```
> restart;#photonF;
> de_orig := r*diff(H(r),r,r) + diff(H(r),r) + (k^2*r - lambda^2/R*r^2)*H(r);
```

$$de\_orig := r\left(\frac{d^2}{dr^2} H(r)\right) + \frac{d}{dr} H(r) + \left(k^2 r - \frac{\lambda^2 r^2}{R}\right) H(r) \quad (1)$$

```
> bc := H(R) = 0, H(1/R) = R;
```

$$bc := H(R) = 0, H\left(\frac{1}{R}\right) = R \quad (2)$$

```
> change_of_vars := { r = (R^2 - 1) * rho / (Pi * R) + 1/R,
        H(r) = G(rho) + R/Pi * (Pi-rho) };
```

$$change\_of\_vars := \left\{ r = \frac{(R^2-1)\rho}{\pi R} + \frac{1}{R}, H(r) = G(\rho) + \frac{R(\pi-\rho)}{\pi} \right\} \quad (3)$$

```
> tmp1 := PDEtools:-dchange(change_of_vars, [de_orig, bc], [G(rho), rho]);
```

$$tmp1 := \left[\frac{\left(\frac{(R^2-1)\rho}{\pi R} + \frac{1}{R}\right)\left(\frac{d^2}{d\rho^2} G(\rho)\right)\pi^2 R^2}{(R^2-1)^2} + \frac{\left(\frac{d}{d\rho} G(\rho) - \frac{R}{\pi}\right)\pi R}{R^2-1} + \left(k^2\left(\frac{(R^2-1)\rho}{\pi R} + \frac{1}{R}\right) - \frac{\lambda^2\left(\frac{(R^2-1)\rho}{\pi R} + \frac{1}{R}\right)^2}{R}\right)\left(G(\rho) + \frac{R(\pi-\rho)}{\pi}\right), G(\pi) = 0, G(0) + R = R \right] \quad (4)$$

```
> de := tmp1[1]
```

$$de := \frac{\left(\frac{(R^2-1)\rho}{\pi R} + \frac{1}{R}\right)\left(\frac{d^2}{d\rho^2} G(\rho)\right)\pi^2 R^2}{(R^2-1)^2} + \frac{\left(\frac{d}{d\rho} G(\rho) - \frac{R}{\pi}\right)\pi R}{R^2-1} + \left(k^2\left(\frac{(R^2-1)\rho}{\pi R} + \frac{1}{R}\right) - \frac{\lambda^2\left(\frac{(R^2-1)\rho}{\pi R} + \frac{1}{R}\right)^2}{R}\right)\left(G(\rho) + \frac{R(\pi-\rho)}{\pi}\right) \quad (5)$$

```
> tmp1[2];
```

$$G(\pi) = 0 \quad (6)$$

```
> tmp1[3];
```

$$G(0) + R = R \quad (7)$$

```
> S := sum(a[n]*sin(n*rho), n = 1..infinity);
```

$$S := \sum_{n=1}^{\infty} a_n \sin(n\rho) \quad (8)$$

```
> eval(de, G(rho) = S) :
  tmp2 := combine(%);
```

$$tmp2 := \frac{1}{\pi^3 R^5 - \pi^3 R^3}\left(\left(\left(\sum_{n=1}^{\infty} \frac{1}{R^2-1}\left(-\pi^4 \sin(n\rho) R^6 n^2 \rho a_n + \pi^2 \sin(n\rho) R^8 k^2 \rho a_n \right.\right.\right.\right.$$
$$- \pi \sin(n\rho) R^8 \lambda^2 \rho^2 a_n - \pi^5 \sin(n\rho) R^4 n^2 a_n + \pi^4 \cos(n\rho) R^6 n a_n$$
$$+ \pi^4 \sin(n\rho) R^4 n^2 \rho a_n + \pi^3 \sin(n\rho) R^6 k^2 a_n - 3\pi^2 \sin(n\rho) R^6 k^2 \rho a_n$$
$$- 2\pi^2 \sin(n\rho) R^6 \lambda^2 \rho a_n + 4\pi \sin(n\rho) R^6 \lambda^2 \rho^2 a_n - R^4 \pi^4 a_n n \cos(n\rho)$$
$$- 2\pi^3 \sin(n\rho) R^4 k^2 a_n - \pi^3 \sin(n\rho) R^4 \lambda^2 a_n + 3\pi^2 R^4 k^2 \rho a_n \sin(n\rho)$$
$$+ 6\pi^2 \sin(n\rho) R^4 \lambda^2 \rho a_n - 6 R^4 \lambda^2 \rho^2 \pi a_n \sin(n\rho) + \pi^3 R^2 k^2 a_n \sin(n\rho)$$
$$+ 2\pi^3 \sin(n\rho) R^2 \lambda^2 a_n - \pi^2 R^2 k^2 \rho a_n \sin(n\rho) - 6\pi^2 R^2 \lambda^2 \rho a_n \sin(n\rho)$$
$$+ 4 R^2 \lambda^2 \rho^2 \pi a_n \sin(n\rho) - \pi^3 \lambda^2 a_n \sin(n\rho) + 2\pi^2 \lambda^2 \rho a_n \sin(n\rho)$$
$$\left.\left. - \lambda^2 \rho^2 \pi a_n \sin(n\rho)\right)\right) + \pi^2 R^7 k^2 \rho - \pi R^7 k^2 \rho^2 - \pi R^7 \lambda^2 \rho^2 + R^7 \lambda^2 \rho^3 + \pi^3 R^5 k^2$$
$$- 3\pi^2 R^5 k^2 \rho - 2\pi^2 R^5 \lambda^2 \rho + 2\pi R^5 k^2 \rho^2 + 5\pi R^5 \lambda^2 \rho^2 - 3 R^5 \lambda^2 \rho^3 - \pi^3 R^5 - \pi^3 R^3 k^2$$
$$- \pi^3 R^3 \lambda^2 + 2\pi^2 R^3 k^2 \rho + 5\pi^2 R^3 \lambda^2 \rho - \pi R^3 k^2 \rho^2 - 7\pi R^3 \lambda^2 \rho^2 + 3 R^3 \lambda^2 \rho^3 + \pi^3 R \lambda^2$$
$$\left. - 3\pi^2 R \lambda^2 \rho + 3\pi R \lambda^2 \rho^2 - R \lambda^2 \rho^3\right) \quad \textbf{(9)}$$

> *restart;# We can neglect the small member* $\frac{\lambda^2 r^2}{R}$ **and** *divide* **by** *r thus we have* **in** *eq*(1);

> $\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}H(r)\right) + k\text{^}2 \cdot H(r) = 0;$

$$\frac{d^2 H(r)}{dr^2} + k^2 H(r) = 0 \quad \textbf{(10)}$$

> $de := 1/r \cdot Diff(r \cdot Diff(H(r), r), r) + k\text{^}2 \cdot H(r);$

$$de := \frac{\frac{d}{dr}\left(r\frac{d}{dr}H(r)\right)}{r} + k^2 H(r) \quad \textbf{(11)}$$

> $bc := H(R) = 0, \mathrm{D}(H)(1/R) = R;$

$$bc := H(R) = 0, \mathrm{D}(H)\left(\frac{1}{R}\right) = R \quad \textbf{(12)}$$

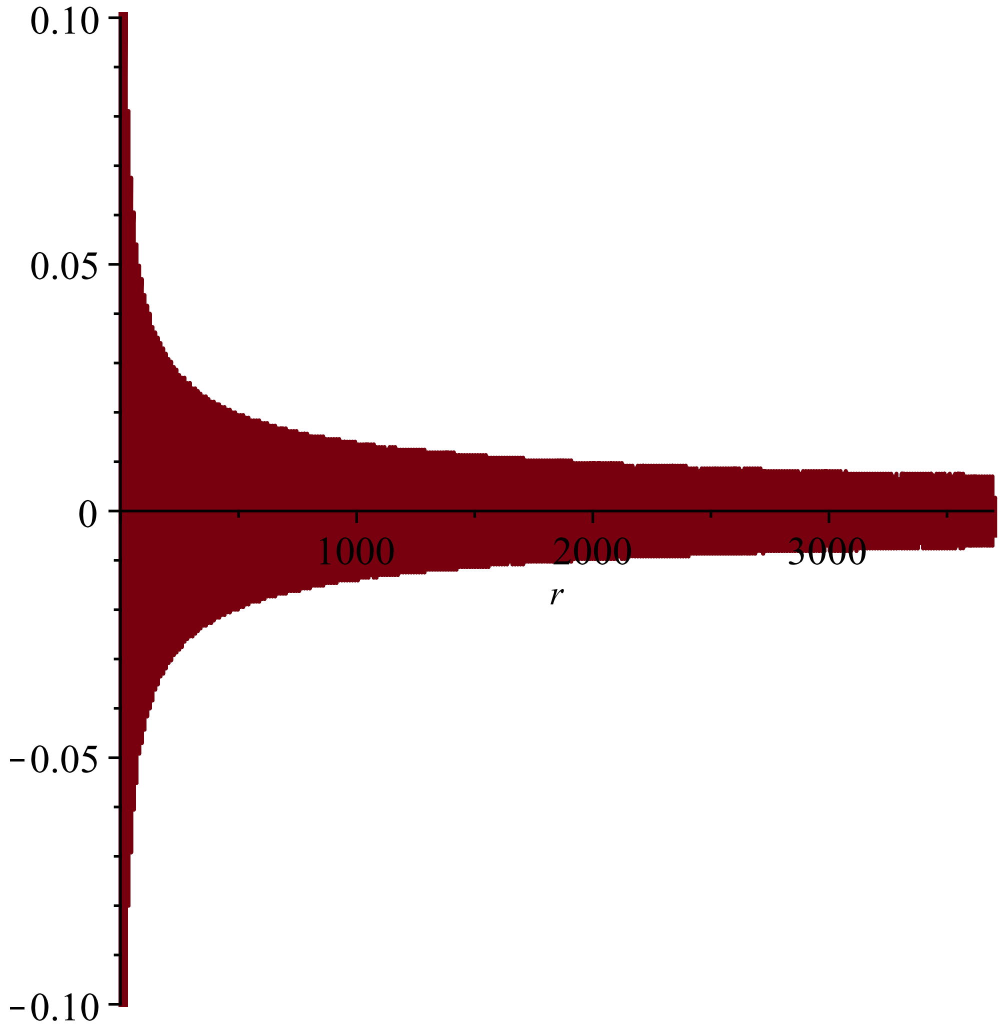
> $dsol := dsolve(\{de, bc\}, H(r));$

$$dsol := H(r) = -\frac{R\,\mathrm{BesselY}(0, kR)\,\mathrm{BesselJ}(0, kr)}{k\left(\mathrm{BesselJ}\left(1, \frac{k}{R}\right)\mathrm{BesselY}(0, kR) - \mathrm{BesselY}\left(1, \frac{k}{R}\right)\mathrm{BesselJ}(0, kR)\right)}$$
$$+ \frac{\mathrm{BesselJ}(0, kR)\,R\,\mathrm{BesselY}(0, kr)}{k\left(\mathrm{BesselJ}\left(1, \frac{k}{R}\right)\mathrm{BesselY}(0, kR) - \mathrm{BesselY}\left(1, \frac{k}{R}\right)\mathrm{BesselJ}(0, kR)\right)} \quad \textbf{(13)}$$

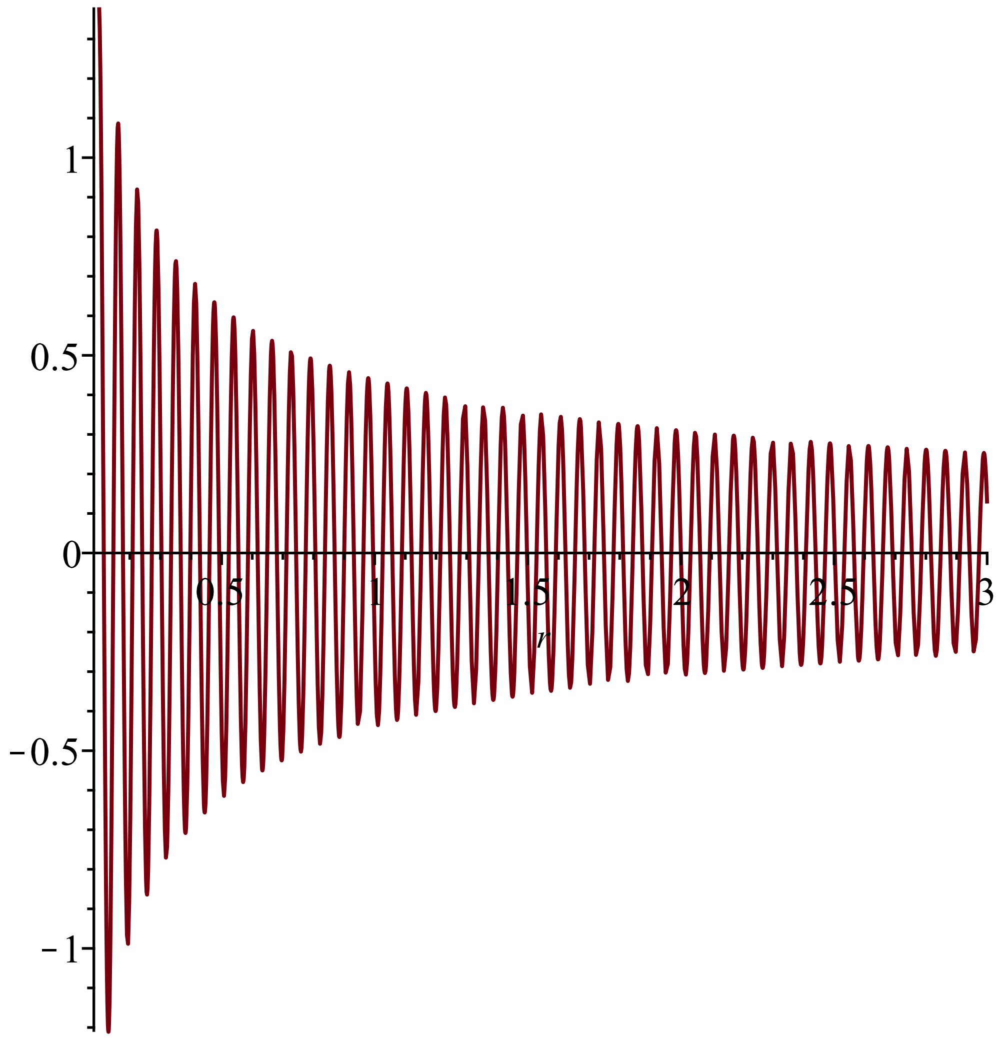
> $R := 3.7 \cdot 10^3;\ k := 10^2;$

$$R := 3700.0$$
$$k := 100 \quad \textbf{(14)}$$

> *plot*(*rhs*(*dsol*), $r = 1/R..R$, *view* = -0.1 .. 0.1, *numpoints* = 10000);



> *plot*(*rhs*(*dsol*), $r = 0.1 .. 3$);*#We cannot see the graph's details because it oscillates so much,*
#but we will see the details if we plot it on a narrow range;


1
0.5
0
−0.5
−1
0.5
1
1.5
2
2.5
3
r

>

>