direction seen is given – on the unprojected position – in figure 2.1.

The mathematics to obtain these angles is rather disturbing, and hardly to the point, so will be omitted here. If you want to redo it, project 2 points of a line part on the plane to a sphere (simple) and calculate the angle they form on this sphere (one needs spherical trigonometry for this, the formulas can e.g. be found in [3]). The results of this are also encoded in the program of appendix A.

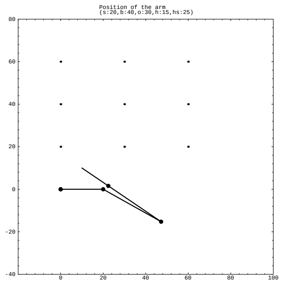
Unfortunately this model is not very good, because the direction of rods (now 'vertical' and 'horizontal' as the most left ones) has an influence on the essential properties of figure 2.1. If one for example 'subtracts' the two figures of 2.1 one doesn't obtain 9 identical angles. We do not have arguments why any to be projected angle could be preferred to another. We also did use one center of the sphere while in reality there are of course two (the eyes), and both will give little different results. Therefore we will choose not to use this model at all, but the one presented in the next section.



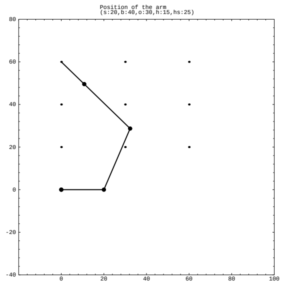
Hand on $(40, 40)$



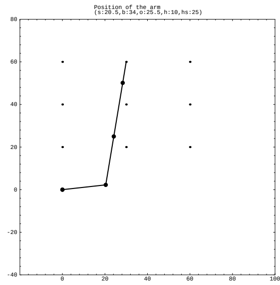
Wrong solution



Another position



And again one



With a turned shoulder

Figure 2.2: Several positions of the model-arm, projected to the table surface.

2.2.2 A model of the arm

In this model the arm is represented by 4 line-parts which are flexibly joined together. These four line-parts are respectively named 'shoulder', 'upper-arm', 'lower-arm' and 'hand'. One now imagines the shoulder to have on one side, the neck, a fixed position in space and puts the 'hand' on the table. Now we want to calculate the direction of the hand as a function of its position. We need however more assumptions than we have right now, because now the problem is indeterminate. One can try a very realistic model in which one takes into account the limitations of the three joints (they are of course certainly not free, e.g. the elbow can only bent in one direction), one could add muscles and heaviness and some general tactics of humans. Such a model, however, will not be presented here. Our goal will be reached by simplification. First of all, we stiffen the wrist and fix the position of the shoulder-joint. Then we limit the position of the elbow to the two dimensional surface of the table. This will cause the complete model to be effectively two dimensional. For a possible position of the hand, now only two directions are left, of which one can be excluded, because in that state the elbow is wrongly turned. The thus described model was implemented in MATHEMATICA [15] and a few drawings of this model-arm are given in figure 2.2.

It is possible that this model cannot reach a certain point in plane whereas a real arm could. This can happen if the point is too far away or too close. In these cases the model

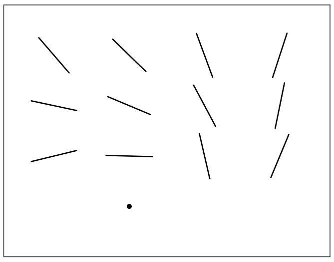


Figure 2.3: Direction of the hand on 12 different positions, according to the presented model. Used was a standard arm (similar to that of the author).

was changed as follows. If the point to be reached is too far, firstly it is tried if it is possible to reach it by fully stretching the elbow and allowing to turn the shoulder. In this case the model doesn't essentially change, just the length of the 2 line-parts and the origin change and the shoulder-joint will play the role of the elbow. Such a case is also presented in figure 2.2.

If then the arm still is too short the upper-arm will be made longer, such that the point gets into reach [1]. One can imagine that the elbow will be lifted in such a case. Something similar will be done if the point is too close to reach (closer than the difference between upper and lower arm).

The mentioned MATHEMATICA program thus includes a function which gives the direction of the hand as a function of position in plane (it is called 'vindhoeken'). This function effectively finds $\{x_1, y_1\}$ out of the following equations:

$$\begin{cases} x_1^2 + y_1^2 & = & a^2 \\ (x - x_1)^2 + (y - y_1)^2 & = & b^2 \end{cases} \tag{2.1}$$

and decides which of the 2 solutions is the correct one (the one with the greatest x_1 value). In these formulas (x, y) is the position of the hand relative to the shoulder and (x_1, y_1) the position of the elbow relative to this same point. a is the length of the upper-arm projected to the plane of the table and b is the length of lower-arm and the hand together. From x_1 and y_1 one then easily, with an arctan-operation, finds the needed angles. If (x, y) is unreachable (too far or too close) a, b, x_1 and y_1 will change their meaning as described before.

The obtained solutions, which can be found analytically, are quite complicated and will not be presented here (refer to the program itself, given in appendix A), but a result, the direction of the hand on 12 different positions, is given in figure 2.3.

We see that this result is quite different from the results from the 'visual model' of figure 2.1, but it is less different than it looks like at first sight. Applying a simple manupulation (multiplying with a (negative) factor and addition of certain number) to the numbers of figure 2.3 one can obtain a figure which looks a lot like figure 2.1.

[1]The mathematica program doesn't display this visually, it uses this only in the calculation

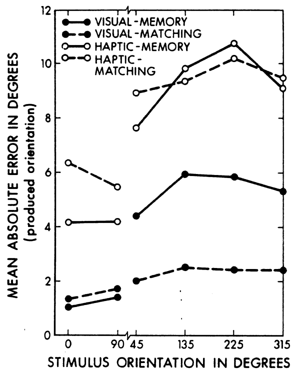


Figure 2.4: Measurements done in the past after the oblique effect. Taken from [10]. We see mean absolute error in produced orientations shown as a function of stimulus orientation for several types of measurement.

2.3 The oblique effect

One of the few already studied phenomena in the study ([2, 9, 10]) of haptic space is the so called *oblique effect*. This effect was first observed in tests in which subjects had to set parallel two rods visually. They could do that more precisely if the orientation of the stimulus rod was vertical or horizontal. They had more difficulties matching an oblique rod. It was shown that this effect did also show if the subject had to do the task haptically. A result of this is shown in figure 2.4, taken from Lechelt [10]. We see in this figure the mean absolute error made in several types of experiments. Assuming that no systematic errors occur, the mean absolute error is a measure for accuracy. Indeed in the non-oblique direction we see smaller values for this number in figure 2.4. In this figure there also appear measurements in which de subjects did have to place rods by memory, so the reference-rod was removed before they could start setting the test-rod. In the experiments described in this work, this was not done. The measurements were done in another plane than the measurements of this work. Because we will not assume not to have a systematic error, we will test for accuracy by taking the mean of standard-deviations.

With the results of the experiments described in the next chapter we will be able to check this.

It was also demonstrated that an oblique effect in the average itself (when averaged results depend on the direction of the stimulus) was not present, which we will also be able to check.

Chapter 3

Experiments

3.1 Setup

The setup exists of a table surface positioned 76.5 cm above the floor, which has several holes in it, in which a rod or a marker can be set. The subjects were seated on a 46 cm high chair. The table surface is the surface of the 2 dimensional coordinate system used in this work. The subject is at the point (0,0).

A drawing of a rod and a marker such as used is given in figure 3.1. A rod is a 19.7 cm

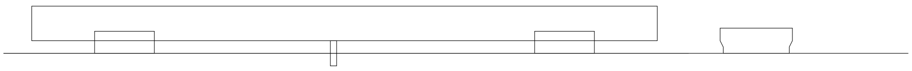


Figure 3.1: Schematic structure of the used rods and marker.

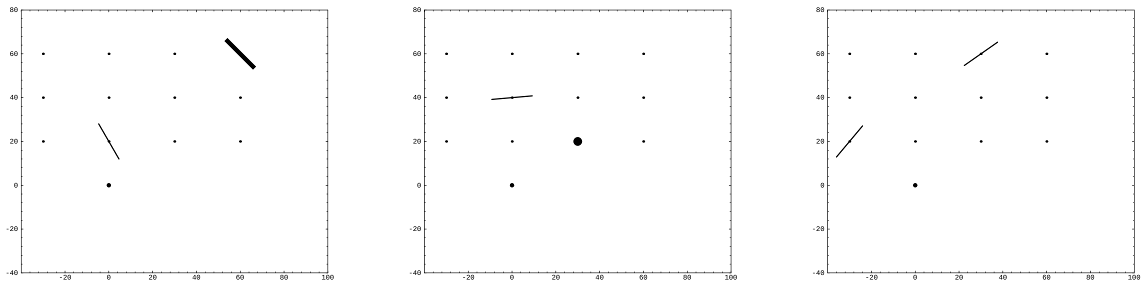
long cylindrical aluminum tube, with a diameter of 1.1 cm. It can rotate around an axis, because a small pin in the middle is in a hole in the table surface. In the tube also are fixed some 'distancers' who keep it 0.4 cm from the table, and stabilizes it. A rod also can have 4 of these with magnets in them so that the rod becomes unturnable and serves as the 'reference'. A rod serves to be a direction on a certain point, which the subject can feel, and eventually set.

A marker is also drawn in figure 3.1. It is round, plastic, has a diameter of 2.3 cm and a thickness of 0.75cm. It also has a pin in its center so it can be positioned fixed on the table surface.

3.2 Description of the experiments

Three kind of measurements were done:

1. *Putting parallel.* In this experiment two rods on two different positions were offered. One of them had a fixed direction and the subject was asked to set the other in the



Putting parallel — Pointing to a marker — Setting in-line

Figure 3.2: The setups. We see three times the plane in which the table-leaf lays. The equation $y = 0$ defines the edge of this table. The small dot at (0,0) is the position of the subject. The even smaller dots are the possible positions of a rod or a marker, and are in reality holes in the table surface. Further we recognize turnable rods (thin line), and an unturnable one (fat line). There is also a marker, indicated with a fat dot. The numbers at the axes are centimeters.

same direction, using only his[1] right hand.

2. *Pointing to a marker.* A marker and a rod were offered on two different positions, and the subject was asked to direct the rod to the marker.

3. *Setting in-line.* Two rods on different positions were offered and both directions were for the subject to set with their right hand. The task was to make them both directed along one line.

Of all these 3 kinds of measurements an example is given in figure 3.2. The locations used are on a two dimensional rectangular lattice with distances of 20 and 30 centimeters. Every measured combination of position of reference and test rod/marker was offered three times and everything in random order.

In the 'marker' and 'in-line' measurements the direction to be set is defined by the location of the marker or the other bar. In the 'parallel' measurement it was defined by the direction of the reference bar which took the values 0, 45, 90 and 135 degrees.

So if n is the number of used positions of the used lattice (usually 9) the number of combinations per task is 3 (repetitions) * 4 (directions) * n (positions of reference) * (n-1) (positions of test) for the parallel measurement; 3 (repetitions) * n (positions of marker) * (n-1) (positions of bar) for the marker task and 3 (repetitions) * (n-1 + n-2 + ... + 1) (combinations of 2 bars on different positions) for the in-line task. For $n = 9$ these numbers are 864, 216 and 108, and this is a good point to thank Astrid Kappers and the subjects for doing this voluminous work.

[1] I use male pronouns when gender is irrelevant.

3.3 Subjects

Three subject were used and of them only their right arm. Some relevant sizes (for the arm model) are given, along with some basic information on the subject, in the following table (age in years, sizes in cm):

sub.	gender	age	upper	lower	shoulder	height	hand
ME	female	22	37	27	26	35	15
NK	female	21	34	25.5	20.5	31	15
RR	male	32	36	27	22	32	12

In appendix B you can see on which positions was measured for every subject. Measurements were always 3 times repeated and offered in random order. Subject NK also did some tasks with her left arm instead of her right.

3.3.1 Tactics of the subjects

The subjects were asked to describe their tactics. In this section their responses will be schematically described. They are presented in the following tabular:

sub.	putting parallel	pointing to marker	setting in-line
ME	First globally set and afterwards with small steps correcting this initial setting. Tried to keep the hand in the same direction.	'Shooting' with the hand to the marker, and continuously improves the setting according to the largeness of missing.	Sometimes imagined a line through the two middle-points. Adjusted in little steps.
NK	Trying to move the hand, while keeping its direction fixed. The path of the hand could be in the beginning along the direction of the fixed rod (not directly to the test-rod).	With spreaded fingers following the rod, aiming at the marker, and 'shooting' the hand, trying to catch it. Hand could be side-wards or along the rod.	First perpendicular, and seek the middle of the one with the other. Or beginning with pointing to each other directly, and optimizing by continuous controlling.
RR	Similar to ME. Imagined parallel lines.	Correcting is done by aiming and shooting the hand.	Each bar after the other.

Chapter 4

Results and their description.

In this chapter first the global results of the three kinds of measurements will be presented. There were three subjects and the results of one will be chosen in the figures, and the others will be mentioned only in words and numbers. The untouched results of every subject one can find in appendix B.

One subject (NK) also did the tasks with her left hand. These results were more or less the same, though possibly somewhat more uncertain. They will be presented together with the other results (all done with right hands). Note also that this subject did the tasks twice with her right hand, one time on other points of the table surface. Take a look at the appendix to get a clear idea of how the used results of the measurements look like.

4.1 Parallel-putting task

A typical result of the parallel-putting task is presented in figure 4.1. In this figure we see 36 squares in which in every one a different position and direction of the reference bar is shown. The other line sections represented the directions the subject has set. These are the results of the subject ME, the others are given in appendix B.1.

The first step taken is to calculate the error the subjects had made in their settings. These errors may be dependent of the following:

1. The position of the reference bar.
2. The position of the test bar.
3. The direction of the reference bar.

and of combined properties, such as the distance between the two bars. It is undoubtable that the position of the test-bar is a relevant factor, as one could already easily notice from figure 4.1, for the pictures do change if one compares pictures from the same column (e.g. the upper right one and the lower right one).

The influence of the properties of the reference bar (direction as well as position) on the error the subjects make is however less clear. Eventually it will be assumed that we can parME.dat

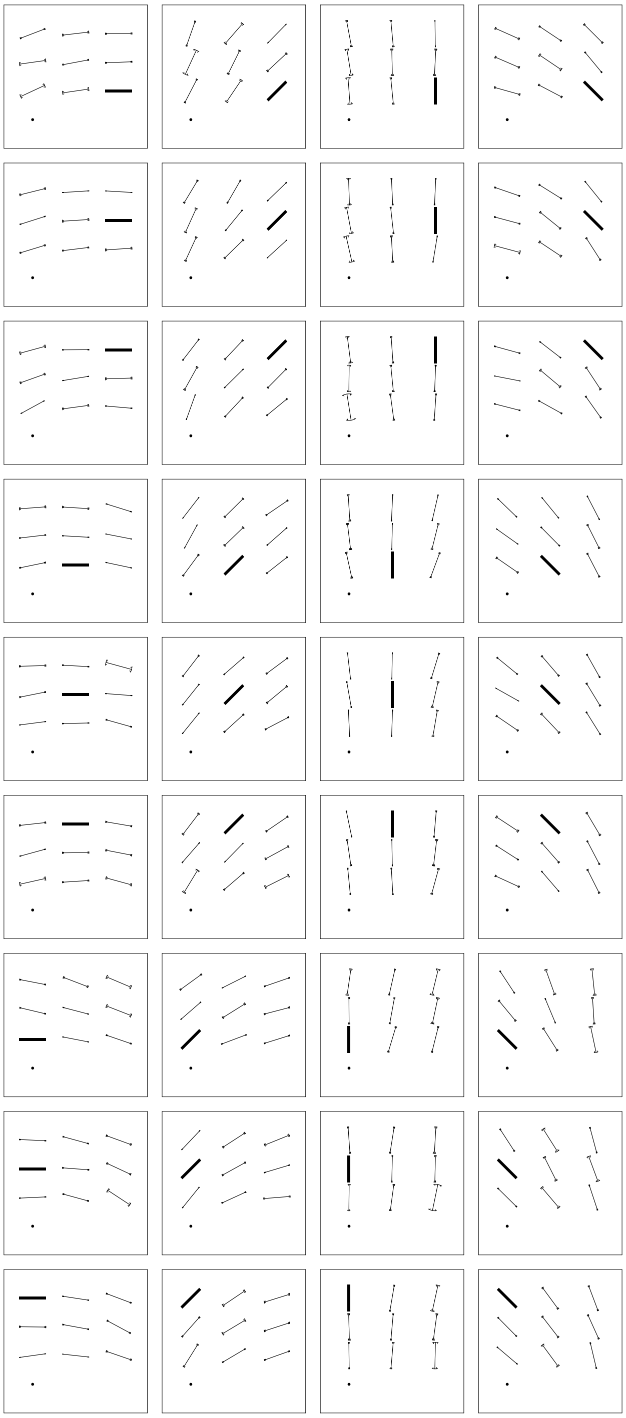


Figure 4.1: A typical measurement of a parallel-putting task. Every square defines the table-leaf surface, and the bar the reference direction. The other eight line-sections represent the averaged direction of the test bar which the subject set parallel to the reference bar. The average was taken over 3 measurements, so in every square 24 measurements are represented. The dot indicates the position of the subject.

define the error in the measurements such that the properties of reference bar have no influence, but first we will look how good or bad this is, and how it is done.

First of all we compare in figure 4.1 the errors horizontally, so we study the influence of the direction of the reference-bar. We therefore for every reference position 8 times (every test position), have to decide if 4 sets (every possible reference direction) of 3 numbers (measurements were repeated 3 times) are similar or not. In fact we only want to know if the mean is the same. A decision technique for such a job is a so called F-test (see [14] chapter 12.2.1 and [8]). The 'test' part of an F-test is to compare the F-value of the set of sets to a certain limit value. The F-value is proportional to an equation in which appear relevant variances:

$$F = \frac{I(J-1)}{I-1} \frac{\text{Variance of the means}}{\text{Mean of the variances}} \tag{4.1}$$

in which I is the number of sets to compare (in this case 4) and J the size of these sets (in this case 3).

For every subject there are 8*9=72 of such F-tests and in the following tabular is presented how often the F-test failed.

Subject	times F-test failed	times F-test succeeded	total
ME	33	39	72
NK	14	58	72
NKR	6	14	20
NKL (left hand)	11	9	20
RR	23	49	72
total	86	170	256

A tolerance level of 10% was used. Therefore approximately 10% of the measurement may fail the test. Many more do, so the direction of the reference-bar does have a certain influence. However not all failed, and certainly not consistently, so we conclude that the influence is small.

Provisionally it is assumed that the direction of the reference is unimportant and in that case the data can be summarized as in figure 4.2. In this figure we see only nine times the table surface with star-like things appearing. These indicate the position of the reference-bar, and are the 4 directions drawn across each other. The other 8 lines then give the averaged mispointings on their respective positions.

This is a good time to say something more about the used representation of averaged angles in these kind of figures. Of course, the whole thing was inspired by an actual little drawing of the 'rods', and their length is scaled correctly indeed. In a figure like figure 4.2, the line-parts do not any more represent any real situation though, but only an averaged number, represented as an angle (if the number is small, multiplied by a factor, of which the value always can be found above the figure). On the endings of this line-parts 2 standard-deviations in the represented angle are indicated (in this figure too small to seen, but you could quickly browse through this report and find more clear examples). The greater one is the standard deviation of the set the mean was taken from. This is only given to check if the averaging does make any sense. It is a property of the imaginary set the measurements are taken from. If the numbers certainly can be averaged, only the other, the standard deviation in the mean is important. They only differ a factor $\sqrt{n-1}$ (n: size of set of measurements). In figure 4.1 for example the averaging certainly

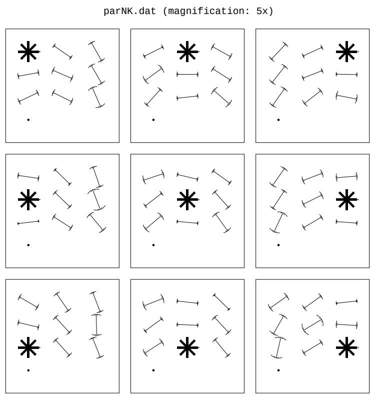


Figure 4.2: In this picture the dependence of the direction of the reference was removed, by calculating the mean error. The position of the reference is indicated with the bars in all the four possible directions, forming a star.

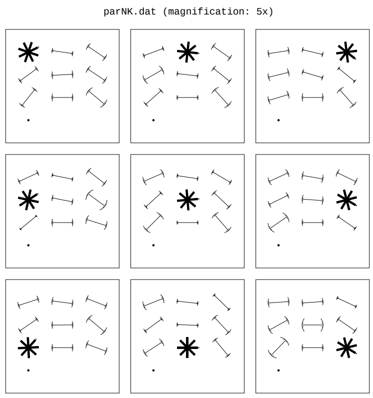


Figure 4.3: The elements are turned such that the one in the lower middle position is horizontal. Now also the need for the little point on one of the 4 bars (look precisely!) in the 'star' becomes clear. It indicates how they were turned.

is justified, and we are really only interested in the standard deviation in the mean.

We proceed now with the real work. We also would like to compare the nine figures of figure 4.2 to each other, because it is clear that they share some properties. In case the subject would not make any error the picture of figure 4.2 would show only horizontal line-sections. The errors are relative to the references. To make the 9 pictures comparable therefore to every one an angle is assigned which turns the lower middle direction horizontal. And each element of this picture is turned with this angle. If the lower middle one is the reference position then in this picture nothing happens, because we assume that the subject could put a test-bar on this position whithout an error. Then pictures as in figure 4.3 arise. Compare it to figure 4.2 if it is not yet clear what happened.

Assuming that these nine figures are comparable, we make the last step and put them into

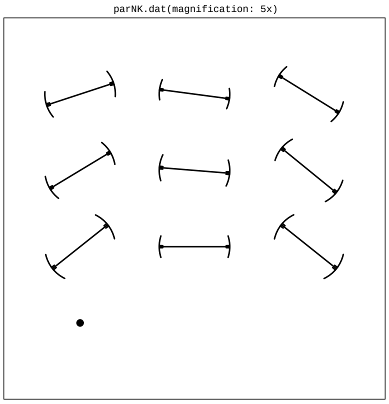


Figure 4.4: Total average of parallel putting task, subject NK

one. We again can use F-tests to say something about wether or not we are allowed to do this. To see if the position is a relevant factor:

Subject	times F-test failed	times F-test succeeded	total
ME	1	7	8
NK	4	4	8
NKR	3	1	4
NKL (left arm)	4	0	4
RR	7	1	8
total	19	13	32

It should be noted that these numbers only have indicating value, because these results depend on the 'normalisation center', the position on which the direction was turned to zero. It is however undoubtable that it is not fully justified doing what we are about doing.

But it is sensible thing to do, because the 9 pictures of figure 4.3 certainly look alike. We therefore do the avaraging, and obtain figure 4.4. The greater of the 2 standard-deviations now has a real value, because it indicates how the numbers are placed around the average they form.

Finally we can compare this figure to the described arm model. Or to another for that matter, but in this work we chose for this one. This can be done in several manners. We want to 'subtract' the results of the model arm (see figure 2.3) from the measured results (as in figure 4.4) such that the structures disappear. We can turn the nine directions on the nine diferent positions in the model such that it is zero on the same position as in the figure 4.4. Then on every position we could find the factor to which the model-angle should be multiplied such that it equals the result angle on that position. The average of these factors can be the taken to multiply the model-angels and the difference of that and the measured result is shown in figure 4.5 (the factor found was 0.15).

This is not a very advanced method and it could be done better. It then becomes however somewhat more complex: We do not turn the model first (however this would do no harm) and we now begin subtracting the model-angles times a factor (arbitrary chosen, for e.g.

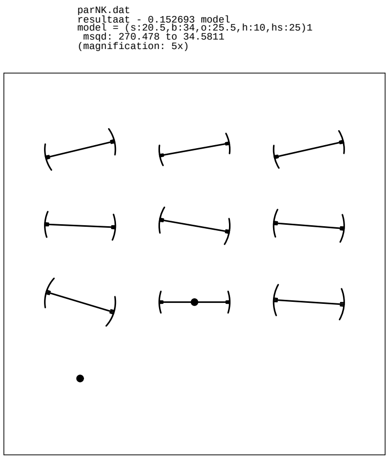


Figure 4.5: The average - 0.15 times model-angles of subject NK in the parallel puting task

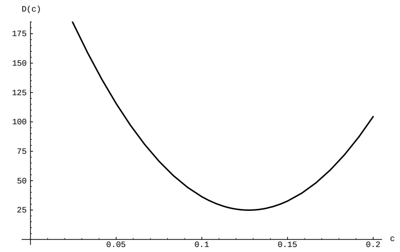


Figure 4.6: The function $D(c)$, which is of course a quadratic function in c.

the result of the first method) from the 9 angles found. We calculate the sum of the squared difference of the mean obtained angle. This sum we minimize with the factor as variable. Afterwards we can, for example, make the average of the nine directions zero or, perhaps more logical, just the lower middle one. Shortly, if we call the resulting angles a_i, b_i the angles of the model and c the factor, we have to minimize the following function:

$$D(c) = \sum_{i=1}^{9} \left(\frac{1}{9} \sum_{j=1}^{9} (a_j - cb_j) - (a_i - cb_i) \right)^2 \tag{4.2}$$

A typical $D(c)$ is shown in figure 4.6. And a result of this procedure is given in figure 4.7. This method is more reliable because it doesn't depend on the chosen 'zero' position and it gives better results. [1] It is even the *best* result if one wants to subtract the model-angles times a factor. There is however no obligation to use this method. It may be possible that other methods of using the model-angles to 'correct' the results of the subject are

[1]We prove that the 'zero' position has in this method no influence by adding a arbitrary turn angle t

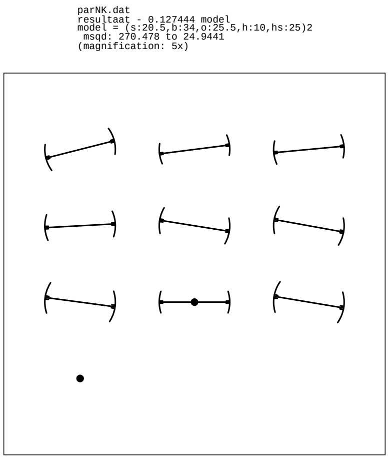


Figure 4.7: The result - 0.13 times model 2 of the subject NK in the parallel putting task.

imaginable. Perhaps a model as the one from section 2.2.1 is usable. With this model we would find negative factors though (which is all right, but the model isn't yet).

We also could have applied the arm model to figure 4.2 (or 4.3, it doesn't matter), because the last averaging step is statistically not justified. We obtain then 9 times a factor for each subject in this task. These factors are graphically shown in figure 4.8. In this picture we do not see any reason why this should be better (nor obvious consistent spatial structure), but we should compare to the other measurements. In every case, it gives an idea about the uncertainty in the factors found.

The results of the correcting with the arm-model the errors the subjects made will be fully presented, also for the other subjects, after the other kinds of measurements were described.

4.2 Pointing to a marker

In the 'pointing to a marker' task a considerably smaller amount of data is obtained, because the 'reference' in this case cannot have a direction, only a position. A typical measurement is summarized in figure 4.9 (again 8 measurements in every square). And

---

to every a_i:

$$D(c,t) = \sum_{i=1}^{9} \left(\frac{1}{9} \sum_{j=1}^{9} (a_j + t - cb_j) - (a_i + t - cb_i) \right)^2 \tag{4.3}$$

And we see that it will disappear immediately:

$$\begin{aligned} D(c,t) &= \sum_{i=1}^{9} \left(\frac{1}{9} \left(9t + \sum_{j=1}^{9} (a_j - cb_j) \right) - t - (a_i - cb_i) \right)^2 \\ &= \sum_{i=1}^{9} \left(\frac{1}{9} \sum_{j=1}^{9} (a_j - cb_j) - (a_i - cb_i) \right)^2 \end{aligned} \tag{4.4}$$

and therefore also the minimum doesn't depend on t.

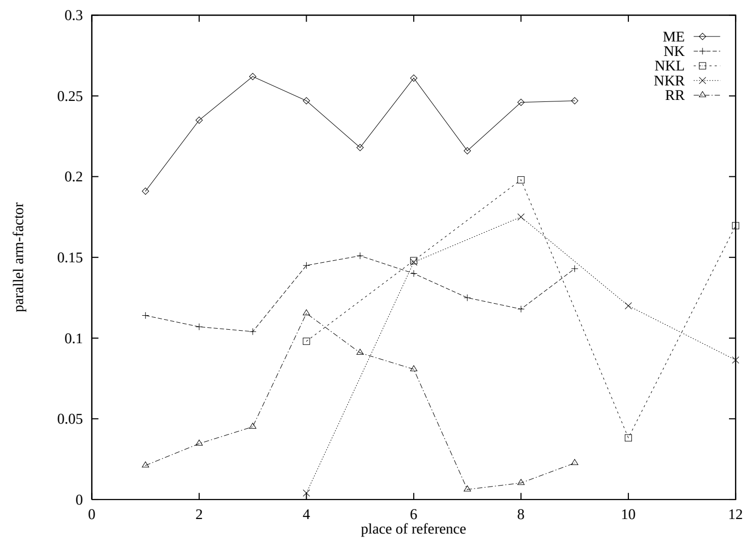


Figure 4.8: The factors on the different positions for the three subjects. The positions are numbered, the lower right one is 1, the upper left one 12, counting from down to up, from right to left.

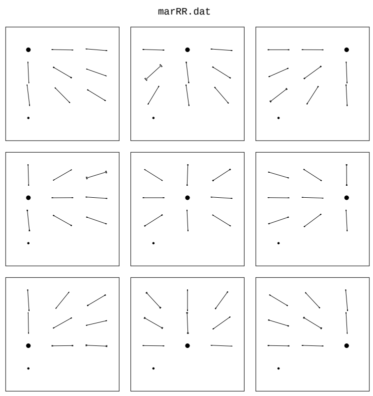


Figure 4.9: A typical measurent of the pointing-to-a-marker task (subject RR).

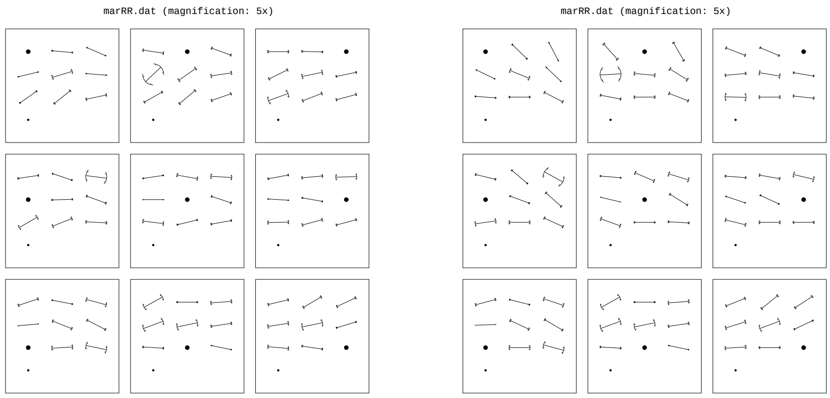
we again can picture the 'error' which was made in figure 4.9 which is shown in figure 4.10A, together with a version in which the angles are turned such that the lower middle is horizontal (4.10B).

The errors in this case may be dependent of:

1. The position of the marker.

2. The position of the test bar.

, or, of course, of some combined effects, such as distance.



A B

Figure 4.10: The errors made in this pointing-to-a-marker task (left). In the right picture every angle is turned back such that the lower middle one is horizontal

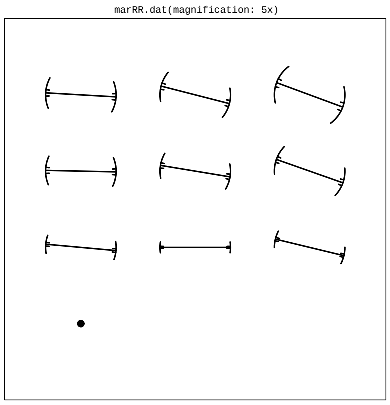


Figure 4.11: Averaged of pointing to marker task of the subject RR

We again can compare the nine figures of figure 4.10B to each other. And we now, by analogy with what we have done in the previous section, assume that the position of the marker is not a relevant factor. We can average, and obtain the picture of figure 4.11. Of course we also test it with an F-test:

Subject	times F-test failed	times F-test succeeded	total
ME	8	0	8
NK	3	5	8
NKL	3	1	4
NKR	3	1	4
RR	5	3	8
total	22	10	32

and see that is again worrying what we're doing, but we don't panic because the figure 4.11 does still show a structure. And this subject didn't have the most beautiful results.

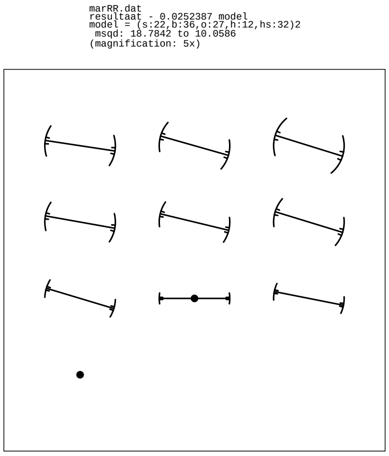


Figure 4.12: Averaged - 0.025 model. This is the subject RR.

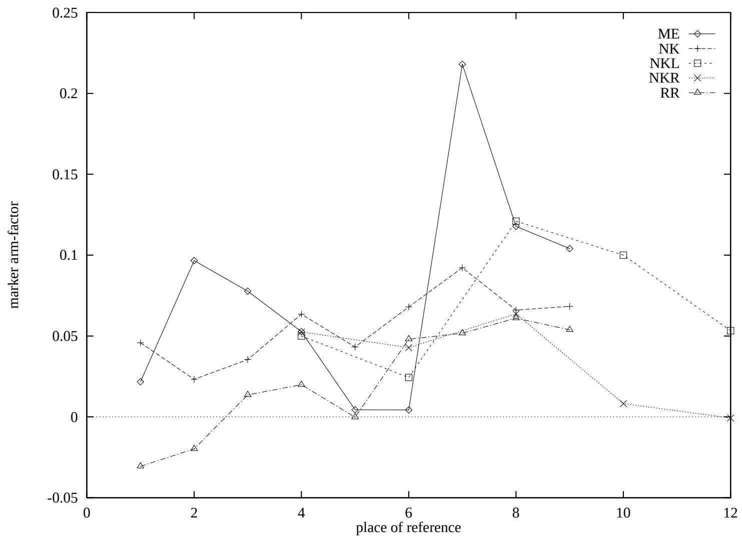


Figure 4.13: Factors of the marker task. Again the position are numbered as in figure 4.8.

And one can again try to compensate with the model, in the same manner as was done before, of which an example is shown in figure 4.12.

This, of course, was also already possible with 4.10A (or B) (9 times) and the factors of these are presented in figure 4.13. The result is quite chaotic, which can be an indication that the input was also chaotic. But at least the factors are nearly always positive, so some properties exist. Later we will compare among the different kinds of measurements.

4.3 Putting in-line

A third task which was done by the subjects was the putting in-line of two bars. Again every combination of two positions was presented to the subject (in random order). A

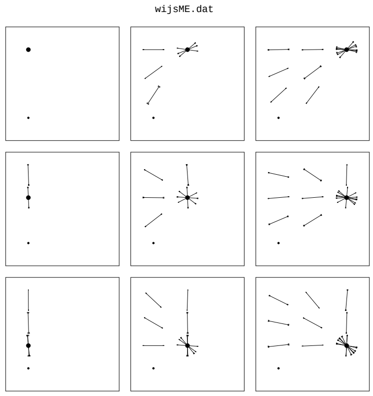


Figure 4.14: A typical measurement of a the putting in-line task. Subject ME. In every of the nine pictures there's one position with a dot, which indicates one of the two bars. On one of the other eight positions is the other bar. On the dotted postions could be 8 bars at the same time. So per picture could be presented 8 measurements. Every measurement is present only once, so some squares are quite sparse, because the measurement belonging to an empty postion is already shown in another square.

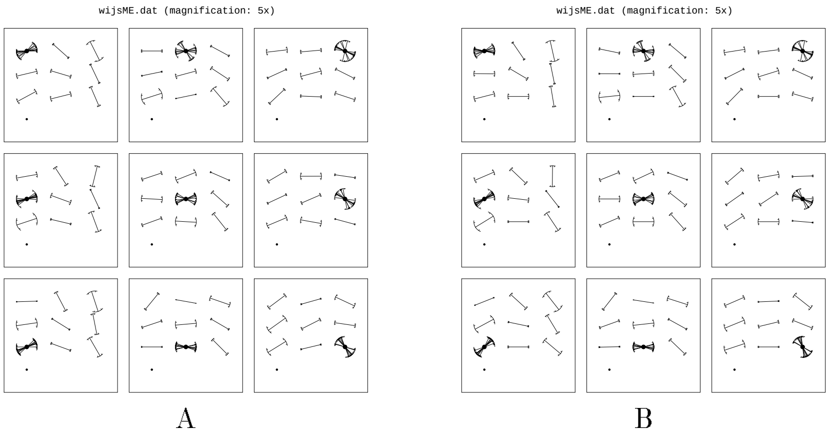


Figure 4.15: Errors made in a setting in-line task. These figures are fuller then those of figure 4.14, because every measurement now is shown 2 times. One time for both bars.

typical result is shown in figure 4.14. We see that the figures are quite sparse and this is of course because in this case two equivalent objects are compared, while in the other two kinds of measurents it always were two different ones (a 'test' and an 'reference') and interchanging their position was relevant.

One could see this measurement as two pointings to a marker at the same time. The role of the marker is taken by the other rod. In every measurement therefore also two times the subject could make an error (a mispointing). These errors are presented in figure 4.15 together, again, with a version of this picture in which the results are presented such that the nine figures have on the same position a bar with athe horizontal direction. The error

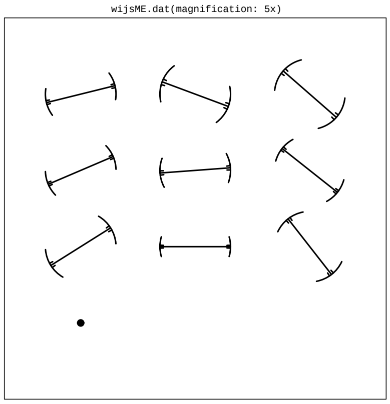


Figure 4.16: Total average of setting in-line task, subject ME

in the setting of the direction of a bar can depend on:

1. The position of that bar.

2. The position of the other (defining the direction).

, and combinations. The nine figures can again be averaged, assuming that the position of the bar is irrelevant in the right figure of 4.15, which results in figure 4.16.

We could again test whether or not this is allowed to do this with an F-test:

Subject	times F-test failed	times F-test succeeded	total
ME	8	0	8
NK	5	3	8
NKL	4	0	4
NKR	3	1	4
RR	7	1	8
total	27	5	32

and according to this we were too careless. We have to keep this in mind, but looking at figure 4.15 and 4.16 we see that the result is not bad. The bad results in the F-test for this subject can for example be caused by the middle upper bars in figure 4.15B, which seems to be behaving quite chaotically (compare them with the neighbours).

Figure 4.16 we can again treat with the arm model and then figure 4.17 appears.

We can do the same as before, applying the arm-model to the step before (figure 4.15), and obtain figure 4.18. In this case, the factors are increasing with the number of the locations. Especially the factors of ME and RR seem to be clearly bigger if front of the subject (positions 7, 8 and 9).

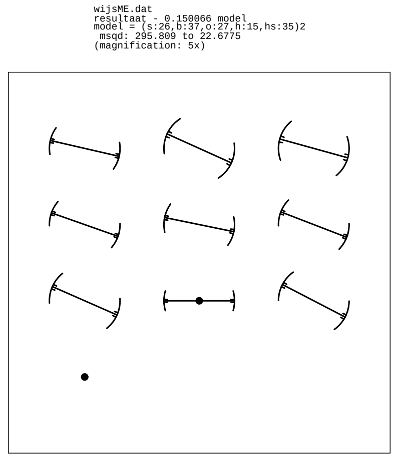


Figure 4.17: Averaged results - 0.15 model. Subject ME.

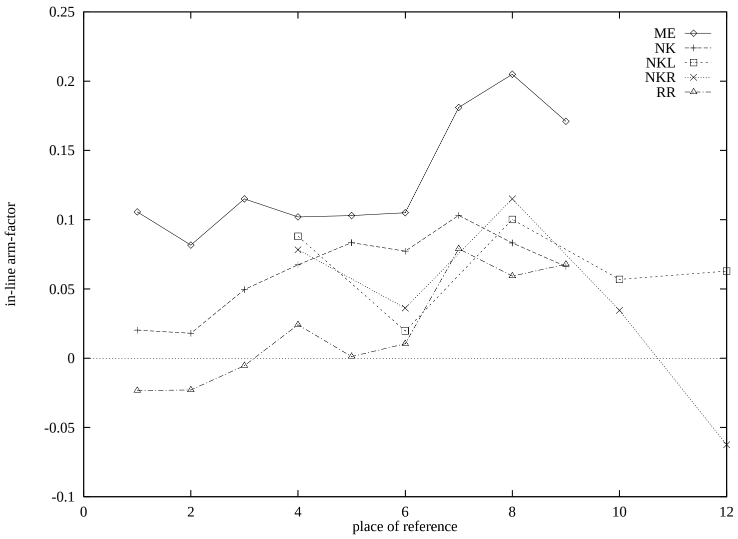


Figure 4.18: Factors of the marker task. Again the positions are numbered as in figure 4.8.

4.4 Summary of results.

All results of the corrrecting the 'averaged ' errors with the arm model are presented in the following table:

subject	factor	before	after	before/after
ME par	0.24	704	31.6	22.3
ME mar	0.084	140	54.5	2.6
ME line	0.15	296	22.7	13
NK par	0.13	270	24.9	11
NKR par	0.14	122	9.91	12
NK mar	0.058	61.9	11.2	5.5
NKR mar	0.072	33.8	5.05	6.7
NK line	0.087	144	28.7	5.0
NKR line	0.096	58.8	7.82	7.5
RR par	0.041	36.2	13.6	2.7
RR mar	0.025	18.8	10.1	1.9
RR line	0.032	37.5	23.2	1.6
NKL par	0.13	108	8.67	12
NKL mar	0.036	13.4	6.35	2.1
NKL line	0.080	65.2	29.4	2.2

The first column gives the subject and the type of measurement. It is a little bit encoded, but easy to understand. In the second column are all the factors ('arm-factors') as found by comparing to the arm-model, described in the previous sections. The other three columns indicate how much the arm model 'helped'. 'Before' and 'after' are the values of $D(0)$ and $D(\text{factor})$, so are dimensionless (squared degrees, if you want).

What have we in fact done, in the previous sections? The described methods for reducing the results to just one picture of the table surface basically only could be done if we assume that the errors can be predicted in the following way.

To every point of the table surface a certain number is assigned by a certain function $\phi(x, y)$. This function was assumed to be the previously described direction of the hand times a factor, but could also be something else (about this later will be said something). Then it was assumed that the error ($M(p_1, p_2)$) in the setting of a direction on a certain point $p_1 = (x_1, y_1)$ relative to another position $p_2(x_2, y_2)$ depends only on ϕ:

$$M(p_1, p_2) = \phi(p_1) - \phi(p_2) \tag{4.5}$$

Which has the obvious properties:

$$M(p_1, p_3) = M(p_1, p_2) - M(p_2, p_3) \tag{4.6}$$

,

$$M(p_3, p_3) = 0 \tag{4.7}$$

and

$$M(p_1, p_2) = -M(p_2, p1) \tag{4.8}$$

Therefore every result ($M(p_1, p_2)$) can be transformed (equation 4.6) such that it is relative to the same position (p_3, the lower middle one). For this, one needs $M(p_2, p_3)$ which was

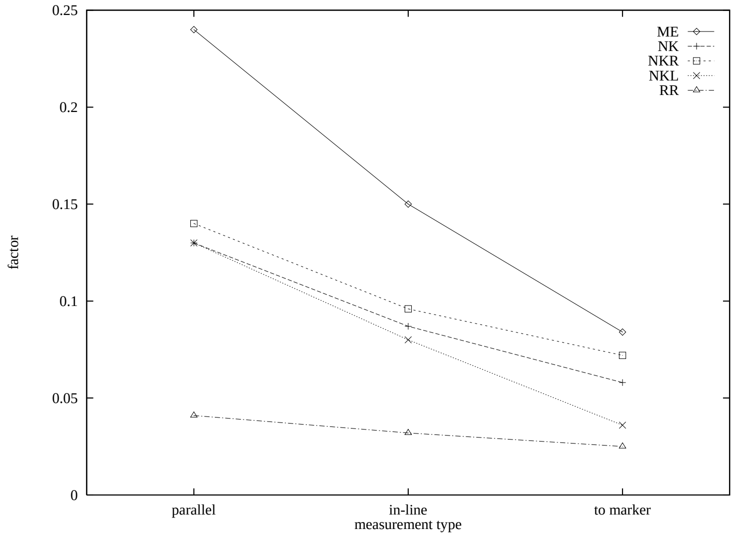


Figure 4.19: The arm-factors found of the tree subjects ME, NK and RR. NKL refers to the left-handed measurement of NK and NKR was a simple right-handed measurement, however in another part of the table-area.

estimated using the measurement of it. This was the 'turning' back of the nine angles such that the lower middle one is horizontal as we did continuously.

We now notice that in the parallel measurements the subjects did not only have to set a direction, but also perceive one. It is expected that errors will be greater by a factor 2. In the pointing-to-a-marker task one only has to set a direction. The setting-in-line case is special, because, in fact it is just two times pointing to a marker, but one must realize that the two bars also have to be parallel when in-line. It would therefore not be strange that these results are a mixed form of the two other types. The numbers in the table are in a wonderful agreement with these remarks. The factor of the parallel task is indeed always the largest from the three, the factor from the pointing-to-a-marker task is the smallest, and the putting-in-line-factor always lies in between. To illustrate this for every subject the three factors are presented also in figure 4.19.

With this figure we see very clearly that the arm-factor depends on the type of measurement. We also see that the three lines belonging to the subject NK are very comparable, and therefore we conclude that hand nor area has an influence.

In words, we can describe the function ϕ, which is scaled by these 'arm-factors', to be the deformation of the 'angle space'. Apparently, finding a direction out of two positions is not influenced, or at least less, because the marker measurements give rise to the smallest factors.

We also presented the factors as a function of the place of the reference. These results are again presented, but now ordered per subject in figure 4.20. We compare the three 'curves' every time to each other. If they differ only in magnitude then we could conclude that the factor is depending on place, or in other words, that the model could be chosen better. Indeed, per subject, we see that the three curves often share properties. In the

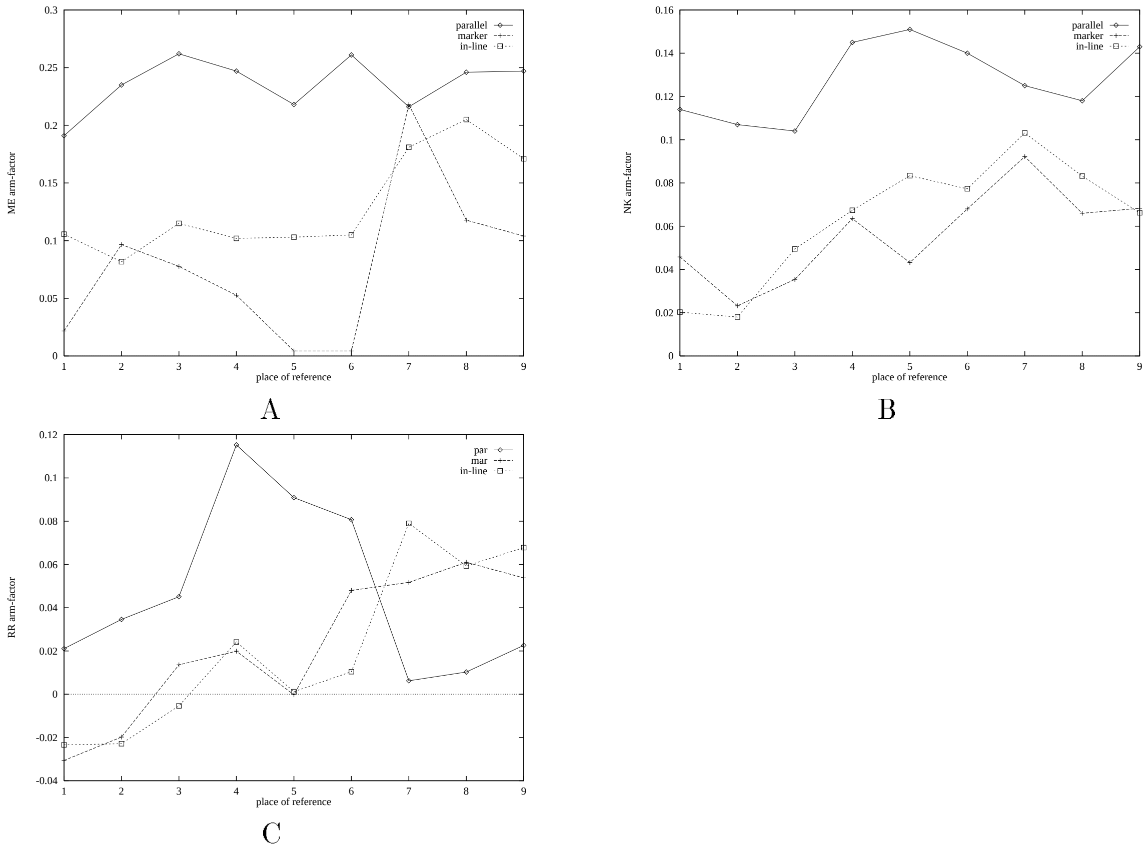


Figure 4.20: The arm-factors as function of reference positions. Positions numbered as before. A: subject ME; B: subject NK; C: subject RR. The results of NKR and NKL are not presented in B, because the picture becomes too messy then.

right positions we find for example that the factors for RR are generally smaller. Between subjects we don't see striking similarities, so possibly the model cannot be fully generalized. Of course we didn't use already the same model for each subject, because it is depending on the sizes of the arm. May be it is possible to define a model which can be modified in a similar way which is better valid generally.

4.5 The oblique effect

Testing for an oblique effect could only be done in the parallel measurements and it was done by calculating the average of the standard deviation on the collected measurements on the 9 different positions. The averages are given in the following table.

subject	oblique orientations	0 and 90 degrees orientations
ME	6.51	7.33
NK + NKR	4.05	3.99
NK (left arm)	4.59	4.34
RR	7.24	5.56

From this simple analysis one can not simply derive that there is an oblique effect. There is however a small effect in the subject NK and a larger in the subject RR. It is however not at all clear that this is significant. The subject ME even gives an anti-oblique effect. A few 'Lechelt'-like pictures are given in figure 4.21. The same numbers as in the upper table appear only split up in the 4 angles now. Also the standard-deviations in the mean of the standarddeviations (so 'degrees' is the unit) appear which led to the conclusion that the results are not significant.

It was assumed that in this kind of measurements the direction of the reference-bar has no influence on the error made in the test-bar. It was however remarked that this assumption is not fully justified. We ask ourselves if an oblique-effect may be the cause. In figure 4.22 we see 6 pictures derived from the 'parallel' measurements on subject NK. The exact description of what is to be seen can be found in the caption. There is no evidence that the F-values become better if we distinguish between oblique and non-oblique directions. This is true generally, so we conclude that there is no 'oblique' effect in this sense.

We also could have done the F-tests of page 12 twice, one time for oblique directions, and one for non-oblique. The results follow:

oblique:

Subject	times F-test failed	times F-test succeeded	total
ME	26	46	72
NK	9	63	72
NKR	5	15	20
NKL (left hand)	4	16	20
RR	18	54	72
total	62	194	256

non-oblique:

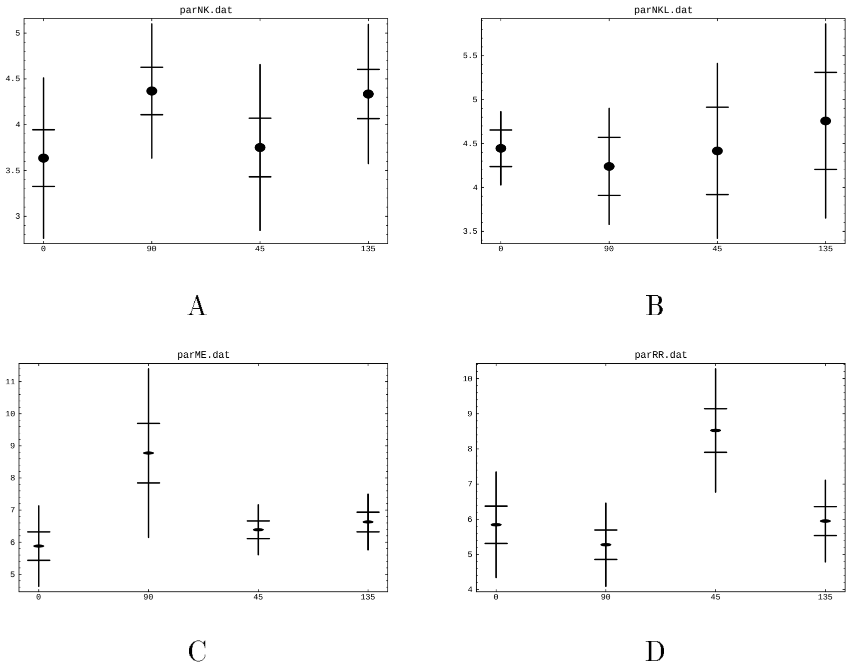


Figure 4.21: Lechelt pictures. Mean of the standard deviations in the settings of the 4 directions of the 3 subjects. A: subject NK, B: subject NK with her left hand, C: subject ME, D: subject RR. Following the habit, again the two standarddeviations are indicated (so standardeviation in standarddeviations).

Subject	times F-test failed	times F-test succeeded	total
ME	12	60	72
NK	8	64	72
NKR	2	18	20
NKL (left hand)	3	17	20
RR	15	57	72
total	40	216	256

and we see that the situation indeed becomes a little better, especially in the non-oblique direction, but still too many numbers fail. Because the gain is only small and together with the fact that the final results hardly change if we seperate ‘oblique’ from ‘non-oblique’ as in figure 4.22 we conclude that it is not useful compicating matters by doing so.

But we have in this an indication that there is an oblique effect in the measured angles itself. We also could test this oblique effect in the mean by making pictures as 2.4 but not average the standard-deviations but the average of absolute value of the error in the measurements. Surely, these values are not from the same set, but we trust that the average of smaller averages is smaller than the average of larger averages. The results are presented in the figures of 4.23.

We see indeed an oblique effect now, and consistently in every subject. Also presented however are the two values ‘in-line’ and ‘not in-line’. There the data was not divided with the direction of the reference bar, but looked was if the two bars were in-line or not. This is a logical step because we already know that the subjects performed better in the in-line

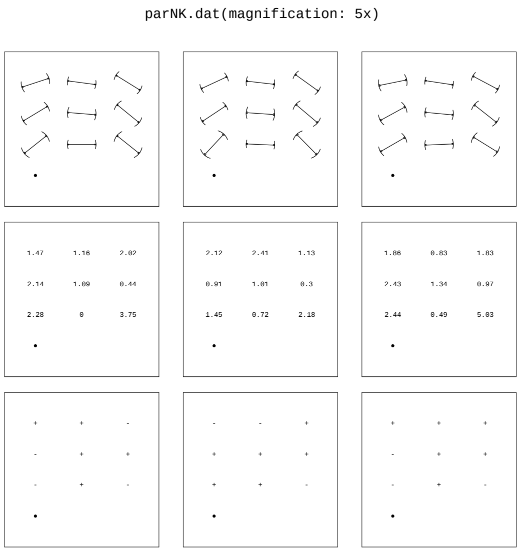


Figure 4.22: Testing for an oblique effect in the mean. The upper three pictures are like figure 4.4. The left is exactly the same. In the middle only the two oblique directions are used and in the right one only the two other direction. In the tree pictures below them are represented the F-values obtained in calculating the tree upper pictures. The lower three finally indicate if the F-value is smaller (+) or greater (-) then a certain threshold.

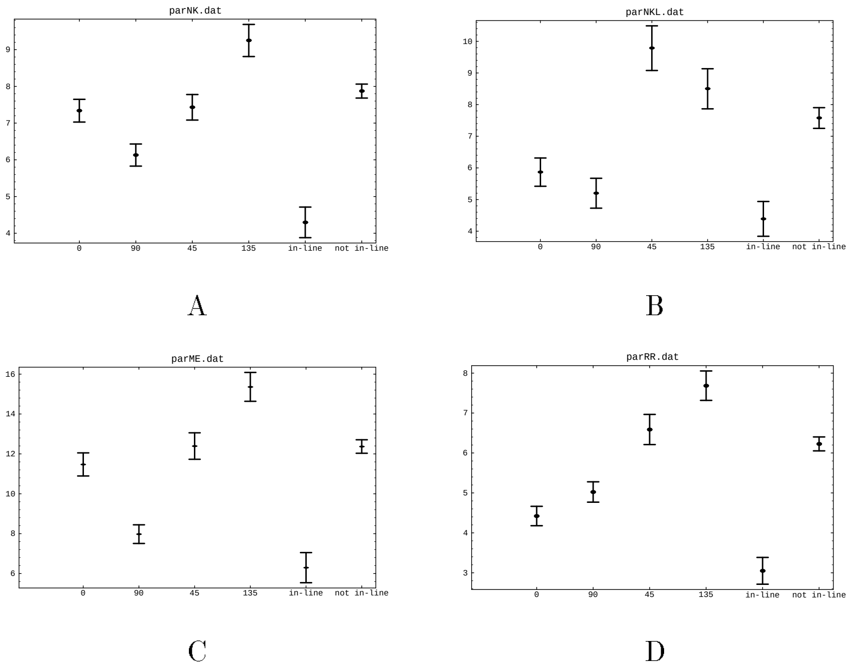


Figure 4.23: Lechelt pictures en test of in-line effect. Mean of the absolute errors in the settings of the 4 directions of the 3 subjects. A: subject NK, B: subject NK with her left hand, C: subject ME, D: subject RR. Only the standard deviation in the mean is given.

tasks (smallest factors were found), so it would not be suprizing if it we would found it also here. Because an rectanglur lattice was used, only non-oblique reference bars can be in-line with their test bars, so an in-line effect could be a cause for the oblique effect found. We see that we indeed see such an in-line effect.. The mean absolute error is always the smallest if looking to only in-line pairs of bars. Because most numbers found on the oblique directions are still greater than the number for ‘not in-line’ even after eliminating the in-line effect, there must be an oblique effect left.

Chapter 5

Conclusions

It was concluded that haptic space, as far as the perception of angles on a table surface is concerned, can be described with a certain potential function, which was called ϕ. Errors in the setting and perception of relative angles can be predicted with the 'potential' difference between the two compared locations. It was assumed that the form of this function can be obtained with a simple model of the arm. For every subject the function had to be scaled with a factor, which was acquired by comparing a simple model of the arm with the acquired measurements.

For doing this all the measurements had to be reduced to a simple function of the surface which was a quite complicated task. For this, we had to make the assumption that the above method really works. We checked it by calculating F-values every time sets must be averaged. Often it was clear that the averaging wasn't justified, but the resulting pictures did give hope. It may be useful to pay more attention to the precise causes of this, but provisionally we have no idea. It may be related to certain tactics of subjects in certain situations, which rises chaos, or may be a totally other treatment will do better.

There were three kinds of measurements and the factors found vary with them. If one had to sense and to set a direction (putting parallel) the factor was greater than when one only had to set one (pointing to a marker). And this is logical, because setting and sensing are two and only setting is one.

It also can be explored if the used arm-model which gives the function is optimal. May be another function would do better. We have suggested already one based on vision, which however we didn't use because it wasn't yet a consistent model (there is not one function we could get, which was better or more logical than others).

Only three subjects were used which did all give more or less comparable results which only differ quantitatively. It is however not at all sure that the results are valid for everybody. It may for example be true that also negative factors are possible. We've seen that our male and oldest subject did produce more realistic results (factors closest to zero, errors may be greater, but less systematic) and we could ask ourselves if age or gender have an influence. It could also be interesting to observe some blind subjects, because they lack the control of vision and are probably more exercised using their haptic perception.

There were already done some other types of measurements, with 2 hands (e.g. paralleling with 2 hands). With these measurements could be checked if also two different functions (those of the two different hands) can be compared to each other.

The oblique effect in the standard-deviation wasn't reproduced convincingly and it should be explored why not, because others did effectively find it in the past (may be because in our case the subject were not aware of the obliqueness of not of the reference bar). On the contrary an oblique effect in the absolute value of the errors made was found. This may be due to the use of a rectangular lattice for the positions on which the bars could be. This causes that oblique orientations are less likely to be also in-line. This is congruent with the fact that the in-line measurement also gives the smallest arm-factors, so the smallest absolute errors. This was tested also, and there was indeed a strong in-line effect found in the parallel measurements. It however can not fully explain the oblique effect so this is probably also present.

It may be possible to explain (partly) the in-line effect by the fact that in-line bars are effectively closer to each other (the endings of the bars are considerably closer to each other than the middles of these bars).

Bibliography

[1] S Appelle, F Gravetter – *Effect of modality-specific experience on visual and judgment of orientation* – Perception **14** (1985) 763

[2] S Appelle, M Countryman – *Eliminating the haptic oblique effect: influence of scanning incongruity and prior knowledge of the standards* – Perception **15** (1986) 325

[3] JN Douglas – *Spherical Trigonometry* – http://utrao.as.utexas.edu/ast307/sphertrig/trig.html

[4] JJ Gibson – *The Senses Considered as Perceptual Systems* – Houghton Mifflin Company, Boston 1966

[5] JJ Gibson – *The Perception of the Visual World* – Houghton Mifflin Company, Boston 1950

[6] N Hogan, BA Kay, ED Fasse, FA Mussa-Ivaldi – *Haptic Illusions: Experiments on Human Manipulation and Perception of "Virtual Objects"* – Cold Spring Harbor Symposia on Quantitative Biology LV, **1990**

[7] JJ Koenderink, AJ van Doorn, J Lappin – *Nexus of exocentric direction in egocentric space* – Utrecht, 1997

[8] B Large – *Math 225 Course Notes, Section 8.2: ANOVA* – http://www.mathcs.duq.edu/larget/math225/8-2.html

[9] EC Lechelt, A Verenka – *Spatial anisotropy in intramodal and cross-modal judgments of stimulus orientation: the stability of the oblique effect* – Perception **9** (1980) 581

[10] EC Lechelt, J Eliuk, G Tanne – *Perception orientational asymmetries: A comparison of visual and haptic space* – Perception & Psychophysics **20** (1976) 463

[11] RK Luneburg – *The metric of binocular visual space* – J. of the Optical Society of America **40.10** (1950) 627

[12] RK Luneburg – *Mathematical Analysis of Binocular Vision* – Princeton University Press, Princeton 1947

[13] Shimbo Masaru, Yamanoi Takahir, Kawaguchi Michiak – *Some metrical aspects of visual space* – Proc. of the int. conf. on Cybernetics & Society **1** (1978) 394

[14] JA Rice – *Mathematical Statistics and Data Analysis* – Wadsworth & Brooks, Belmont

1988

[15] S Wolfram – *Computer program. With the book: Mathematica; A System for Doing Mathematics by Computer;* – Second Edition

List of Figures

Appendix A

The arm-model program

The following MATHEMATICA program (arm.m) was used in relation to the in this work presented model of the arm. Other programs (ruimte.m, par.m, mar.m and wijs.m) will not be fully presented in this report because they are even more uninteresting to read and I don't want to spill to much paper. The author will happily supply them to you after a request, even digitally, which is more useful anyway.

```
(*

  Haptic Perception.
  model of an arm, projection on visual sphere
posvingers[narm,{<hoeken>}]
tekenarm[narm,{hoeken}]
testarm[narm]
vindhoeken[narm,pos]
vindvergelijkingen
tekenbereik[narm]
testarm2[narm]
tekengevolg[narm]
modelhoeken[narm]
tekenminmodel[narm,h]
 b: tekenminmodelmethod=1 or 2 or 3 (derichting).
    info[[usedhand]]=rechts or links (also in other relevant functions)
-----------------
projectpoint[p, narm]
alfa[p, p]
alfaa[p1,p2]
testalfa
findprojangl[p1,p2,narm]
drieD[p1]
tekenprojangl[narm,richting[0]]
*)
If[!NumberQ[reedsanim],
<< Graphics`Animation`;
Print["(a) Graphics`Animation` has been loaded"];
reedsanim=1;
];
Switch[$OperatingSystem,
       "Windows",
       (* << "c:/users/michiel/hapt/prog/ruimte.m"; *)
       << "c:/temp/ruimte.m";
       ,
       "MacOS",
       Print["Not yet implemented"];
       ,
       "Unix",
       << "~/Hapt/prog/ruimte.m";
      ];
constanten :=
Module[{i},
factor=0.5;
(* info[[filenaam]]="Model"; *)
sset = Table[{},{i,5}];
(* informatie: lengte, vrijheid) *)
              l=1;    v=2;
          schouder=1;       barm=2;          oarm=3;        hand=4;       vingers=5;  hschouder=6;
narm = {  {20, {-10,10}},   {40,{-35,135}},  {30,{0,160}},  {15,{-20,20}}, {0,{}},     25};
      (*          /\ :vrijheid van het bijbehorende gewricht
                      (deze info. gebruikt het model (nog?) niet
      *)
(*gemeten: *)
RRarm ={  {22, {-10,10}},   {36,{-35,135}},  {27,{0,160}},  {12,{-20,20}}, {0,{}},     32};
(* NKarm ={  {20.5, {-10,10}},   {34,{-25,135}},  {25.5,{0,160}},  {18.7,{-20,20}}, {0,{}},     25};   *)
NKarm ={  {20.5, {-10,10}},   {34,{-25,135}},  {25.5,{0,160}},  {10,{-20,20}}, {0,{}},     25};
MEarm ={  {26, {-10,10}},   {37,{-25,135}},  {27,{0,160}},  {15,{-20,20}}, {0,{}},     35};
posschouder = {0,0};
](*constanten*);
If[!NumberQ[l], Print["(a) constants have been set"]; constanten;];
pm := Switch[info[[usedhand]],rechts,1,links,-1,_,1];
```

```
cs[hoek_] := {pm Cos[hoek Degree], Sin[hoek Degree]} //N;
(*  The following function gives the position of the end of the hand as a
    function of the angles of the joints. *)
posvingers[i_, h_] := posschouder +
   i[[schouder,1]]  cs[h[[schouder]]]  +
   i[[barm,1]]      cs[h[[schouder]]+h[[barm]]] +
   i[[oarm,1]]      cs[h[[schouder]]+h[[barm]]+h[[oarm]]] +
   i[[hand,1]]      cs[h[[schouder]]+h[[barm]]+h[[oarm]]+h[[hand]]];
gewricht[r_]  = Disk[r,1];
been[r1_,r2_] = {Line[{r1,r2}],gewricht[r1]};
projbarm[i_] := Sqrt[i[[barm,1]]^2 - i[[hschouder]]^2];
tekenarm[i_,inh_] :=
Module[{pos=sset,j},
If[Length[inh]==2, h=vindhoeken[i,inh]; Print["(a!) Converted position to angles"];
                  , h=inh
  ];
pos[[schouder]]=posschouder;
pos[[barm]]    =pos[[schouder]] + i[[schouder,1]] cs[h[[schouder]]];
pos[[oarm]]    =pos[[barm]]     + projbarm[i] cs[h[[schouder]] + h[[barm]]];
pos[[hand]]    =pos[[oarm]]     + i[[oarm,1]] cs[h[[schouder]] + h[[barm]] + h[[oarm]]];
pos[[vingers]] =pos[[hand]]     + i[[hand,1]] cs[h[[schouder]] + h[[barm]] + h[[oarm]] + h[[hand]]];
Show[Graphics[{stip,
               been[pos[[schouder]], pos[[barm]]],
               been[pos[[barm]],     pos[[oarm]]],
               been[pos[[oarm]],     pos[[hand]]],
               been[pos[[hand]],     pos[[vingers]]],
               Table[Point[positie[j]],{j,1,9}]
              },
              PlotRange ->
              Switch[info[[usedhand]],rechts,{{-20,100},{-40,80}},
                                links, {{-100,20},{-40,80}},
                                _,{{-20,100},{-40,80}}
                    ],
              AspectRatio -> Automatic,
              Frame -> True
             ],
      PlotLabel -> "Position of the arm" <> "\n" <> vatsamen[i]
     ]
];
tekenbereik[i_] :=
Show[Graphics[{stip,Table[Point[positie[j]],{j,1,9}],
               Circle[{i[[schouder,1]],0}, projbarm[i]+i[[oarm,1]]+i[[hand,1]]]
               },
               PlotRange  -> {{-20,100},{-40,80}},
               AspectRatio -> Automatic,
                              Frame -> True
             ]
    ]
(* swinging arm, calculated from angles *)
testarm[i_] := Animate[tekenarm[i,{
 i[[schouder,v,1]] + (i[[schouder,v,2]] - i[[schouder,v,1]]) (Sin[t]+1)/2,
 i[[barm,v,1]] + (i[[barm,v,2]] - i[[barm,v,1]]) (Sin[t]+1)/2,
 i[[oarm,v,1]] + (i[[oarm,v,2]] - i[[oarm,v,1]]) (Sin[t]+1)/2,
 i[[hand,v,1]] + (i[[hand,v,2]] - i[[hand,v,1]]) (Sin[t]+1)/2}],
 {t,0, 2 Pi}, Frames-> 20, Closed -> True];
vindhoeken[i_, pos_] :=
Module[{
 x=pm pos[[1]]-i[[schouder,1]],
 y=pos[[2]],
 a=projbarm[i], b=i[[oarm,1]] + i[[hand,1]],
 hulp1,hulp2,
 schouderbeweeg=False
},
If[(a+b)<Sqrt[x^2+y^2] // N,
   schouderbeweeg=True;
   b=a+b;
   a=i[[schouder,1]];
   x=pm pos[[1]];
   If[(a+b)<Sqrt[x^2+y^2] //N,
      Print["(a!) Too far. Elbow will be lifted (reaching ("<> ToString[x] <> "," ToString[y] <> ")) "];
      (* This is not graphicly implemented *)
      (* the ! means: warning! *)
      b=Sqrt[x^2+y^2]-a+0.01 //N;
     ];
  ];
hx=x; hy=y; ha=a; hb=b;
 If[Abs[(b-a)]>Sqrt[x^2+y^2] //N,
      Print["(a!) Too close. Elbow will be lifted (reaching ("<> ToString[pos[[1]]] <> "," ToString[y] <> ")) "];
      If[b>a//N, b=Sqrt[x^2+y^2]+a-0.01 //N,
              b=a-Sqrt[x^2+y^2]+0.01 //N
         ];
       (* does't properly function *)
   ];
hx1=x; hy1=y; ha1=a; hb1=b;
op1 =If[x!=0,
      {(a^2*x^2 - b^2*x^2 + x^4 + x^2*y^2 -
      y*(x^2*(-a^4 + 2*a^2*b^2 - b^4 + 2*a^2*x^2 + 2*b^2*x^2 - x^4 +
            2*a^2*y^2 + 2*b^2*y^2 - 2*x^2*y^2 - y^4))^(1/2))/
    (2*x^3 + 2*x*y^2), (a^2*y - b^2*y + x^2*y + y^3 +
      (x^2*(-a^4 + 2*a^2*b^2 - b^4 + 2*a^2*x^2 + 2*b^2*x^2 - x^4 +
          2*a^2*y^2 + 2*b^2*y^2 - 2*x^2*y^2 - y^4))^(1/2))/(2*(x^2 + y^2))},
     (* x==0 *)
        {-((-a^4 + 2*a^2*b^2 - b^4 + 2*a^2*y^2 + 2*b^2*y^2 - y^4)/y^2)^(1/2)/2,
           (a^2 - b^2 + y^2)/(2*y)}
       ] //N;
op2 = If[x!=0,
       {(a^2*x^2 - b^2*x^2 + x^4 + x^2*y^2 +
      y*(x^2*(-a^4 + 2*a^2*b^2 - b^4 + 2*a^2*x^2 + 2*b^2*x^2 - x^4 +
            2*a^2*y^2 + 2*b^2*y^2 - 2*x^2*y^2 - y^4))^(1/2))/
```

```
    (2*x^3 + 2*x*y^2), (a^2*y - b^2*y + x^2*y + y^3 -
      (x^2*(-a^4 + 2*a^2*b^2 - b^4 + 2*a^2*x^2 + 2*b^2*x^2 - x^4 +
           2*a^2*y^2 + 2*b^2*y^2 - 2*x^2*y^2 - y^4))^(1/2))/(2*(x^2 + y^2))},
       (* x==0 *)
           {((-a^4 + 2*a^2*b^2 - b^4 + 2*a^2*y^2 + 2*b^2*y^2 - y^4)/y^2)^(1/2)/2,
              (a^2 - b^2 + y^2)/(2*y)}
          ] //N;
deop = If[op1[[1]] > op2[[1]], op1, op2];
hulp1 = ArcTan[deop[[1]],deop[[2]]];
hulp2 = ArcTan[(x-deop[[1]]), y-deop[[2]]]-hulp1;
If[schouderbeweeg,{hulp1,hulp2,0,0},{0,hulp1,hulp2,0}] / Degree //N
];
(* With the following module the quite complicated equations before were find  *)
vindvergelijkingen:=
Module[{eq1,eq2},
eq1 = x1^2+y1^2 == a^2;
eq2 = (x-x1)^2 + (y-y1)^2 == b^2;
opl = Solve[{eq1,eq2},{x1,y1}];
op1 = Simplify[{x1,y1} /. opl[[1]]];
op2 = Simplify[{x1,y1} /. opl[[2]]];
deop= If[op1[[1]]>op2[[1]],op1,op2];
InputForm[FullDefinition[deop]] >> hulpje;
];
(* moving arm, calculated from position of hand; it makes a circle *)
testarm2[i_]:= Animate[
tekenarm[i,vindhoeken[i,{30,40}+30 cs[t]]],{t,0,360},
Frames -> 24,
Closed -> True
]
hoekhand[h_] := -90(pm-1) + pm (h[[schouder]] + h[[barm]] + h[[oarm]] + h[[hand]]);
vatsamen[dearm_] := "(s:"<>ToString[dearm[[schouder,l]]]<>
                    ",b:"<>ToString[dearm[[barm,l]]]<>
                    ",o:"<>ToString[dearm[[oarm,l]]]<>
                    ",h:"<>ToString[dearm[[hand,l]]]<>
                    ",hs:"<>ToString[dearm[[hschouder]]] <>
                    ")";
tekengevolg[dearm_]:=
Module[{aantalPos=12,cor},plot={stip};
(* cor=hoekhand[vindhoeken[dearm,positie[normcentrum[info]]]]; *)
 cor=0; (* For report *)
For[i=1, i<=aantalPos, i++,
    AppendTo[plot,lijn[hoekfactor (hoekhand[vindhoeken[dearm,positie[i]]]-cor), positie[i]]];
   ];
laatZien1[plot,"of  an arm " <> "\n" <> vatsamen[dearm] <> showfactor];
];
(* --------------
   here begins the real thing
*)
modelhoeken[dearm_] :=
Module[{aantalPos=9,cor,h},
cor=hoekhand[vindhoeken[dearm,positie[normcentrum[info]]]];
h={};
For[i=1, i<=aantalPos,i++,
    AppendTo[h, hoekhand[vindhoeken[dearm,positie[i]]]-cor]
   ];
h //N
]
makemodelh[dearm_] :=
Module[{},
 modelh = Table[hoekhand[vindhoeken[dearm,positie[i]]],{i,1,9}] //N;
Print["(a) The list 'modelh' was created, and can be used by the function DD"];
 ];
DD[c_,dearm_,h_] :=
Module[{mean,ih},
ih=infosetset[h];
For[i=1, i<=Length[ih], i++,
     If[h[[ih[[i]]]][[1]] == Indeterminate, ih=Delete[ih,i]
       ]
   ];
mean = Sum[h[[ih[[i]],1]] - c* modelh[[ih[[i]]]],{i,1,Length[ih]}] / Length[ih] //N;
Sum[ (mean - (h[[ih[[i]],1]] - c*modelh[[ih[[i]]]]))^2,{i,1,Length[ih]}] //N
];
testDD :=
(* it is also possible analitically: *)
Module[{},
g=2;
aa=Array[a,g];
bb=Array[b,g];
ddd[c_] := Sum[ (Sum[ a[j] + t - c b[j],{j,1,g}]/g - (a[i] + t - c b[i]))^2,{i,1,g}];
deeq := D[ddd[c],c]==0;
];
meansquaredif[h_] :=
Module[{mean,ih},
ih=infosetset[h];
For[i=1, i<=Length[ih], i++,
     If[h[[ih[[i]]]][[1]] == Indeterminate, ih=Delete[ih,i]
       ]
   ];
mean = Sum[h[[ih[[i]],1]],{i,1,Length[ih]}]/Length[ih] //N;
Sum[ (mean - h[[ih[[i]],1]])^2, {i,1,Length[ih]}] //N
];
tekenminmodelmethod=2;
tekenminmodel[dearm_,h_] :=
(* depends on the value of normcentrum *)
Module[{cor,ih,i},
ih=infosetset[h];
plot= {stip};
Switch[tekenminmodelmethod,
 1, modelh=modelhoeken[dearm]; (* gedraaide hoeken van het model *)
    factors={};
    For[i=1, i<=Length[ih], i++,
```

```
          If[ih[[i]]==normcentrum[info],1,
          AppendTo[factors,h[[ih[[i]]]][[1]]/modelh[[ih[[i]]]]]];
       ];
    gemfactor=Mean[factors] //N;
    cor=0,
 2, makemodelh[dearm];
    gemfactor= c /. FindMinimum[DD[c,dearm,h],{c,0.1}][[2]];
    (* cor=0 *)
    cor=h[[normcentrum[info]]][[1]]-(modelh[[normcentrum[info]]] gemfactor),
         (* dit gaat nog mis als bij 9x testen we op het normcentrum zitten,
            echter alleen bij tekenen, de getallen zijn ok.
         *)
 3, makepersh[dearm];
    gemfactor= c /. FindMinimum[DD[c,dearm,h],{c,0.1}][[2]];
    (* cor=0 *)
    cor=h[[normcentrum[info]]][[1]]-(modelh[[normcentrum[info]]] gemfactor),
 _, Print["method " <> ToString[tekenminmodelmethod] <> " not implemented (what did you want?)"]
  ];
AppendTo[plot,Disk[positie[normcentrum[info]],1]];
For[i=1, i<=Length[ih], i++,
    AppendTo[plot,lijn[hoekfactor {h[[ih[[i]]]][[1]]-(modelh[[ih[[i]]]] gemfactor)-cor,h[[ih[[i]]]][[2]],h[[ih[[i]]]][[3]]},positie[ih[[i]]]]];
   ];
laatZien1[plot,"resultaat - "<> ToString[gemfactor] <> " model \n"
             <> "model = " <> vatsamen[dearm] <> ToString[tekenminmodelmethod]
             <> If[tekenminmodelmethod==3,"("<>ToString[derichting]<>")"]
             <> "\n msqd: " <> ToString[meansquaredif[h]]
             <> " to "   <>      ToString[DD[gemfactor,NKarm,h]]
             <> "\n" <> showfactor];
(* nodig voor dat stomme windows: *)
Print["resultaat - "<> ToString[gemfactor] <> " model \n"
             <> "model = " <> vatsamen[dearm] <> ToString[tekenminmodelmethod]
             <> If[tekenminmodelmethod==3,"("<>ToString[derichting]<>")"]
             <> "\n msqd: " <> ToString[meansquaredif[h]]
             <> " to "   <>      ToString[DD[gemfactor,NKarm,h]]
             <> "\n" <> showfactor];
](* tekenminmodel *);
(* ------------------------------------------------------------------
   Projection on sphere
(alternative for arm)
---------------------------------------------------------------------
*)
(* standard distance shoulders-eyes:
*)
hoofd=30; (* cm *)
projectpoint[p_, dearm_] :=
Module[{x,y,z},
 x=p[[1]]; y=p[[2]]; z=p[[3]];
 phi = ArcTan[x,y];
 theta = ArcTan[Sqrt[y^2+x^2],(z-dearm[[hschouder]]-hoofd)];
 {theta,phi} //N
];
mekka ={ 21 Degree, 40 Degree};
utrecht = {52 Degree, 5 Degree};
nuuk = { 65 Degree, -52 Degree};
oost = 90 Degree;
west = -90 Degree;
noord = 0 Degree;
zuid = 180 Degree;
oosten = {0,0.0001 Degree};
westen = -oosten;
noorden = {0.0001 Degree,0};
zuiden  = -noorden;
alfa[p1_, p2_]:=
Module[{t1,t2,phi1,phi2,phi,a},
   t1:=p1[[1]];
   t2:=p2[[1]];
   phi1:=p1[[2]];
   phi2:=p2[[2]];
   phi:=phi2-phi1;
   a=  ArcSin [  Sin[phi] Cos[t2]
             / ( Sin [ ArcCos [ Sin[t1]  Sin[t2] +
                                Cos[t1]  Cos[t2] * Cos[phi]
                              ]
                     ]
               )
            ];
   da= 1;
   If[da<0, a = 180-a];
   a //N
 ];
(* alfa has a range of only Pi, something must be found to let it be 2 Pi
   (pretty puzzling to get that right)
    <- Solved! Use vectors, implementated in mekka2.cpp
*)
rondje[p_] := Plot[alfa[{Sin[x Degree],Cos[x Degree]}*1 + p,p] / Degree,{x,0,360}];
alfaa[p1_,p2_] := D[alfa[{x Degree,p1[[2]]},p2],x] /. x-> p1[[1]];
testalfa := Plot[Evaluate[Table[alfa[{x Degree,phi Degree},mekka]/Degree //N,{phi,0,360,10}]],{x,-90,90}];
findprojangl[p1_,p2_,dearm_] :=
alfa[projectpoint[p1,dearm],projectpoint[p2,dearm]];
drieD[p1_] := {p1[[1]],p1[[2]],0}
richting[t_]:= {0.001 Cos[t], 0.001 Sin[t],0} //N
persmodelangl[dearm_,p_,persrichting_] :=
   Re[(findprojangl[drieD[p],drieD[p]+persrichting,dearm] / Degree
     +90)] //N; (* 0 position is in alfa different then in lijn[..] *)
testpersmodelangl :=
 Plot[Evaluate[Table[persmodelangl[narm,positie[i],richting[t Degree]],{i,1,9}]],{t,0, 360}];
tekenprojangl[dearm_,persrichting_]:=
Module[{aantalPos=9,cor},plot={stip};
For[i=1, i<=aantalPos, i++,
    AppendTo[plot,
```

```
             lijn[ hoekfactor *
                         Re[
                         (persmodelangl[dearm,positie[i],persrichting]-
                          If[normaliseer,
                             persmodelangl[dearm,positie[normcentrum[info]],persrichting],
                             0
                            ]
                         )  ]
                   ,positie[i]
                 ]
            ];
   ];
AppendTo[plot,lijn[ArcTan[persrichting[[1]],persrichting[[2]]]/ Degree//N,{30,-10}]];
laatZien1[plot,"'visual' projection" <> showfactor]
];
testrichting :=
Animate[ tekenprojangl[narm,richting[t]],
         {t,0, 2 Pi}, Frames -> 20, Closed -> True
       ];
derichting = 0;
makepersh[dearm_] :=
Module[{},
 modelh = Table[persmodelangl[dearm,positie[i],richting[derichting Degree]],{i,1,9}] //N;
Print["(a) The list 'modelh' was created, and can be used by the function DD"];
 ];
(*


*)
```

Appendix B

The raw results

In this appendix the not yet shown unadapted results are shown.

B.1 Putting parallel

Not yet shown were the results of the subjects NK and RR. They are in the figures B.1, B.2, B.3 and B.4.

B.2 Pointing to a marker

Of the subject NK and ME the results of this tasks weren't already shown. See figure B.5.

B.3 Putting in line

Still to be presented of this task are the results of the subjects NK and RR, they are in figure B.6.

parNK.dat

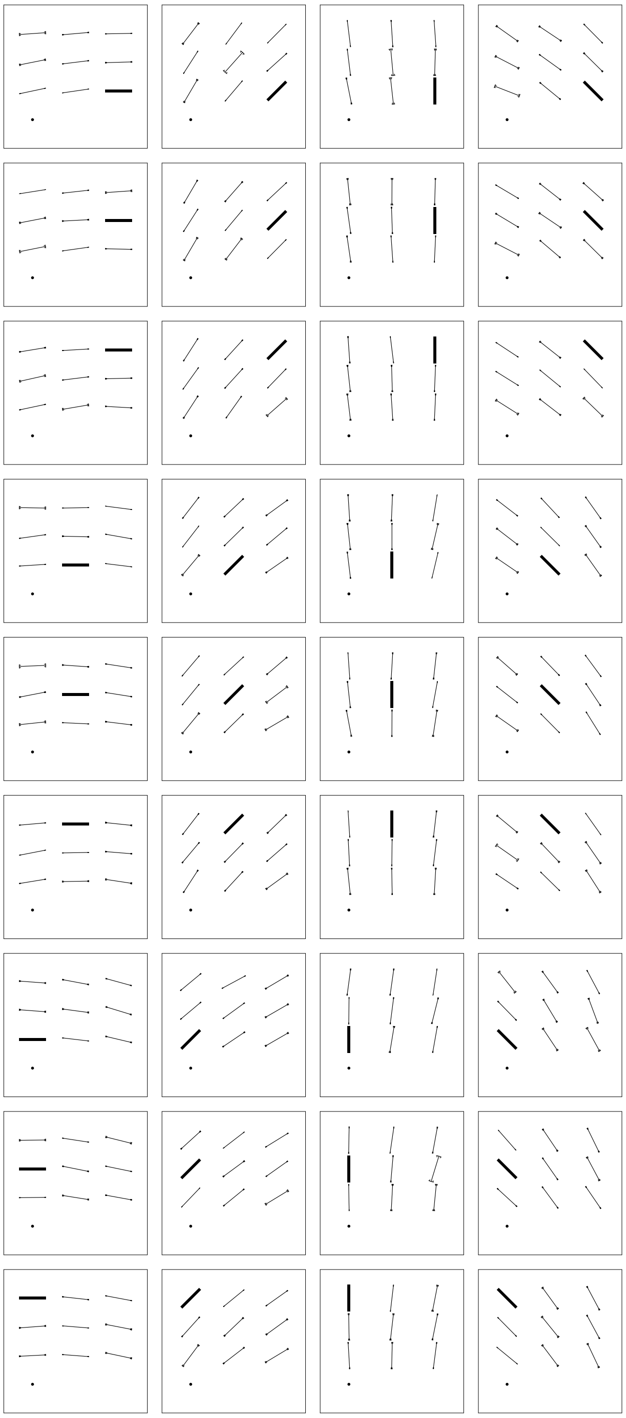


Figure B.1: The putting-parallel results of the subject NK.

parNKR.dat

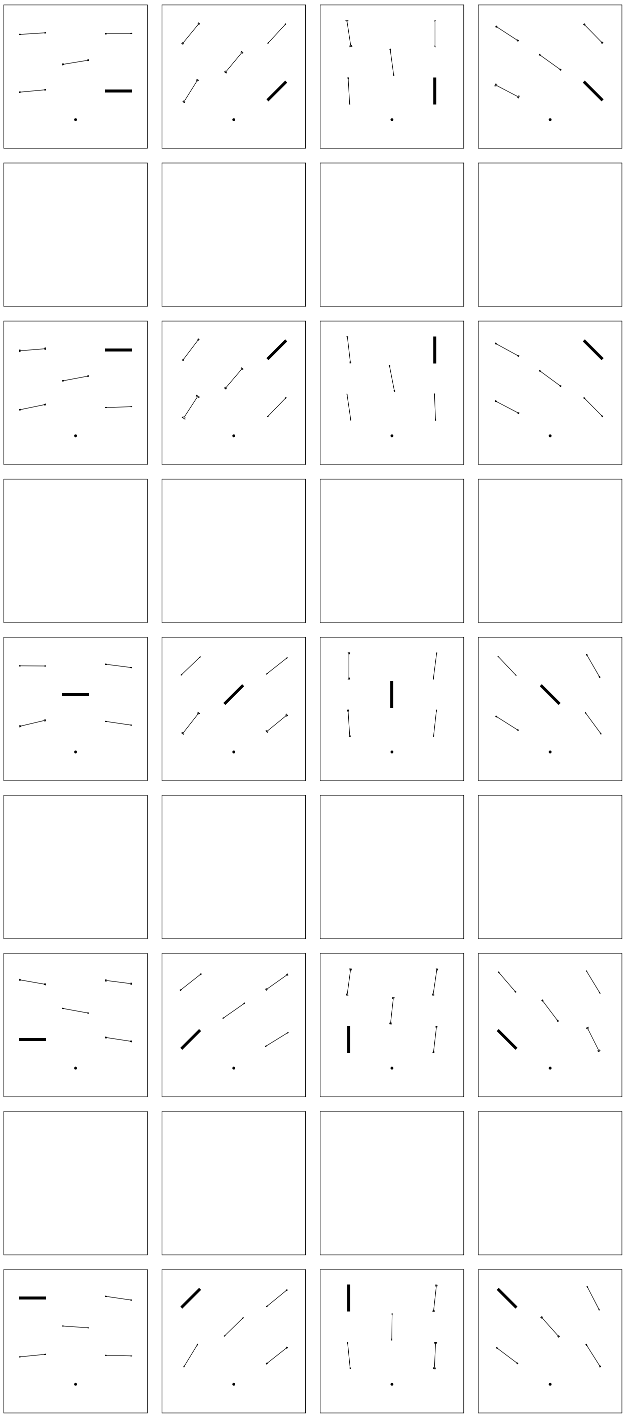


Figure B.2: More putting-parallel results of the subject NK.

parNKL.dat

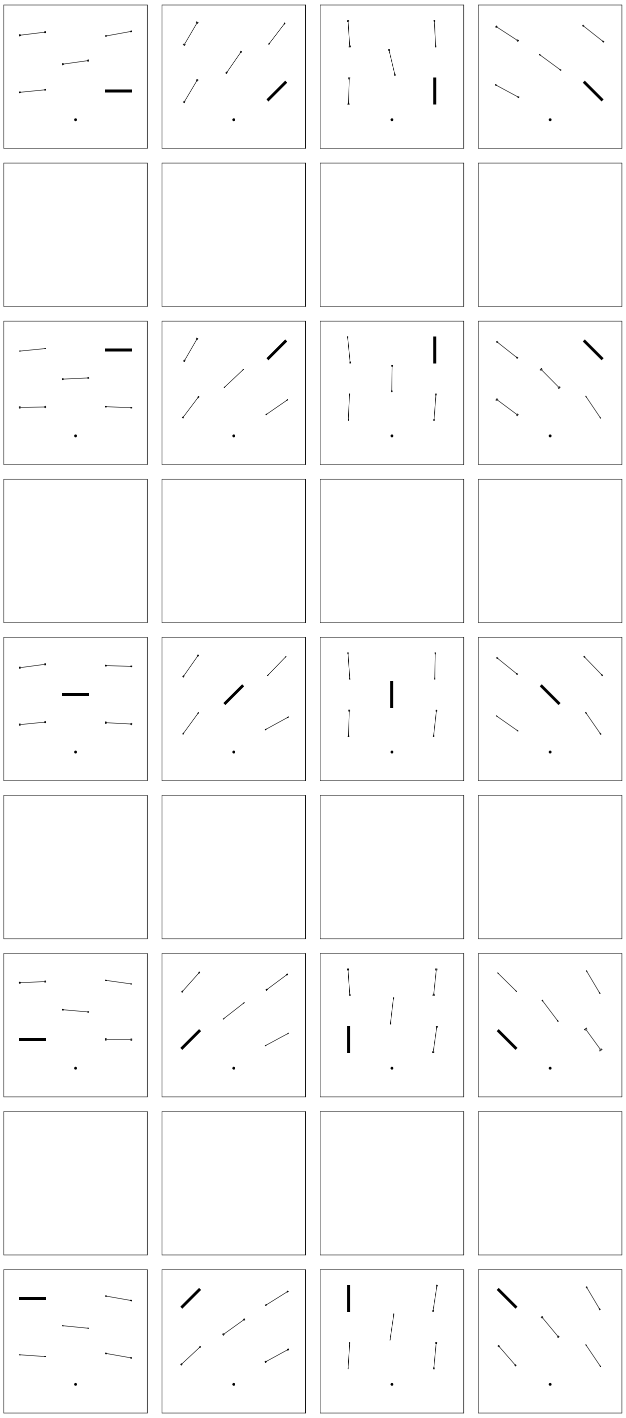


Figure B.3: More putting-parallel results of the subject NK. This time with the left hand in stead of the right one.

parRR.dat

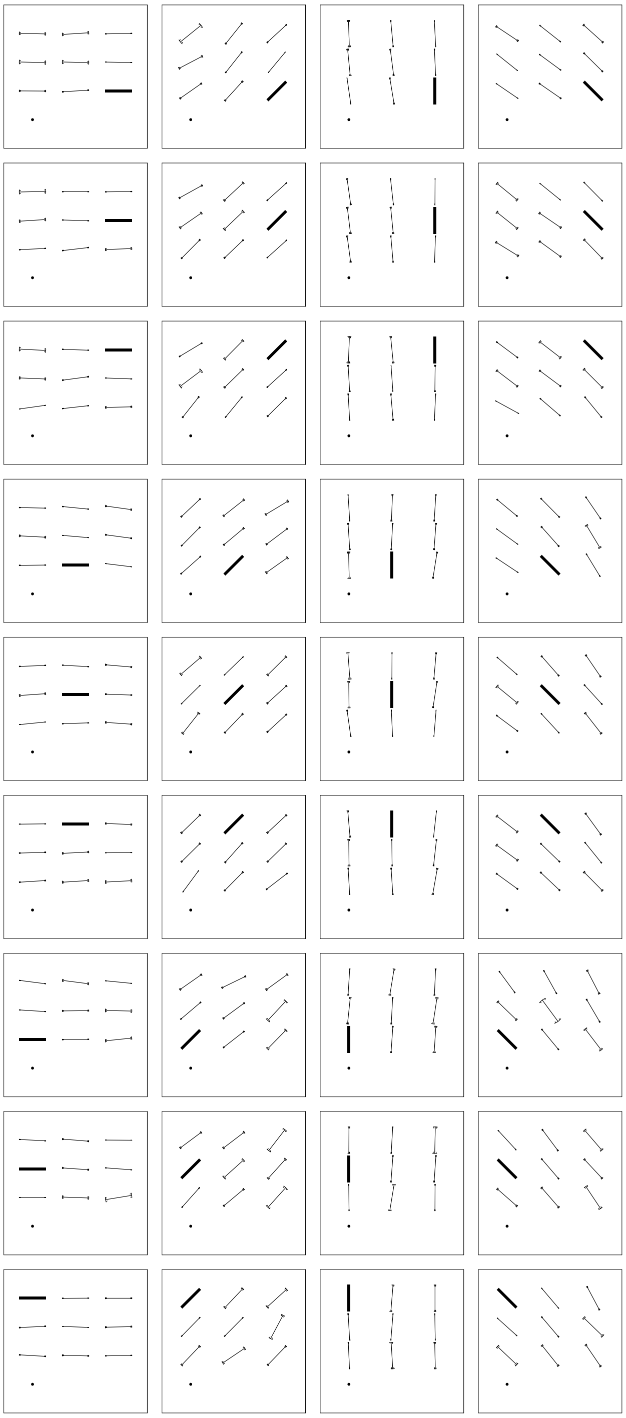
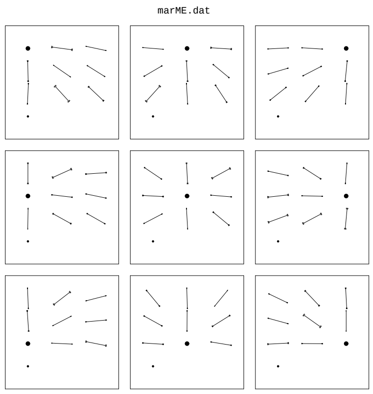
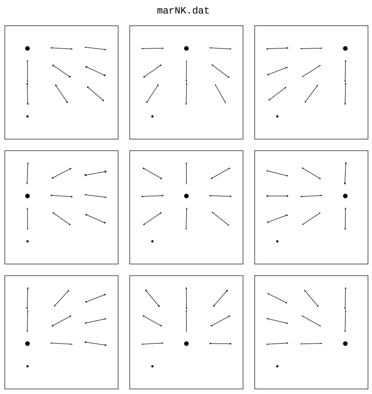


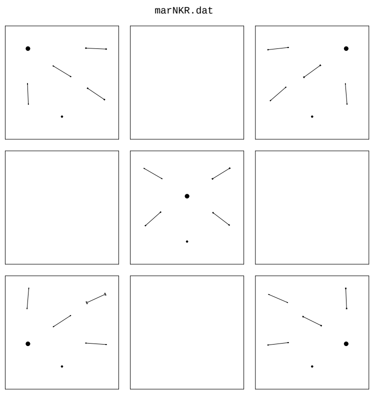
Figure B.4: The putting-parallel results of the subject RR



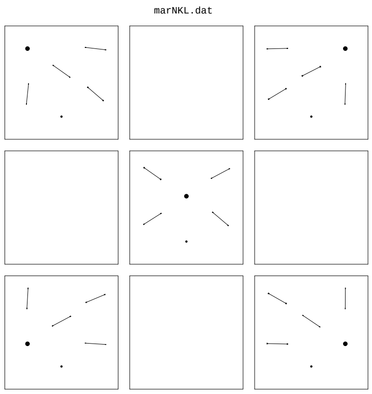
ME



NK

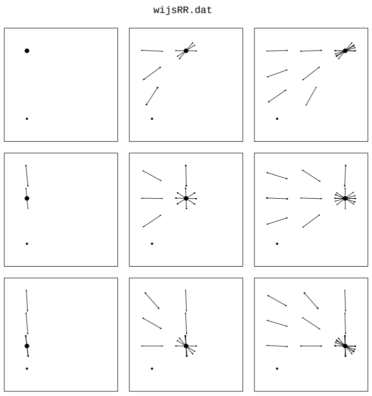


again NK (NKR)

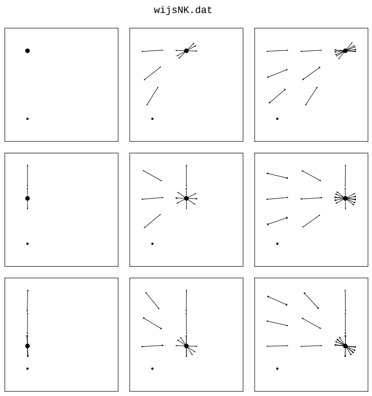


left hand NK (NKL)

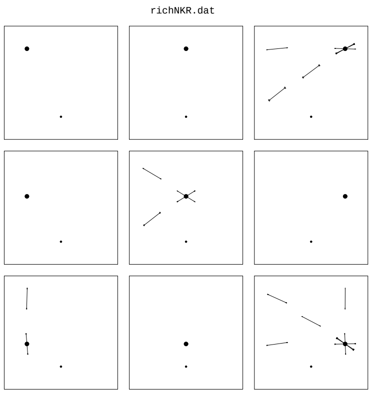
Figure B.5: Results of the pointing to a marker task



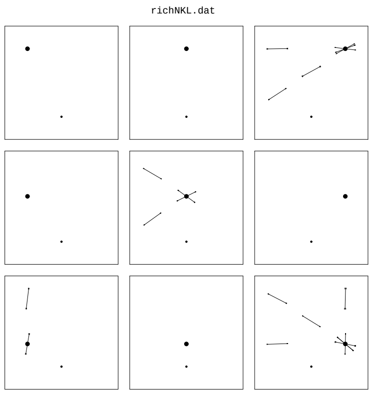
RR



NK



again NK (NKR)



left hand NK (NKL)

Figure B.6: Results of the setting in-line task

Index