

> restart; #Ond Scalat where hbar noted as h because is more covenable;

> eq6 := $\frac{2 m}{h^2} \cdot \left(E + \frac{e^2}{\rho} + \frac{L^2}{2 \cdot m \cdot \rho^2} - \frac{h^2}{2 \cdot m} k \right) \cdot \frac{\rho^2}{R} \cdot f(\rho) + \text{diff}(\rho \cdot f(\rho), \rho, \rho)$; #Onduscular;

$$\text{eq6} := \frac{2 m \left(E + \frac{e^2}{\rho} + \frac{L^2}{2 m \rho^2} - \frac{h^2 k}{2 m} \right) \rho^2 f(\rho)}{h^2 R} + 2 \frac{d}{d\rho} f(\rho) + \rho \left(\frac{d^2}{d\rho^2} f(\rho) \right) \quad (1)$$

> change_of_vars := { rho=rb·r, f(rho) =g(r)};

$$\text{change_of_vars} := \{ \rho = rb r, f(\rho) = g(r) \} \quad (2)$$

> PDEtools:-dchange(change_of_vars, eq6, [g(r), r]);

$$\frac{2 m \left(E + \frac{e^2}{rb r} + \frac{L^2}{2 m rb^2 r^2} - \frac{h^2 k}{2 m} \right) rb^2 r^2 g(r)}{h^2 R} + \frac{2 \left(\frac{d}{dr} g(r) \right)}{rb} + \frac{r \left(\frac{d^2}{dr^2} g(r) \right)}{rb} \quad (3)$$

> eq1 := $\frac{2 m \left(E + \frac{e^2}{rb r} + \frac{L^2}{2 m rb^2 r^2} - \frac{h^2 k}{2 m} \right) rb^2 r^2 g(r)}{h^2 R} + \frac{2 \left(\frac{d}{dr} g(r) \right)}{rb} + \frac{r \left(\frac{d^2}{dr^2} g(r) \right)}{rb}$;

$$\text{eq1} := \frac{2 m \left(E + \frac{e^2}{rb r} + \frac{L^2}{2 m rb^2 r^2} - \frac{h^2 k}{2 m} \right) rb^2 r^2 g(r)}{h^2 R} + \frac{2 \left(\frac{d}{dr} g(r) \right)}{rb} + \frac{r \left(\frac{d^2}{dr^2} g(r) \right)}{rb} \quad (4)$$

>

> subs $\left(E = \frac{m \cdot e^4}{32 \cdot \text{Pi}^2 \cdot \epsilon^2 \cdot h^2} \cdot E0, rb = \frac{4 \cdot \text{Pi} \cdot \epsilon \cdot h^2}{m \cdot e^2}, \text{eq1} \right)$;

$$\frac{32 h^2 \left(\frac{m e^4 E0}{32 \pi^2 \epsilon^2 h^2} + \frac{e^4 m}{4 \pi \epsilon h^2 r} + \frac{L^2 m e^4}{32 \pi^2 \epsilon^2 h^4 r^2} - \frac{h^2 k}{2 m} \right) \pi^2 \epsilon^2 r^2 g(r)}{m e^4 R} + \frac{\left(\frac{d}{dr} g(r) \right) m e^2}{2 \pi \epsilon h^2} \quad (5)$$

$$+ \frac{m e^2 r \left(\frac{d^2}{dr^2} g(r) \right)}{4 \pi \epsilon h^2}$$

>

> eq2 := $\left(E0 + \frac{8 \cdot \text{Pi} \cdot \epsilon}{r} + \frac{L^2}{h^2 \cdot r^2} - 16 \cdot \frac{h^4 \cdot k \cdot \pi^2 \cdot \epsilon^2}{m^2 \cdot e^4} \right) \cdot \frac{r^2}{R} \cdot g(r) + \text{diff}(r \cdot g(r), r, r)$;

$$\text{eq2} := \frac{\left(E0 + \frac{8 \pi \epsilon}{r} + \frac{L^2}{h^2 r^2} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4} \right) r^2 g(r)}{R} + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (6)$$

>

> eq2 := $\left(E0 + \frac{8 \pi \epsilon}{r} + \frac{L^2}{h^2 \cdot r^2} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4} \right) \cdot \frac{r^2}{R} \cdot g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right)$;

$$\text{eq2} := \frac{\left(E0 + \frac{8 \pi \epsilon}{r} + \frac{L^2}{h^2 r^2} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4} \right) r^2 g(r)}{R} + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (7)$$

>

> TFreeOrd := $\left(\frac{E0}{R} + \frac{8 \pi \epsilon}{r \cdot R} + \frac{L^2}{h^2 r^2 \cdot R} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4 \cdot R} \right) \cdot r^2 \cdot g(r)$;

$$\text{TFreeOrd} := \left(\frac{E0}{R} + \frac{8 \pi \epsilon}{r \cdot R} + \frac{L^2}{h^2 r^2 \cdot R} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4 \cdot R} \right) \cdot r^2 \cdot g(r); \quad (8)$$

$$TFreeOrd := \left(\frac{E0}{R} + \frac{8 \pi \epsilon}{r R} + \frac{L^2}{h^2 r^2 R} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4 R} \right) r^2 g(r) \quad (8)$$

$$> sort \left(\left(\frac{E0}{R} r^2 + \frac{8 \pi \epsilon}{R} r + \frac{L^2}{h^2 \cdot R} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4 R} r^2 \right), r \right);$$

$$\frac{E0 r^2}{R} - \frac{16 h^4 k \pi^2 \epsilon^2 r^2}{m^2 e^4 R} + \frac{8 \pi \epsilon r}{R} + \frac{L^2}{h^2 R} \quad (9)$$

$$> eq2 := \left(\frac{E0 r^2}{R} - \frac{16 h^4 k \pi^2 \epsilon^2 r^2}{m^2 e^4 R} + \frac{8 \pi \epsilon r}{R} + \frac{L^2}{h^2 R} \right) \cdot g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right);$$

#Onduscular equation.

$$eq2 := \left(\frac{E0 r^2}{R} - \frac{16 h^4 k \pi^2 \epsilon^2 r^2}{m^2 e^4 R} + \frac{8 \pi \epsilon r}{R} + \frac{L^2}{h^2 R} \right) g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (10)$$

> restart;

#ScrodingerScal where hbar noted as h because is more covenable; we start from equation 19.8 Feynman

$$> eq1 := \frac{2 m}{h^2} \cdot \left(E + \frac{e^2}{\rho} \right) \cdot \rho \cdot f(\rho) + diff(\rho \cdot f(\rho), \rho, \rho); \#Schrodinger;$$

$$eq1 := \frac{2 m \left(E + \frac{e^2}{\rho} \right) \rho f(\rho)}{h^2} + 2 \frac{d}{d\rho} f(\rho) + \rho \left(\frac{d^2}{d\rho^2} f(\rho) \right) \quad (11)$$

$$> change_of_vars := \{ \rho = r b \cdot r, f(\rho) = g(r) \};$$

$$change_of_vars := \{ \rho = r b r, f(\rho) = g(r) \} \quad (12)$$

$$> PDEtools:-dchange(change_of_vars, eq1, [g(r), r]);$$

$$\frac{2 m \left(E + \frac{e^2}{r b r} \right) r b r g(r)}{h^2} + \frac{2 \left(\frac{d}{dr} g(r) \right)}{r b} + \frac{r \left(\frac{d^2}{dr^2} g(r) \right)}{r b} \quad (13)$$

$$> eq2 := \frac{2 m \left(E + \frac{e^2}{r b r} \right) r b^2 r g(r)}{h^2} + \frac{d^2}{dr^2} (r \cdot g(r)); \#we multiply rb ;$$

$$eq2 := \frac{2 m \left(E + \frac{e^2}{r b r} \right) r b^2 r g(r)}{h^2} + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (14)$$

$$> subs \left(E = \frac{m \cdot e^4}{32 \cdot \pi^2 \cdot \epsilon^2 \cdot h^2} \cdot E0, rb = \frac{4 \cdot \pi \cdot \epsilon \cdot h^2}{m \cdot e^2}, eq2 \right);$$

#with wacuum permittivity ϵ and $E0$ =energy in Rythberg unit

$$\frac{32 \left(\frac{m e^4 E0}{32 \pi^2 \epsilon^2 h^2} + \frac{e^4 m}{4 \pi \epsilon h^2 r} \right) \pi^2 \epsilon^2 h^2 r g(r)}{m e^4} + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (15)$$

$$> eq2 := 32 \cdot \left(\frac{E0}{32 \cdot h^2} + \frac{\pi \cdot \epsilon}{4 \cdot h^2 \cdot r} \right) \cdot h^2 \cdot r \cdot g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right);$$

$$eq2 := 32 \left(\frac{E0}{32 h^2} + \frac{\pi \epsilon}{4 h^2 r} \right) h^2 r g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (16)$$

$$> eq2 := 32 \left(\frac{E0}{32} + \frac{\pi \epsilon}{4 \cdot r} \right) \cdot r \cdot g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right);$$

$$eq2 := 32 \left(\frac{E0}{32} + \frac{\pi \epsilon}{4r} \right) r g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (17)$$

$$> eq2 := \left(E0 + \frac{8 \cdot \pi \cdot \epsilon}{r} \right) \cdot r \cdot g(r) + \frac{d^2}{dr^2} (r \cdot g(r));$$

$$eq2 := \left(E0 + \frac{8 \pi \epsilon}{r} \right) r g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r) \right) \quad (18)$$

> restart;# we have the Schrodinger Scaled equation with wacuum permittivity ϵ

$$> eq3 := \left(E0 + \frac{8 \cdot \pi \cdot \epsilon}{\rho} \right) \cdot \rho \cdot f(\rho) + \frac{d^2}{d\rho^2} (\rho \cdot f(\rho)); \#eqation (1) with wacuum permittivity $\epsilon$$$

$$eq3 := \left(E0 + \frac{8 \pi \epsilon}{\rho} \right) \rho f(\rho) + 2 \frac{d}{d\rho} f(\rho) + \rho \left(\frac{d^2}{d\rho^2} f(\rho) \right) \quad (19)$$

> restart;#now we have the Schrodinger Scaled eq1 with wacuum permittivity ϵ ;

$$> eq1 := \left(E0 + \frac{8 \cdot \pi \cdot \epsilon}{\rho} \right) \cdot \rho \cdot f(\rho) + \frac{d^2}{d\rho^2} (\rho \cdot f(\rho));$$

$$eq1 := \left(E0 + \frac{8 \pi \epsilon}{\rho} \right) \rho f(\rho) + 2 \frac{d}{d\rho} f(\rho) + \rho \left(\frac{d^2}{d\rho^2} f(\rho) \right) \quad (20)$$

$$> change_of_vars := \left\{ \rho = rb \cdot r, f(\rho) = g(r) \cdot \frac{2 \cdot R - r \cdot rb}{2 \cdot R} \right\};$$

$$change_of_vars := \left\{ \rho = rb r, f(\rho) = \frac{g(r) (-rb r + 2 R)}{2 R} \right\} \quad (21)$$

> PDEtools:-dchange(change_of_vars, eq1, [g(r), r]);

$$\frac{\left(E0 + \frac{8 \pi \epsilon}{rb r} \right) rb r g(r) (-rb r + 2 R)}{2 R} + \frac{2 \left(\frac{\frac{d}{dr} g(r)}{2 R} (-rb r + 2 R) - \frac{g(r) rb}{2 R} \right)}{rb} \quad (22)$$

$$+ \frac{r \left(\frac{\frac{d^2}{dr^2} g(r)}{2 R} (-rb r + 2 R) - \frac{\left(\frac{d}{dr} g(r) \right) rb}{R} \right)}{rb}$$

$$> eq2 := simplify \left(\frac{\left(E0 + \frac{8 \pi \epsilon}{rb r} \right) rb r g(r) (-rb r + 2 R)}{2 R} + \frac{2 \left(\frac{\left(\frac{d}{dr} g(r) \right) (-rb r + 2 R)}{2 R} - \frac{g(r) rb}{2 R} \right)}{rb} + \frac{r \left(\frac{\left(\frac{d^2}{dr^2} g(r) \right) (-rb r + 2 R)}{2 R} - \frac{\left(\frac{d}{dr} g(r) \right) rb}{R} \right)}{rb} \right);$$

$$eq2 := \frac{1}{2 rb R} \left((-r^2 rb + 2 R r) \left(\frac{d^2}{dr^2} g(r) \right) + (-4 rb r + 4 R) \left(\frac{d}{dr} g(r) \right) + 2 \left(-\frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right) rb g(r) \right) \quad (23)$$

$$\begin{aligned} > \text{eq2} := \frac{(-r^2 rb + 2 R r) \cdot r}{2 rb R} \cdot \left(\frac{d^2}{dr^2} g(r) \right) + \frac{(-4 rb r + 4 R)}{2 rb R} \cdot \left(\frac{d}{dr} g(r) \right) + \frac{2}{R} g(r) \left(\right. \\ & \left. - \frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right); \end{aligned}$$

$$\begin{aligned} \text{eq2} := & \frac{(-r^2 rb + 2 R r) r \left(\frac{d^2}{dr^2} g(r) \right)}{2 rb R} + \frac{(-4 rb r + 4 R) \left(\frac{d}{dr} g(r) \right)}{2 rb R} \\ & + \frac{2 g(r) \left(-\frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right)}{R} \end{aligned} \quad (24)$$

$$\begin{aligned} > \text{eq2} := \frac{(-r rb + 2 R) \cdot r}{2 rb R} \cdot \left(\frac{d^2}{dr^2} g(r) \right) + \frac{(-4 rb r + 4 R)}{2 rb R} \cdot \left(\frac{d}{dr} g(r) \right) + \frac{2}{R} g(r) \cdot \left(-\frac{r^2 E0 rb^2}{2} \right. \\ & \left. + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right); \end{aligned}$$

$$\begin{aligned} \text{eq2} := & \frac{(-r rb + 2 R) r \left(\frac{d^2}{dr^2} g(r) \right)}{2 rb R} + \frac{(-4 rb r + 4 R) \left(\frac{d}{dr} g(r) \right)}{2 rb R} \\ & + \frac{2 g(r) \left(-\frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right)}{R} \end{aligned} \quad (25)$$

$$\begin{aligned} > \text{eq2} := \frac{r}{2 rb R} \cdot \left(\frac{d^2}{dr^2} g(r) \right) + \frac{(-4 rb r + 4 R)}{2 rb R \cdot (-r rb + 2 R)} \cdot \left(\frac{d}{dr} g(r) \right) + \frac{2 \cdot g(r)}{R \cdot (-r rb + 2 R)} \cdot \left(\right. \\ & \left. - \frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right); \# \frac{(-4 rb r + 4 R)}{(-r rb + 2 R)} \sim 2; \end{aligned}$$

$$\begin{aligned} \text{eq2} := & \frac{r \left(\frac{d^2}{dr^2} g(r) \right)}{2 rb R} + \frac{(-4 rb r + 4 R) \left(\frac{d}{dr} g(r) \right)}{2 rb R (-r rb + 2 R)} \\ & + \frac{2 g(r) \left(-\frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right)}{R (-r rb + 2 R)} \end{aligned} \quad (26)$$

$$\begin{aligned} > \text{eq2} := r \left(\frac{d^2}{dr^2} g(r) \right) + 2 \frac{d}{dr} g(r) \\ & + \frac{2 rb \left(-\frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right)}{-r rb + 2 R} g(r); \end{aligned}$$

$$\begin{aligned} \text{eq2} := & r \left(\frac{d^2}{dr^2} g(r) \right) + 2 \frac{d}{dr} g(r) \\ & + \frac{2 rb \left(-\frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right) g(r)}{-r rb + 2 R} \end{aligned} \quad (27)$$

$$\begin{aligned} > \text{TFreeOrd} := \text{simplify} \left(\frac{2 rb \left(-\frac{r^2 E0 rb^2}{2} + r (E0 R - 4 \pi \epsilon) rb + 8 R \pi \epsilon - 1 \right)}{-r rb + 2 R} \right); \\ \text{TFreeOrd} := & \frac{rb \left(-r^2 E0 rb^2 + 2 E0 R r rb - 8 \pi \epsilon r rb + 16 R \pi \epsilon - 2 \right)}{-r rb + 2 R} \end{aligned} \quad (28)$$

$$\begin{aligned}
&> \text{series}\left(\frac{rb(-r^2 E0 rb^2 + 2 E0 R r rb - 8 \pi \epsilon r rb + 16 R \pi \epsilon - 2)}{-rb r + 2 R}, r\right); \\
&\frac{rb(16 R \pi \epsilon - 2)}{2 R} + \frac{1}{2} \frac{rb(2 E0 R rb - 8 \pi \epsilon rb) + \frac{rb^2(8 R \pi \epsilon - 1)}{R}}{R} r \\
&+ \frac{1}{2} \frac{-rb^3 E0 + \frac{rb^3(2 E0 R^2 - 1)}{2 R^2}}{R} r^2 - \frac{1}{8} \frac{rb^4}{R^4} r^3 - \frac{1}{16} \frac{rb^5}{R^5} r^4 - \frac{1}{32} \frac{rb^6}{R^6} r^5 + O(r^6)
\end{aligned} \tag{29}$$

$$\begin{aligned}
&> \text{eq3} := r \left(\frac{d^2}{dr^2} g(r) \right) + 2 \frac{d}{dr} g(r) + \left(\frac{rb(16 R \pi \epsilon - 2)}{2 R} \right. \\
&+ \frac{1}{2} \frac{rb(2 E0 R rb - 8 \pi \epsilon rb) + \frac{rb^2(8 R \pi \epsilon - 1)}{R}}{R} r \\
&+ \left. \frac{1}{2} \frac{-rb^3 E0 + \frac{rb^3(2 E0 R^2 - 1)}{2 R^2}}{R} r^2 \right) \cdot g(r); \#Aproximative
\end{aligned}$$

$$\begin{aligned}
\text{eq3} := r \left(\frac{d^2}{dr^2} g(r) \right) + 2 \frac{d}{dr} g(r) + \left(\frac{rb(16 R \pi \epsilon - 2)}{2 R} \right. \\
+ \left. \frac{rb(2 E0 R rb - 8 \pi \epsilon rb) + \frac{rb^2(8 R \pi \epsilon - 1)}{R}}{2 R} r \right. \\
+ \left. \frac{\left(-rb^3 E0 + \frac{rb^3(2 E0 R^2 - 1)}{2 R^2}\right) r^2}{2 R} \right) g(r)
\end{aligned} \tag{30}$$

$$\begin{aligned}
&> \frac{rb(16 R \pi \epsilon - 2)}{2 R} + \frac{1}{2} \frac{rb(2 E0 R rb - 8 \pi \epsilon rb) + \frac{rb^2(8 R \pi \epsilon - 1)}{R}}{R} r \\
&+ \frac{1}{2} \frac{-rb^3 E0 + \frac{rb^3(2 E0 R^2 - 1)}{2 R^2}}{R} r^2 = \frac{E0 r^2}{R} - \frac{16 h^4 k \pi^2 \epsilon^2 r^2}{m^2 e^4 R} + \frac{8 \pi \epsilon r}{R} + \frac{L^2}{h^2 R}; \\
&\#Aproximation;
\end{aligned}$$

$$\begin{aligned}
&\frac{rb(16 R \pi \epsilon - 2)}{2 R} + \frac{rb(2 E0 R rb - 8 \pi \epsilon rb) + \frac{rb^2(8 R \pi \epsilon - 1)}{R}}{2 R} r \\
&+ \frac{\left(-rb^3 E0 + \frac{rb^3(2 E0 R^2 - 1)}{2 R^2}\right) r^2}{2 R} = \frac{E0 r^2}{R} - \frac{16 h^4 k \pi^2 \epsilon^2 r^2}{m^2 e^4 R} + \frac{8 \pi \epsilon r}{R} + \frac{L^2}{h^2 R}
\end{aligned} \tag{31}$$

$$\begin{aligned}
&> \text{solve} \left\{ \left[\frac{1}{2} \frac{-rb^3 E0 + \frac{rb^3(2 E0 R^2 - 1)}{2 R^2}}{R} = \frac{E0}{R} - \frac{16 h^4 k \pi^2 \epsilon^2}{m^2 e^4 R} \right. \right. \\
&\left. \left. \right. \right.
\end{aligned}$$

$$\begin{aligned} &> \text{evalf}\left(\frac{4 \pi \epsilon \hbar \text{bar}^4 + a \cdot m e^2}{32 \pi^2 \epsilon^2 \hbar \text{bar}^4}\right); \\ & \qquad \qquad \qquad 3.566211881 \cdot 10^{22} \end{aligned} \tag{37}$$

$$\begin{aligned} &> \text{evalf}\left(\frac{4 \pi \epsilon \hbar \text{bar}^4 + l \cdot (l + 1) \cdot \hbar \text{bar}^2 \cdot m \cdot e^2}{32 \pi^2 \epsilon^2 \hbar \text{bar}^4}\right); \\ & \qquad \qquad \qquad 3.566211881 \cdot 10^{22} \end{aligned} \tag{38}$$

$$\begin{aligned} &> \text{restart}; \\ &> \text{eqOnd} := \left(\frac{E0 r^2}{R} - \frac{16 h^4 k \pi^2 \epsilon^2 r^2}{m^2 e^4 R} + \frac{8 \pi \epsilon r}{R} + \frac{l \cdot (l + 1)}{R}\right) \cdot g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r)\right); \# \text{Onduscular equation.} \\ \text{eq2} := (7. \cdot 10^{-44} r^2 + 6.014324624 \cdot 10^{-33} r + 1.135135135 \cdot 10^{-20}) g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r)\right) \end{aligned} \tag{39}$$

$$\begin{aligned} &> \text{eqSchro} := \frac{rb (-r^2 E0 rb^2 + 2 E0 \cdot R \cdot r \cdot rb - 8 \cdot 3.14159 \cdot \epsilon \cdot rb \cdot r + 16 R \cdot \text{Pi} \cdot \epsilon - 2)}{-rb r + 2 R} \cdot g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r)\right); \# \text{Schrodinger scaled aproximation;} \\ \text{eq3} := \frac{5.292410932 \cdot 10^{-11} (-6.014324624 \cdot 10^{-33} r^2 + 8.409400328 r + 1.646722082 \cdot 10^{13}) g(r)}{-5.292410932 \cdot 10^{-11} r + 7.400000000 \cdot 10^{22}} \\ + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r)\right) \end{aligned} \tag{40}$$

$$\begin{aligned} &> \text{series}\left(\frac{5.292410932 \cdot 10^{-11} (-6.014324624 \cdot 10^{-33} r^2 + 8.409400328 r + 1.646722082 \cdot 10^{13})}{-5.292410932 \cdot 10^{-11} r + 7.400000000 \cdot 10^{22}}, r\right); \\ 1.177720263 \cdot 10^{-20} + 6.014324627 \cdot 10^{-33} r + 2.145571999 \cdot 10^{-75} r^2 + 1.534493068 \cdot 10^{-108} r^3 \\ + 1.097455120 \cdot 10^{-141} r^4 + 7.848896585 \cdot 10^{-175} r^5 + O(r^6) \end{aligned} \tag{41}$$

$$\begin{aligned} &> \text{eqSchro} := (2.145571999 \cdot 10^{-75} r^2 + 6.014324627 \cdot 10^{-33} r + 1.177720263 \cdot 10^{-20}) \cdot g(r) + 2 \frac{d}{dr} g(r) + r \left(\frac{d^2}{dr^2} g(r)\right); \# \text{Schrodinger scaled aproximation;} \\ \text{Ser} := 2.145571999 \cdot 10^{-75} r^2 + 1.534493068 \cdot 10^{-108} r^3 + 1.097455120 \cdot 10^{-141} r^4 \\ + 7.848896585 \cdot 10^{-175} r^5; \\ \text{Ser} := 2.145571999 \cdot 10^{-75} r^2 + 1.534493068 \cdot 10^{-108} r^3 + 1.097455120 \cdot 10^{-141} r^4 \\ + 7.848896585 \cdot 10^{-175} r^5 \end{aligned} \tag{42}$$

$$\begin{aligned} &> \text{subs}(r = rb, \text{Ser}); \\ & \qquad \qquad \qquad 6.009664236 \cdot 10^{-96} \end{aligned} \tag{43}$$

$$\begin{aligned} &> \text{Ser1} := \text{evalf}(7. \cdot 10^{-44} \cdot rb^2); \\ & \qquad \qquad \qquad \text{Ser1} := 1.960672943 \cdot 10^{-64} \end{aligned} \tag{44}$$