THE ECONOMIC IMPLICATIONS OF OUTSOURCING

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Abstract

In this paper we develop a Cournot model in which firms have the option to outsource part of their production. The model has a unique Cournot-Nash equilibrium in which partial outsourcing occurs. Outsourcing reduces costs, increases competition and may lead to higher production levels and lower per-firm profit. Distinguishing between more and less flexible firms, we then explain how under low competition outsourcing is positively related to performance while this is reversed when competition increases, a pattern that we illustrate by using the recent development of the global car industry.

Keywords: outsourcing, firm performance, flexibility, competition

JEL classification: L22, L11, D23

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1. Introduction

Outsourcing of services and manufacturing activities has truly taken off over the past one or two decades. While among management scholars some part of this increase is attributed to a loss in popularity of vertical integration with its heavy costs of bureaucracy (D'Aveni and Ravenscraft, 1994), it seems fair to say that the positive attributes of outsourcing itself are at least equally important in explaining the trend. In the management literature outsourcing is associated with a wide range of positive performance aspects like increased nimbleness, the ability to focus on the core specializations of the firm and a decrease in production cost levels (e.g. Quinn, 1999). Thus outsourcing is a key element of firms' strategies and what to outsource is an extremely important decision that trickles down into many other decision areas.

The economic literature has developed a set of useful concepts and tools to explain make-or-buy decisions, in particular transaction cost economics (TCE, see Williamson, 1985; Hennart, 1988), measurement and information approaches (Milgrom and Roberts, 1987), and resource-based explanations (Barney, 1991). TCE's central contention is that governance decisions, like the decision what to outsource, are aligned with transaction attributes, particularly asset specificity, uncertainty and frequency. Measurement approaches suggest that the degree to which the relation between inputs and outputs can be specified, determines what activities are usefully outsourced. The resource-based view of the firm focuses on the set of resources controlled by the firm and uses heterogeneity between firms in their sets as an explanatory mechanism for make-or-buy decisions.

In recent years a variety of modeling approaches have been used to describe outsourcing (Shy and Stenbacka, 2003). Most of these models, however, have focused on explaining a particular make-or-buy decision or buyer-supplier setting. While such individual transactions are important to analyze from a performance perspective, this type of analysis might not be sufficient. In particular, we contend there is also a need to analyze the overall degree of outsourcing of a firm, otherwise known as its vertical structure (McLaren, 2000). This is in line with the 'firm as a nexus of treaties' argument of TCE: the overall structure of the firm is a reflection of all of its governance and contracting choices combined. Thus we will look not at decisions on individual products but rather at vertical structures of firms. There are three primary arguments for our choice to adopt this aggregate level of analysis. First, the individual decisions may not be unrelated. For instance the outsourcing of a manufacturing activity may imply some supporting services will also be outsourced. Thus there is the possibility of economies of scope related to performing different but related activities. Second, comparing the overall level of outsourcing of one firm with that of another, or comparing various levels a firm can choose from, will lead to additional insights in the performance implications of outsourcing. Finally, it may well be the case that a firm determines its overall level of production in conjunction with its outsourcing level. If outsourcing lowers the firm's overall costs it will cause upward adjustments in its productive capacity. Since outsourcing is thought to be a competition-enhancing mechanism (Cachon and Harker, 2002) it is also likely to be found in more competitive environments. This implies that the level of outsourcing may be both a consequence of the competitive setting in which the firm operates and a predictor of production levels in an industry.

In the Grossman and Helpman (2002) model the focus is on determining when particular differentiated products ought to be supplied by one vertically integrated firm rather than two specialized producers. They argue that while specialized suppliers have lower production costs, they also face higher governance (transaction) costs. In the face of strong competition the likelihood of holdup problems and search frictions increases, such that relatively large production cost advantages are needed for an outsourcing decision to be taken. Shy and Stenbacka (2003) look at how competition in input markets helps to lower the costs of inputs to buyers by promoting the willingness of suppliers to supply at average costs. Their setting is a single component that can either be outsourced or produced in-house. Van Mieghem (1999) discusses the relation between subcontracting and production decisions, in particular the option value of varying contract modes. Nickerson and Vanden Bergh (1999) present a duopoly model based on a Cournot setting through which they investigate economizing decisions. Their primary concern is with firms' choices of asset specificity levels and governance modes.

In this paper we develop a Cournot oligopoly model that includes a firm's choice for a level of outsourcing. Our model includes both the positive and the negative aspects of outsourcing. It is also consistent with TCE reasoning and reasonably flexible in terms of defining cost and revenue functions. The exact shapes of these cost and revenue functions are determined by firm and industry specifics. The cost function is specified such that for increasingly specific assets the costs of outsourcing become higher relative to the costs of vertical integration. Thus there is less to gain from outsourcing non-standardized inputs. Similarly the revenue function is defined in our model such that the added value of integration is markedly higher near the core of the firm's activities. Thus the basic premises of the model are in line with managerial and economic logic. Our model deviates from the Nickerson and Vanden Bergh (1999) Cournot model in various important ways.

We do not investigate a single make-or-buy decision, but rather a whole series of decisions. Our model is not restricted to a duopoly setting but allows for an arbitrary number of firms. We show that our model has a unique equilibrium that is symmetric when firms are symmetric. We also allow for differentiation among firms and analyze the effects when they compete in the same market. Finally, in our model profitability is not only a function of production levels but also of outsourcing levels, which are endogenous.

AIM OF THE PAPER

The goal of this paper is threefold. First of all we develop a Cournot oligopoly model in which each firm, besides the usual strategic choice for production level, also has the option to outsource part of its production. We show that this model has a unique Cournot-Nash equilibrium and derive a characterization in terms of the underlying parameters of the model. Next we use this analysis for two purposes. First of all we offset the behavior of the firms in this equilibrium with their behavior in the corresponding classical Cournot setting. It turns out that the option to outsource creates additional competitive pressures in the industry. Firms maintain a higher production level, but nevertheless generate less profit than in the classical setting. Finally, we use our model to give an explanation of why the outsourcing - performance relation has been seen to shift from positive to negative or vice versa. When distinguishing between highly flexible firms and their less flexible competitors, it turns out that when competition within the industry is fairly low, flexible firms tend to outsource more and also generate the highest profits. However, when competition increases, less flexible firms move towards higher levels of outsourcing than flexible firms, although the latter still perform better. Thus, the relationship between outsourcing and performance is reversed for higher levels of outsourcing.

The paper starts in section two by describing the model. In the third section we determine individually optimal behavior. Section four discusses the existence and uniqueness of the Cournot-Nash equilibrium. In the fifth section we then discuss the performance effects of outsourcing levels in the model. In section six we explain how performance shifts may occur and illustrate this through the example of the world car industry. The conclusions discuss possible extensions of our model.

As a prefatory note to the remainder of the paper it is empirically highly unlikely, if not impossible, to find either full outsourcing or complete vertical integration across the range of transactions. Full outsourcing implies there is no decision-making center within the firm to take make-or-buy decisions and monitor their effects. Full vertical integration, on the other hand, seems to sometimes be legally impossible (like is the case with the legal obligation to assign external accountants to certifation of balance sheets and profit and loss statements) and in other cases highly impractical (for instance in the case of electricity or medical services supplied to the firm).

2. The model

We suppose there are n firms competing on a market for one homogeneous good. Firm i has to make two strategic choices. It has to decide which quantity q_i to produce and what level of outsourcing r_i to maintain. Thus, q_i is a non-negative real number and r_i is an element of the closed interval $[0,1]$. We assume that firms make their respective decisions simultaneously without any prior knowledge concerning the choices the other firms are about to make. In other words, we assume the firms face a pure Cournot setting in their end markets.

Given the choices $q = (q_i)_{i=1}^n$ and $r = (r_i)_{i=1}^n$ made by the firms, the profit of firm i is given by

$$\Pi_i(q,r) = \lambda_i(r_i)P(Q)q_i - c_i(r_i)q_i$$

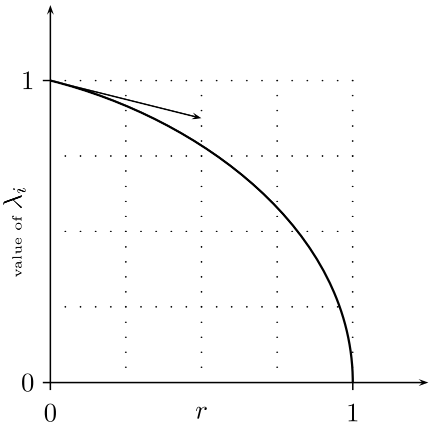
where $Q = q_1 + \cdots + q_n$ is the total quantity produced, $P(Q) = a - Q$ is the price of the good given that total production equals Q, and λ_i and c_i are exogenously determined parameter functions that govern the effect of the level of outsourcing on revenue and production costs, respectively.

THE EFFECTS OF OUTSOURCING Outsourcing is often believed to lead to lower production costs, particularly where non-specific assets are concerned (Williamson, 1985). Outsourcing may, however, also undermine output streams when opportunistic behavior is not kept in check (Williamson, 1985). We model these effects of outsourcing on performance of firms through parameter functions c_i for the effects on production costs and λ_i for the effects on output streams, respectively. In this section we will briefly discuss these parameter functions λ_i and c_i and clarify the restrictions our interpretation puts on the shape of these functions.

First we will discuss the parameter function λ_i. Given the quantities $q = (q_i)_{i=1}^n$ produced and outsourcing level r_i of firm i, we assume that the total revenue of firm i equals

$$R_i(q, r_i) = \lambda_i(r_i)P(Q)q_i.$$

Thus, for each level of outsourcing r_i the parameter $\lambda_i(r_i)$ measures the fraction of revenue that firm i obtains compared to the revenue obtained if it would not outsource. This way we lump all effects of outsourcing on revenue together and consider λ_i as the basic parameter, without any further internal structure.



Nevertheless, it is crucial to obtain a good understanding of the nature of λ_i. We suggest λ_i is an index of the outsourcability of the range of inputs required for production. We could think of r as consisting of the range of activities required to produce a product. As r increases from 0 to 1, activities will become increasingly difficult to outsource. As a simplified practical example, suppose three activities need to be undertaken for a car producer to produce its cars, which are production, strategic decision-making, and obtaining raw materials for production. A modern firm is probably unlikely to integrate backwards and produce all raw materials in-house. We would find this activity close to $r = 0$, implying the firm is probably better off by outsourcing it. Likewise it is unlikely to outsource its strategic decision-making to any outside party. This activity will be found near $r = 1$. Where production is concerned, however, the outcome may not be as clear-cut. It could either be outsourced or maintained in-house. In other words this activity is found somewhere in the middle of the range between $r = 0$ and $r =$. In reality many more activities are obviously required to produce the car. More generally the increasing bargaining power of suppliers of more specific assets and the incomplete nature of contracts with these suppliers will increase transaction costs as r approaches 1. Also note that loss of quality will grow even in terms of percentages as the level of outsourcing increases, because activities closer to the firm's core, that is activities closer to $r = 1$, provide more potential economies of scope. Thus it is reasonable to assume that costs of outsourcing will be at least linear in the level of outsourcing and we assume λ_i to be decreasing and concave, that $\lambda_i'(0) < 0$, and that $\lambda_i'(r_i) \to -\infty$ as $r_i \to 1$. This assumption is consistent with Williamson's (1991) graphical display of the advantages of markets and hierarchies.

Concerning the cost function $c_i \colon [0, 1] \to \mathbb{R}_+$ that specifies the effect of outsourcing on the in-house marginal production cost, we will assume that it is a decreasing and convex function. Again this is consistent with the manner in which Williamson (1991) has portrayed it. Moreover, we will employ the functional form

$$c_i(r_i) = d_i(r_i) \cdot \lambda_i(r_i)$$

where $d_i\colon [0,1] \to \mathbb{R}_+$ is decreasing and convex, and $d_i'(r_i) \to -\infty$ as $r_i \to 0$. These assumptions are the technical equivalents of the assumptions that the effect of outsourcing on cost reduction dwindles as the level of outsourcing goes up, and that the cost reduction of outsourcing initially outweighs the effect it has on revenue. Furthermore, we assume that $d_i(0) \leq a$ and that $d_i(1) = 0$.

The function d_i does in fact have a very natural interpretation, namely as being the cost level relative to the level of λ_i. Just like we can write the profit function of firm i in the classical oligopoly model as $\Pi_i = (P(Q) - c_i)q_i$, we can write firm i's profit function as

$$\Pi_i(q,r) = \lambda_i(r_i)\big(P(Q) - d_i(r_i)\big)q_i$$

in our model. Thus, just like the factor $P(Q) - c_i$ measures the marginal profit of an extra unit of production in the oligopoly model, the factor $P(Q) - d_i(r_i)$ is a measure for the marginal profit at a fixed level r_i of outsourcing activity.

Finally notice that, even if the current price is zero (or even negative if we would allow that) firm i could choose to produce, and still guarantee itself a zero profit by setting $r_i = 1$. In other words, full outsourcing of all of firm i's activities would be optimal. It is, however, very difficult to imagine what a firm that outsources *all* its activities would look like. It would be an empty shell, possessing no value added, even to the extent that for all purposes it might as well not exist. Therefore we make the additional assumption that the choice $r_i = 1$ is not feasible for firm i. Nevertheless, for ease of exposition, we will sometimes consider the values of the parameter functions λ_i, c_i and d_i at outsourcing level $r_i = 1$.

3. Optimal behavior

Before we turn to equilibrium considerations in the next section, we will first systematically analyze the best response behavior of a firm. We will derive a full characterization of firm i's best response to an arbitrary profile of decisions of the other firms.

In order to do this, let $(q_j, r_j)_{j \neq i}$ be an arbitrary fixed profile of strategy decisions of the other firms. Write $Q_{-i} = \sum_{j \neq i} q_j$. First we have the following simple observation.

Proposition 1. *Suppose that $a - Q_{-i} \leq 0$. Then any pair of decisions of the form $(0, r_i)$ with $0 \leq r_i < 1$ is a best response for firm i.*

Proof. Notice that, given the decisions $(q_j, r_j)_{j \neq i}$, the profit function is given by

$$\Pi_i(q_i, r_i) = \lambda_i(r_i)\big(a - Q_{-i} - q_i - d_i(r_i)\big)q_i.$$

Now notice that, since $r_i = 1$ is not a feasible choice, $\lambda_i(r_i) > 0$ for any r_i. Furthermore, since $a - Q_{-i} \leq 0$ and $d_i(r_i) > 0$, the second factor $a - Q_{-i} - q_i - d_i(r_i)$ is strictly smaller than zero.

Hence, the only optimal choice for q_i is $q_i = 0$. Given, this choice, the outsourcing level can be chosen arbitrarily. $\triangleleft$

Now suppose that $a - Q_{-i} > 0$. In this case we will show that there exists a unique best response in which firm i produces, and also outsources a positive fraction of its production. To see why this is so, first notice that proposition 7 in the appendix states that boundary points of the feasible region are not optimal in this case. Thus a strategically optimal choice (q_i, r_i) must satisfy the first order conditions

$$\begin{cases} \dfrac{\partial \Pi_i}{\partial q_i} = \lambda_i(r_i)\big(a - Q_{-i} - 2q_i - d_i(r_i)\big) = 0 \\ \dfrac{\partial \Pi_i}{\partial r_i} = \Big[\lambda_i'(r_i)\big(a - Q_{-i} - q_i - d_i(r_i)\big) - \lambda_i(r_i)d_i'(r_i)\Big]q_i = 0, \end{cases}$$

which is equivalent to

$$\begin{cases} r_i = 1 & \text{or} & 2q_i = a - Q_{-i} - d_i(r_i) \\ q_i = 0 & \text{or} & a - Q_{-i} - q_i = d_i(r_i) + \frac{\lambda_i(r_i)d_i'(r_i)}{\lambda_i'(r_i)} \end{cases}$$

Now $r_i = 1$, full outsourcing of activities, is not feasible for firm i, and $q_i = 0$ is not optimal by proposition 7. So, writing

$$f_i(r_i) := \frac{\lambda_i(r_i)d_i'(r_i)}{\lambda_i'(r_i)}$$

and $Q = q_1 + \cdots + q_n$, it follows that *any* best response (q_i, r_i) will necessarily satisfy

$$(*) \qquad \begin{cases} q_i = f_i(r_i) \\ a - Q = d_i(r_i) + f_i(r_i). \end{cases}$$

In words, a firm chooses its optimal plan (q_i, r_i) in such a way that (1) its production level q_i equals $f_i(r_i)$, and (2) the price level $P = a - Q$ will be equal to $d_i(r_i) + f_i(r_i)$. But the reverse statement is also true. However, since the profit function $\Pi_i(q_i, r_i)$ is not concave, the proof of this statement requires some effort.

Proposition 2. *A solution to* $(*)$ *is a best response to the choices* $(q_j, r_j)_{j \neq i}$ *of the other firms.*

Proof. Let (q_i^*, r_i^*) be a solution to the system $(*)$. First notice that this automatically implies that $a - Q_{-i} = d_i(r_i^*) > 0$. We will show that (q_i^*, r_i^*) maximizes the profit function

$$\Pi_i(q_i, r_i) = \lambda_i(r_i)\big(a - Q_{-i} - q_i - d_i(r_i)\big)q_i.$$

Now, given an arbitrary level r_i of outsourcing, the profit function is quadratic in the quantity q_i. So, it is clear that the decision to produce

$$q_i(r_i) = \frac{1}{2}(a - Q_{-i} - d_i(r_i)) \vee 0$$

maximizes the profit function, given the fixed level r_i of outsourcing. Define

$$H_i(r_i) := \frac{1}{4}\lambda_i(r_i)\big(a - Q_{-i} - d_i(r_i)\big)^2.$$

We need to show that r_i^* maximizes the function

$$\Pi_i(q_i(r_i), r_i) = \begin{cases} H_i(r_i) & \text{if } a - Q_{-i} \geq d_i(r_i) \\ 0 & \text{else.} \end{cases}$$

To this end, notice that the first order condition $H_i'(r_i) = 0$ yields

$$(a - Q_{-i} - d(r_i))\left[\lambda'(r_i)(a - Q_{-i} - d(r_i)) - 2d'(r_i)\lambda(r_i)\right] = 0.$$

This is equivalent to the requirement that r_i satisfies one of the two equations

$$a - Q_{-i} = d_i(r_i)$$

and

$$a - Q_{-i} = d_i(r_i) + 2f_i(r_i).$$

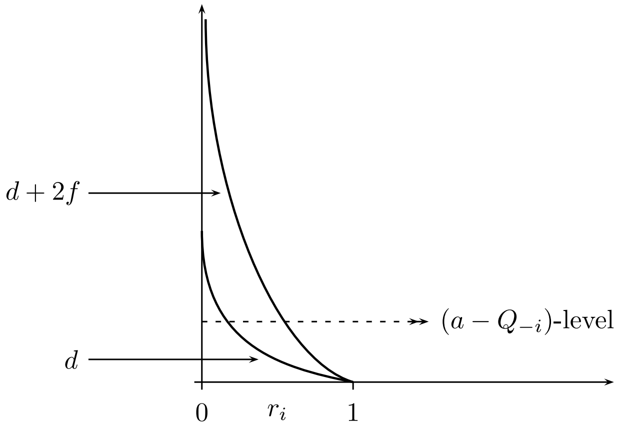
Now, since (q_i^*, r_i^*) is a solution to $(*)$, we know that $q_i^* = f_i(r_i^*)$. Thus, from the second equation in $(*)$ we get that

$$d_i(r_i^*) + f_i(r_i^*) = a - Q_{-i} - q_i^* = a - Q_{-i} - f_i(r_i^*)$$

which shows that r_i^* does indeed satisfy the equation

$$a - Q_{-i} = d_i(r_i) + 2f_i(r_i).$$

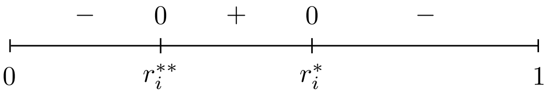
We will show that the solution to this equation always maximizes H_i.



By proposition 6 and the assumptions concerning d it is clear that the graphs of d and $d + 2f$ look as depicted above. Both are positive and decreasing functions, and $d + 2f$ tends to infinity close to zero. Thus, since we assumed that $a - Q_{-i} > 0$ there are two possibilities.

A. Only the second equality has a (unique) solution. In this case $a - Q_{-i} > d(r_i)$ everywhere. From this it easily follows that H_i is increasing on the interval $[0, r_i^*)$ and decreasing on $(r_i^*, 1)$. Hence, r_i^* maximizes H_i.

B. The first equation also has a solution, say r_i^{**}. Clearly, $r_i^{**} < r_i^*$. It is straightforward to check that the sign schedule of H_i' looks as follows.



So, the maximum is attained either in r_i^* or in $r_i = 0$. However, in this case $a - Q_{-i} - d_i(0) \leq 0$, so the maximum payoff for $r_i = 0$ is zero. While

$$H_i(r_i^*) = \frac{1}{4}\lambda_i(r_i^*)\big(a - Q_{-i} - d_i(r_i^*)\big)^2 = \frac{1}{4}\lambda_i(r_i)\big(2f_i(r_i^*)\big)^2 > 0.$$

Hence, the maximum is attained in r_i^*. $\triangleleft$

This shows that there is a single optimum outsourcing level that neither requires full outsourcing nor full integration but rather maintains some intermediate value of r. Thus in equilibrium firms automatically outsource some but not all of their activities.

4. Existence and uniqueness of Cournot-Nash equilibrium

In this section we will show that our model has a unique Cournot-Nash equilibrium. We will also briefly discuss the symmetric case and show in an example how the equilibrium can be computed in that case. First notice that from proposition 2 and its preceding discussion we can immediately conclude the following statement.

Theorem 1. *A strategy profile $(q_i, r_i)_{i=1}^n$ is a Cournot-Nash equilibrium if and only if it satisfies the system of equalities*

$$(*) \quad \begin{cases} q_i = f_i(r_i) \\ a - Q = d_i(r_i) + f_i(r_i) \end{cases}$$

for all i.

We will explain why this system has a unique solution. Consider the function F from the open interval $(0, a)$ to the collection of positive reals constructed in the following way. Take a number Q in $(0, a)$. By proposition 6 we know that $d_i + f_i$ is a continuous and strictly decreasing function whose range is $(0, \infty)$. So, there is a unique number $R_i(Q) \in (0, 1)$ such that

$$a - Q = d_i(R_i(Q)) + f_i(R_i(Q)).$$

Furthermore, since $d_i + f_i$ is continuous and decreasing, we know that R_i, viewed as a function of Q, is continuous and increasing in Q. Now define

$$F(Q) := f_1(R_1(Q)) + \cdots + f_n(R_n(Q)).$$

Clearly F is continuous. Furthermore, since all functions R_i are increasing, and all functions f_i are decreasing, F is (strictly!) decreasing in Q. Moreover, from proposition 6 we know that $F(Q) \to 0$ as $Q \to a$. Hence, there exists a unique $Q^* \in (0, a)$ such that $F(Q^*) = Q^*$. Now, from the construction it is immediately clear that $(q_i^*, r_i^*)_{i=1}^n$ with

$$r_i^* := R_i(Q^*)$$

and

$$q_i^* := f_i(r_i^*)$$

is the unique solution of the system $(*)$. Hence, by theorem 1 it is also the unique Cournot-Nash equilibrium of our model.

Corollary 1. *The equilibrium price* $P(Q^*) = a - Q^*$ *is strictly larger than zero and strictly smaller than* a.

Proof. Follows immediately from the fact that the domain of F is the open interval $(0, a)$. ◁

Thus far we have shown that in general there exists a unique Cournot-Nash equilibrium, and we have derived a system of equations that fully determine the unique equilibrium. However, due to the non-linearity of this system it is not clear how, if at all, this system can be solved. Nevertheless, as we will argue now, in the symmetric case we can take our computations one step further.

THE SYMMETRIC CASE Suppose that all characteristics are the same for all firms. So, for all firms i we have $d_i = d$ and $\lambda_i = \lambda$ for certain functions d and λ. Then, since in this case also all functions f_i and therefore also all functions R_i are identical, we find that all equilibrium outsourcing levels r_i^* and also all equilibrium output levels q_i^* are identical across firms. Hence, the equilibrium is of the form $(q^*, r^*)_{i=1}^n$.

Once we have observed this, it is easy to see that a strategy profile $(q, r)_{i=1}^n$ is a Cournot-Nash equilibrium if and only if it satisfies the pair of equalities

$$(**) \quad \begin{cases} q = f(r) \\ a - nq = d(r) + f(r) \end{cases}$$

or, alternatively,

$$(***) \quad \begin{cases} q = f(r) \\ a - d(r) = (n+1)f(r). \end{cases}$$

In this system the second equation only depends on the level r of outsourcing activities. Once this equation is solved, the corresponding equilibrium level of production is simply given by the first equation. Thus, solvability of this system only depends on the question whether we can solve the second equation. The next example shows that sometimes the second equation can indeed be solved.

Example 1. Using the above characterization of the Cournot-Nash equilibrium for the symmetric situation we can compute the equilibrium as follows. Take for example $\lambda(r) = \sqrt{1-r}$ and $d(r) = 1 - \sqrt{r}$. Since in this case $\lambda'(r) = \frac{-1}{2\sqrt{1-r}}$ and $d'(r) = \frac{-1}{2\sqrt{r}}$, we find that the equilibrium condition

$$a - d(r) = (n+1)f(r)$$

from $(***)$ yields

$$\frac{a - 1 + \sqrt{r}}{n+1} = \frac{1-r}{\sqrt{r}}.$$

Clearly the left-hand expression is positive and increasing in r, while the right-hand expression is decreasing and maps the interval $(0,1)$ onto the entire positive halfline $(0,\infty)$. Thus there must be a unique solution $0 < r < 1$ to the above equation. This becomes even more obvious upon realizing that the above equation can be rewritten to

$$(a-1)\sqrt{r} = (n+1) - (n+2)r.$$

A straightforward computation shows that the solution to this equation is

$$\begin{aligned} r^* &= \left(\frac{-(a-1) + \sqrt{(a-1)^2 + 4(n+1)(n+2)}}{2(n+2)} \right)^2 \\ &= \frac{n+1}{n+2} + \frac{a-1}{2(n+2)^2} \left(a - 1 - \sqrt{(a-1)^2 + 4(n+1)(n+2)} \right) \end{aligned}$$

which is indeed a number strictly between zero and one. The corresponding equilibrium quantity q^* can now be computed using the equilibrium condition $q^* = f(r^*)$. ◁

5. A comparison with the Cournot model

In this section we will compare equilibrium behavior of firms in our model with their equilibrium behavior in the Cournot oligopoly model without the option to outsource part of the production. In fact we will discuss a few intuitive results that follow directly from our model. These results thus confirm the reliability of the conclusions that can be drawn from the model.

First consider the effect of outsourcing when we compare the equilibrium with the benchmark oligopoly situation in the classical Cournot model without outsourcing (which is equivalent to setting $r_i = 0$ in our model). The equilibrium quantity in the classical model is equal to

$$q^e = \frac{1}{n+1}(a - d(0)) = \frac{1}{n+1}(a - c(0))$$

which is a smaller quantity than the equilibrium quantity

$$q^* = \frac{1}{n+1}(a - d(r^*))$$

produced when outsourcing is allowed. Moreover, as we will see in the next example, also the per-firm profit may be reduced when outsourcing is an option.

Example 2. In the classical setting, as is well known, the per-firm profit is equal to

$$\Pi^e = (q^e)^2 = \left(\frac{a - c(0)}{n+1}\right)^2$$

while for our setting we find that the equilibrium per-firm profit Π^* equals

$$\Pi^* = \Pi_i(q^*, r^*) = \frac{1}{4}\lambda(r^*)\big(a - Q^*_{-i} - d(r^*)\big)^2 = \lambda(r^*)\,(q^*)^2 = \lambda(r^*)\left(\frac{a - d(r^*_i)}{n+1}\right)^2.$$

Thus,

$$\Pi^* = \lambda(r^*)\left(\frac{a - d(r^*)}{a - c(0)}\right)^2 \cdot \Pi^e.$$

Taking λ and d as in example 1, we know that $r^* \approx \frac{n+1}{n+2}$ when the number of firms is sufficiently high. Thus, since $\lambda(r)$ and $d(r)$ both converge to zero as $r \to 1$, we get that the factor

$$\lambda(r^*)\left(\frac{a - d(r^*)}{a - c(0)}\right)^2$$

goes to zero in this case, and $\Pi^* < \Pi^e$ when competition is sufficiently intense. $\triangleleft$

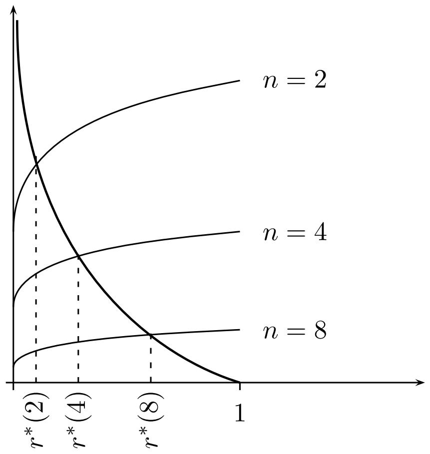
This is even true in general: when competition is sufficiently intense (so, n is sufficiently large) the per-firm profit Π^* in the presence of outsourcing is less than the per-firm profit Π^e in the classical Cournot model. Outsourcing implies firms transfer part of their profits to outside suppliers. In cases of sufficient competition this transfer will involve substantial amounts. To see this, notice that $\Pi^* < \Pi^e$ exactly when

$$\lambda(r^*)\left(\frac{a - d(r^*)}{a - c(0)}\right)^2 < 1.$$

Now write $r^*(n)$ for the equilibrium level of outsourcing when n is the number of firms. Recall that $r^*(n)$ is the unique solution to the equation

$$\frac{a - d(r)}{n+1} = f(r).$$

Graphically this looks as follows.



The function $r \mapsto \frac{a-d(r)}{n+1}$ is increasing in r. However, if n increases, the graph of this function will be closer to the horizontal axis. Thus, since f is a decreasing function, the point of intersection $r^*(n)$ of the two curves will move towards $r = 1$. Consequently the factor

$$\lambda(r^*(n)) \left(\frac{a - d(r^*(n))}{a - c(0)} \right)^2$$

will converge to zero as n becomes large. Hence, also in general outsourcing will have a negative effect on the equilibrium per-firm profit when competition is intense.

Although outsourcing initially reduces production costs it also tends to enlarge competition. Firms have lower costs when they outsource part of their production, which encourages them to increase production. The increase in production lowers the price in equilibrium which offsets the initial gain of production cost reduction.

6. A possible explanation for performance switches

In this section we will discuss why it is feasible that outsourcing is sometime positively and sometimes negatively related to performance of the firm (Capon, Farley, and Hoenig, 1990). The basis for the explanation lies in the effect we observed in the previous section. Consider the differences between US and Japanese automobile producers as they are generally conceived. US producers have become known for being proficient in mass production (Chandler, 1964) while their Japanese counterparts, Toyota in particular, are known for flexibility in their lean production systems (Womack, Roos and Jones, 1990). Toyota has demonstrated an ability to create competitive advantage by building long-term relations with key suppliers such as Nippondenso. In addition it relies much more on the teamwork model where US firms tend to display very high degrees of specialization in the form of an extensive division of labor. As we will see, firms with higher flexibility levels –in our context firms for which outsourcing results in a relatively large cost reduction initially – will, when competition is low, produce more and also outsource a larger fraction of their production given the assumed cost pattern. Thus, since the more flexible firms will also have higher profits, performance will be positively correlated to level of outsourcing. However, when competition in the market increases (either by an increase in the number of firms or by a reduction of the price intercept), high asset specificity firms will decrease their outsourcing level, even below the equilibrium level maintained by low asset specificity firms. This means a reversal of the impact of the level of outsourcing on performance and we get a negative correlation between these two indicators. Our explanation is that in such highly competitive markets flexibility helps firms to differentiate themselves from competitors much like it has been ascribed to provide Toyota with a competitive advantage (Womack et al, 1990). Note that although cars are not a homogeneous product in the strictest sense of the Cournot model, they come pretty close nowadays. Car producers produce fairly similar models but apply different sets of resources to do so. The fact that a homogeneous product is being produced does not quantitate qua require a single way of producing that product. Thus it is feasible to speak of less and more flexible firms while using a Cournot model as we do here.

In order to see how performance switches can occur, suppose there are n firms that can be of two types. Type 1 firms are supposed to have a low flexibility (LF). Their cost structure is given by λ and d_{lf}. Type 2 firms are supposed to have a high flexibility (HF). The effects of outsourcing on the cost characteristic d_{hf} for HF firms is supposed to be more dramatic than it is for LF firms. We model this by the assumption that

$$d_{hf}(r) < d_{lf}(r)$$

for all $r > 0$. For simplicity we assume that $d_{hf}(0) = d_{lf}(0)$ and that the λ-characteristic is the same for both types of firms. We assume further that there are $K(n)$ LF firms in the market. The number of HF firms is denoted by $L(n)$. So, $K(n) + L(n) = n$. Moreover, we will assume that both K and L are increasing in n.

ASSUMPTION (single crossing) For ease of exposition we will also assume that $d_{lf} + f_{lf}$ and $d_{hf} + f_{hf}$ only cross once. More specifically, there exists an r_* such that $d_{hf}(r) + f_{hf}(r) > d_{lf}(r) + f_{lf}(r)$ for all $r < r_*$, and $d_{hf}(r) + f_{hf}(r) < d_{lf}(r) + f_{lf}(r)$ for all $r > r_*$. (1)

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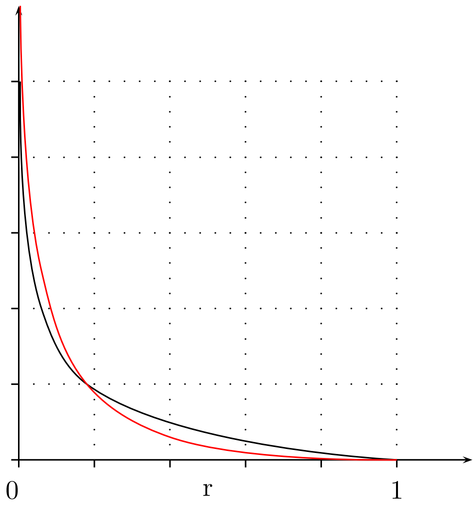
(1) This assumption is not essential to the conclusion. It does make the analysis more straightforward though. With the assumption the two predominant market structures we will identify in this section are located at two different intervals. Without the above assumption they might alternate more often.

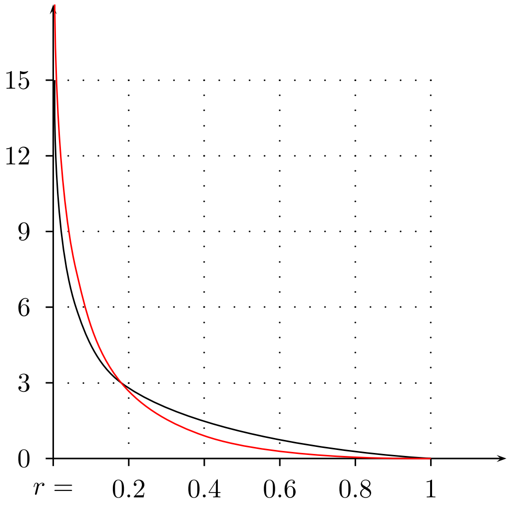


The point r_* where $d_{lf} + f_{lf}$ and $d_{hf} + f_{hf}$ intersect will be called *the crossover level.*

Example 3. In order to show that this is not really a very stringent assumption, and to get some feeling for what it implies, suppose that LF firms have characteristics $d_{lf}(r) = 1 - \sqrt{r}$ and $\lambda(r) = \sqrt{1-r}$, and HF firms have characteristic $d_{hf}(r) = d_{lf}^2(r) = (1-\sqrt{r})^2$ while their λ is identical to the LF characteristic. It is easy to check that

$$d_{lf} + f_{lf} = (1-\sqrt{r})\left(\frac{1+2\sqrt{r}}{\sqrt{r}}\right) \quad \text{and that} \quad d_{hf} + f_{hf} = (1-\sqrt{r})\left(\frac{2+\sqrt{r}-3r}{\sqrt{r}}\right)$$

It is straightforward to check that these two curves do indeed have a single point of intersection, namely at $r_* = \frac{1}{36}(\sqrt{13}-1)^2 \approx 0.1886$. Thus in this case the respective graphs of $d_{lf} + f_{lf}$ (in black) and $d_{hf} + f_{hf}$ (in gray) look as follows



So, we do have single crossing in this example and the crossover level r_* equals $\frac{1}{36}(\sqrt{13}-1)^2$. ◁

From the equilibrium conditions $(*)$ in theorem 1 it is clear that firms of the same type employ the same level of outsourcing, and hence also the same output level in equilibrium. Let (r^*_{lf}, q^*_{lf}) be the equilibrium strategy of a LF firm, and (r^*_{hf}, q^*_{hf}) the equilibrium strategy of a HF firm. Write Π^*_{lf} for the profit in equilibrium of a LF firm and Π^*_{hf} for a HF firm.

Proposition 3. *Suppose that* $q^*_{hf} > q^*_{lf}$*. Then also* $\Pi^*_{hf} > \Pi^*_{lf}$.

Proof. Let Q^* denote the total quantity produced in equilibrium. Write $Q^*_{-lf} = Q^* - q^*_{lf}$ and $Q^*_{-hf} = Q^* - q^*_{hf}$. Then, since $q^*_{hf} > q^*_{lf}$, it is clear that $Q^*_{-lf} > Q^*_{-hf}$. So, since also $d_{lf}(r) \geq d_{hf}(r)$ for any r, we get for any pair (q, r) with $q > 0$ that

$$\begin{aligned}\Pi_{lf}(q,r) &= \lambda(r)\big(a - Q^*_{-lf} - q - d_{lf}(r)\big)q \\ &< \lambda(r)\big(a - Q^*_{-hf} - q - d_{hf}(r)\big)q \\ &= \Pi_{hf}(q,r).\end{aligned}$$

Hence a best response will certainly give a HF firm a higher profit than a LF type firm. ◁

Proposition 4. *Suppose that* $r^*_{hf} \geq r^*_{lf}$*. Then* $q^*_{hf} > q^*_{lf}$.

Proof. From $r^*_{hf} \geq r^*_{lf}$ we get that

$$d_{hf}(r^*_{hf}) < d_{lf}(r^*_{hf}) \leq d_{lf}(r^*_{lf}).$$

Hence, by the equilibrium condition $q^*_i = a - Q^* - d_i(r^*_i)$, we get that

$$q^*_{hf} = a - Q^* - d_{hf}(r^*_{hf}) > a - Q^* - d_{lf}(r^*_{lf}) = q^*_{lf}. \qquad ◁$$

Given this proposition, we see that there are basically three qualitatively different market structures in equilibrium, namely

(1) the market structure in which $r^*_{hf} \geq r^*_{lf}$ and $q^*_{hf} > q^*_{lf}$, which we will call MS I,
(2) the one in which $r^*_{hf} < r^*_{lf}$ and $q^*_{hf} > q^*_{lf}$, called MS II, and
(3) the one in which $q^*_{hf} \leq q^*_{lf}$ (and automatically $r^*_{hf} < r^*_{lf}$), called MS III.

From the above two propositions it is clear that in MS I a HF type firm has the higher level of outsourcing and also the higher level of performance. Historically the big three US car manufacturers (GM, Ford, and Chrysler) have operated highly vertically integrated production systems (Chandler, 1964). At the same time these firms were pretty much shielded from competition since foreign firms had not entered the US market at any scale while domestic competition was absent. Japanese firms, on the other hand, had already been using their cooperative Keiretsu system for some time (Nishiguchi, 1994), outsourcing substantial amounts of production to affiliated and trusted outside suppliers. In MS II, although the HF type firms still perform better, they have a lower level of outsourcing than the LF type firms due to the pressure to alleviate competition and decrease total output. This resembles the recent history of the car industry and perhaps its present as well. During the 1980s and 1990s US car manufacturers embarked on large-scale outsourcing of manufacturing operations in an effort to improve efficiency (see amongst others Kotabe, 1998, for a sketch). They partly did so in a belief that Japanese firms had benefited from higher outsourcing levels. In the meantime competition in their home market in North America had increased substantially with the entry of not just Japanese but also European competitors. Given this increase in competition it makes much sense to observe more outsourcing in this market, as we argued before. But in the end the US producers still found themselves being outcompeted by some of the foreign competition. Our findings here seem to suggest that this is due to the structurally different characteristics of some of these competitors that imply they need not outsource as much as their US counterparts under current highly competitive conditions and can still be more profitable. In MS III the situation is less clear-cut. Although it seems plausible that even in this situation the HF type firms perform better, we were not able to demonstrate this. Nevertheless, we are still able to make our argument by focusing only on market structures MS I and MS II and by showing that these two structures are quite often prevalent (in some models that fall within our framework MS III does not occur at all). We will try to explain our line of reasoning.

In which market structure a specific industry finds itself in equilibrium is governed by two exogenous parameters, namely the price intercept a and the total number of firms n. In order to determine exactly when these various market structures occur, let $(q^*_{lf}(n,a), r^*_{lf}(n,a))$ denote the equilibrium strategy of a LF type firm, and $(q^*_{hf}(n,a), r^*_{hf}(n,a))$ the equilibrium strategy of a HF type firm.

We can show the following facts. Let r_* be the crossover level, and let q_{**} be the lowest production level for which

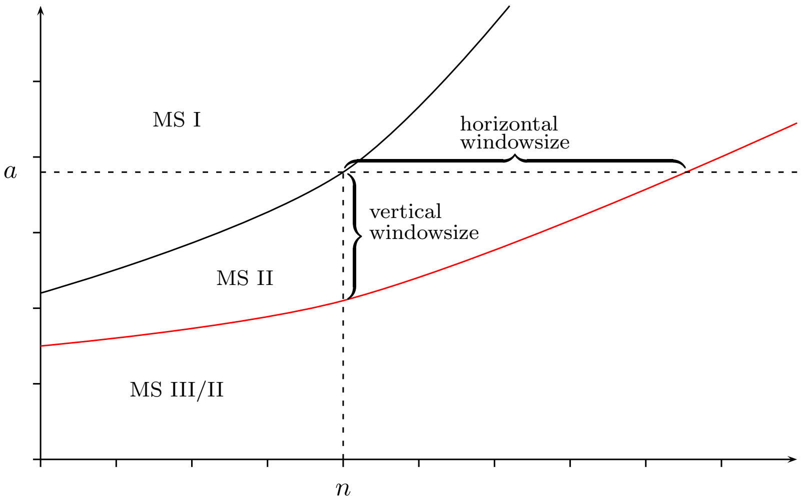
$$q^*_{hf}(n,a) = q^*_{lf}(n,a),$$

that is, the lowest level of q for which MS III is realized. Consider the figure depicted below. The upper graph depicts the division line between market structures MS I and MS II. So, it is the set of parameter settings (n,a) for which

$$r^*_{lf}(n,a) = r_* = r^*_{hf}(n,a).$$

The lower graph depicts the division line between MS II and MS III (MS II may occur below this line as well, but it is the first time MS III occurs). It is the set of parameter settings for which

$$f_{lf}(r^*_{lf}(n,a)) = q_{**} = f_{hf}(r^*_{hf}(n,a)).$$



In appendix B it is shown that the upper graph is monotonically increasing. Moreover, in appendix C it is shown that the horizontal as well as the vertical distance (the “windows” depicted in the figure) between both graphs tend to infinity when the number of firms becomes large. This shows that both market structures MS I and MS II are frequently prevalent. Indeed, as example 4 below shows, in many models MS III never occurs.

All in all, this seems to offer a plausible explanation for the underlying reasons why US car manufacturers were never able to outperform their Japanese competitors even in the face of serious efforts on the part of the US manufacturers to decrease their production costs by means of large-scale outsourcing of operations. US manufacturers traditionally employed, and still employ, less flexible production methods than their Japanese counterparts which as our model shows can only lead to a better performance for the US manufacturers under extremely competitive circumstances, in which case profit margins are in absolute terms obviously already modest in the first place.

Example 4. In this example we will show why MS III does not always occur. First notice that on the boundary between the MS II and MS III/II regions the equations

$$f_{lf}(r_{lf}) = f_{hf}(r_{hf})$$
$$d_{lf}(r_{lf}) = d_{hf}(r_{hf})$$

necessarily hold, where the second equation simply follows from the first equation above combined with the second equilibrium equation from the system $(*)$ in theorem 1. Consider the model discussed in examples 1 and 3. Writing $r_h := r_{lf}$ and $r_l := r_{hf}$ the equations in this case are

$$\begin{aligned} 1-\sqrt{r_h} &= (1-\sqrt{r_l})^2 \\ \frac{1-r_h}{\sqrt{r_h}} &= 2(1-\sqrt{r_l})\frac{1-r_l}{\sqrt{r_l}}. \end{aligned}$$

Now notice that we can use the first equation to simplify the second one to

$$\frac{1+\sqrt{r_h}}{\sqrt{r_h}} = 2\frac{1+\sqrt{r_l}}{\sqrt{r_l}}.$$

From this we get that

$$\begin{gathered}\frac{1+\sqrt{r_h}}{\sqrt{r_h}} = 2\frac{1+\sqrt{r_l}}{\sqrt{r_l}} \\ \Leftrightarrow \\ \frac{1}{\sqrt{r_h}} = 1 + \frac{2}{\sqrt{r_l}} \\ \Leftrightarrow \\ \sqrt{r_h} = \frac{1}{1+\frac{2}{\sqrt{r_l}}} \\ \Leftrightarrow \\ \sqrt{r_h} = \frac{\sqrt{r_l}}{\sqrt{r_l}+2}\end{gathered}$$

On the other hand, from the first equation we can easily deduce that

$$\sqrt{r_h} = 2\sqrt{r_l} - r_l.$$

However, since

$$\sqrt{r_l} < 4\sqrt{r_l} - r_l\sqrt{r_l} = (2+\sqrt{r_l})(2\sqrt{r_l} - r_l)$$

for all $0 < r_l < 1$, we get that

$$\frac{\sqrt{r_l}}{\sqrt{r_l}+2} < 2\sqrt{r_l} - r_l$$

which shows that the above equalities cannot be satisfied simultaneously. Hence, MS III does not occur in this model, and we will only observe MS I and MS II. ◁

Remarks. When there are more than two types, a similar result can be obtained. What will make the result less clear-cut in this setting is that the crossover points of different types of firms need not coincide. Nonetheless, when enough firms are in MS II in a pairwise comparison, the trend in a plot of outsourcing level versus performance will still be significantly downwards. ◁

Conclusions

We have presented a Cournot model with the option to outsource some of the firm's activities. The optimal behavior of firms was described. The outsourcing optimum is never located at one of the two extremes, complete outsourcing or full integration, but rather somewhere in between. As more firms compete in a market, it becomes beneficial to outsource more. For monopolists outsourcing is generally a less preferred option. The shapes of cost and revenue functions are both determined by average transaction characteristics. We have demonstrated that differences in such shapes, for instance when the level of flexibility of one industry is higher than that of another industry, imply differences in the optimal degree of outsourcing. We have also shown the presence of a Cournot-Nash equilibrium in our model. Then we allowed for differences in the cost functions of competing firms, by introducing producers with different levels of flexibility. While some firms seek to specialize their resources towards a limited set of markets, others will want to create a wider range of alternative uses. We showed how initially highly flexible firms outsource more than their counterparts in equilibrium but also how there is a switching point, dependent on the market size and number of competitors, where this situation is reversed. We also showed that this reversal in level of outsourcing is not necessarily, and even quite often not, accompanied by a reversal of performance. The more flexible firm will usually keep its competitive edge even when less flexible firms are driven to a high degree of outsourcing of operations.

There are many inputs that are natural make or natural buy decisions. For instance most organizations are very unlikely to produce their own electricity, given the associated costs of misalignment. Therefore for these types of decisions the penalty for wrong decision-making is much more severe. From an economic point of view these are probably less interesting decisions to investigate at the activity level since firms will not normally make such mistakes. Indeed, most of the literature seems to focus on decisions that could end in either make or buy. Yet these decisions do form part of a firm's overall vertical structure. Our model includes such 'obvious' decisions as well as the less obvious ones.

Furthermore the model presented here is reasonably flexible. We can accommodate how technological change affects the choice for a level of outsourcing. For instance in the IT outsourcing area it has been suggested that the introduction of electronic markets might increase the relative attractiveness of outsourcing (Malone, Yates and Benjamin, 1988). A similar effect might occur when the level of institutional voids in a country decreases due to liberalization of the economy or improved property rights regimes. Another possible effect our model could capture is the occurrence of internal or external learning. A final future extension of this model might focus on introducing additional options such as firms differing not only in their cost functions but also in their revenue functions due to differences in their underlying resources. Or one might introduce, along the lines of some recent literature on subcontracting (van Mieghem, 1999), differing contract types that refine contracting beyond make or buy. For instance it is conceivable to introduce into the current model a third type of contracting, a buyer-supplier alliance, with differing characteristics. In addition we would like to stress the value of empirical investigations that can corroborate the existence of an optimum level of outsourcing among firms. From a decision-maker's perspective the current paper serves as a confirmation of existing intuitions. It helps to demonstrate the presence of optimal strategies and the implications of differing transaction characteristics and competitive settings on outsourcing strategy. This can perhaps best be phrased as the balancing of complexity and flexibility. While outsourcing increases the number of external suppliers firms deal with, thus raising coordination costs, it also improves firms' abilities to switch between suppliers and allows for more frequent reconfigurations of supply chains. In current times this offers much needed potential for differentiation from competitors. Appropriate outsourcing will facilitate decision-making elsewhere in the firm.

Appendix A.

This appendix contains some of the technical proofs of statements in this paper.

Proposition 5. *On the open interval* $(0, 1)$ *the function* $r \mapsto d'(r)\lambda(r)$ *is strictly increasing.*

Proof. We will show that $(d' \cdot \lambda)' > 0$ on the open interval $(0, 1)$. To this end notice that for any $0 < r < 1$,

$$(d' \cdot \lambda)' = d'' \cdot \lambda + d' \cdot \lambda'.$$

Now the assertion follows from the observation that d is decreasing and convex on $(0, 1)$, and λ is positive and decreasing on this interval. ◁

Moreover, a useful tool in the following analysis is the function

$$f(r) := \frac{d'(r)\lambda(r)}{\lambda'(r)}$$

defined for all $0 < r < 1$. We will frequently use the following facts about this function.

Proposition 6. *The function* f *is decreasing. Furthermore,* $f(\varepsilon) \to \infty$ *as* $\varepsilon \to 0$ *and* $f(1-\varepsilon) \to 0$ *as* $\varepsilon \to 0$.

Proof. First we will show that $f'(r) < 0$ for all $0 < r < 1$. Since d is decreasing and convex and λ is positive, decreasing and concave, using proposition 5 we get that

$$f'(r) = (d'\lambda)'(r)\frac{1}{\lambda'(r)} - d'(r)\lambda(r)\lambda''(r)\left(\frac{1}{\lambda'(r)}\right)^2 < 0.$$

Furthermore, since $d'(\varepsilon) \to -\infty$ as $\varepsilon \to 0$ and $\lambda'(0) < 0$, we get that $f(\varepsilon) \to \infty$ as $\varepsilon \to 0$. Similarly it follows from our assumptions that $f(1 - \varepsilon) \to 0$ as $\varepsilon \to 0$. ◁

Proposition 7. *Suppose that* $a - Q^*_{-i} > 0$. *Then the choice* $q_i = 0$ *is not optimal. Neither is* $r_i = 0$.

Proof. Recall that the profit function of firm i looks like

$$\Pi_i(q_i, r_i) = \lambda(r_i)\big(a - Q^*_{-i} - q_i - d(r_i)\big)q_i.$$

So, $q_i = 0$ can only be part of an optimal choice for firm i when

$$a - Q^*_{-i} \leq 0$$

since it reduces profits to zero. This however contradicts our assumption that $a - Q^*_{-i} > 0$.

Next, let (q_i, r_i) be an optimal choice. So, it maximizes

$$\Pi_i(q_i, r_i) = \lambda(r_i)\big(a - Q^*_{-i} - q_i - d(r_i)\big)q_i.$$

Then, as we just argued, $q_i > 0$. Hence,

$$\frac{\partial \Pi_i}{\partial r_i}(q_i, \varepsilon) = \Big[\lambda'(\varepsilon)\big(a - Q^*_{-i} - q_i - d(\varepsilon)\big) - \lambda(\varepsilon)d'(\varepsilon)\Big]q_i$$

will be strictly positive for sufficiently small ε, because the first term between the square brackets converges to zero, while $\lambda(\varepsilon)d'(\varepsilon) \to -\infty$ as $\varepsilon \to 0$. So, close to $r_i = 0$, Π_i is a strictly increasing function in r_i. ◁

Appendix B. The parameter dependence of equilibrium

In this appendix the parameter functions d_i and λ_i are assumed to be fixed for all firms. We will analyze the dependence of the Nash-Cournot equilibrium on the number of firms n and the price intercept a. In order to do this we need some notation.

Let n and a be given. Let $q^*_i(n, a)$ be the quantity produced by firm i in equilibrium and $r^*_i(n, a)$ its level of outsourcing. Let $Q^*(n, a)$ be the total amount produced.

Proposition 8. *Let n be fixed. The function $q^*_i(n, a)$ is continuous and increasing in a.*

Proof. Take two values a_{low} and a_{high} of a and suppose that $a_{low} < a_{high}$. First we will show that $q^*(n, a_{low}) < q^*(n, a_{high})$.

For clarity we will index all variables and functions in the old situation with the subindex *low* and those in the new situation with the subindex *high*. Only parameters like d_i and f_i, that are not changed by raising a_{low} to a_{high}, will not be subindexed.

First notice that $Q^*_{low} := Q^*(n, a)$ is determined by the intersection of the decreasing function

$$F_{low}(Q) = (f_1 \circ R_{1,low})(Q) + \cdots + (f_n \circ R_{n,low})(Q)$$

with the 45^o line, while each function $R_{i,low}$ is determined by the equation

$$a_{low} - Q = d_i(R_{i,low}(Q)) + f_i(R_{i,low}(Q)).$$

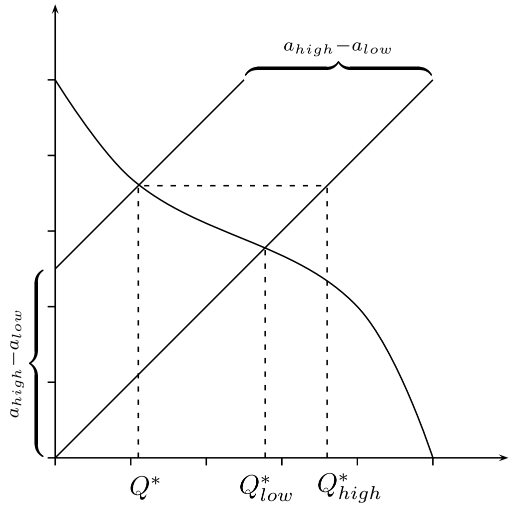
Now, in the new situation, each $R_{i,high}$ is determined by the equation

$$a_{low} - Q + (a_{high} - a_{low}) = a_{high} - Q = d_i(R_{i,high}(Q)) + f_i(R_{i,high}(Q))$$

which shows that $R_{i,high}(Q) = R_{i,low}(Q - (a_{high} - a_{low}))$. This implies that Q^*_{high} can be found by shifting the graph of F_{low} horizontally by an amount of $a_{high} - a_{low}$ to the right and subsequently computing the intersection with the 45^o line. Equivalently, we can also compute the point of intersection Q^* of the graph of F with the line

$$Q \mapsto Q + (a_{high} - a_{low})$$

and subsequently add $a_{high} - a_{low}$ to the result (2).



From this construction it is clear that $Q^* < Q^*_{low}$. Thus, using the equilibrium conditions in the first and last step, we get that

$$\begin{aligned} q_i^*(n, a_{high}) &= f_i(R_{i,high}(Q^*_{high})) \\ &= f_i(R_{i,low}(Q^*_{high} - (a_{high} - a_{low}))) \\ &= f_i(R_{i,low}(Q^*)) \\ &> f_i(R_{i,low}(Q^*_{low})) \\ &= q_i^*(n, a_{low}). \end{aligned}$$

Hence, $q_i^*(n, a_{high}) > q_i^*(n, a_{low})$.

Finally notice that the continuity of $q_i^*(n, a)$ in a can easily be seen in the above graphical representation. $\triangleleft$

Theorem 2. *Both $Q^*(n, a)$ and $a - Q^*(n, a)$ are continuous and monotonically increasing in a. Moreover, both $Q^*(n, a)$ and $a - Q^*(n, a)$ diverge to infinity when $a \to \infty$.*

Proof. Let $q_i^*(n, a)$ denote the quantity that firm i produces in equilibrium, and let $r_i^*(n, a)$ be its level of outsourcing. By proposition 8 we know that each $q_i^*(n, a)$ increases when a increases.

---

(2) In order to make this into an airtight construction, we need to define F also for negative values of Q. This does not raise any further technical problems though.

Hence, this also holds for

$$Q^*(n,a) = q_1^*(n,a) + \cdots + q_n^*(n,a).$$

Furthermore, since $r_i^*(n,a)$ is decreasing in a, it follows from the equilibrium condition

$$a - Q^*(n,a) = d_i(r_i^*(n,a)) + f_i(r_i^*(n,a))$$

that also $a - Q^*(n,a)$ is increasing in a.

In order to establish the limiting behavior, notice that in equilibrium we have

$$a - Q^*(n,a) = d_i(r_i^*(n,a)) + q_i^*(n,a).$$

Summing on both sides over all firms yields the equilibrium condition

$$n(a - Q^*(n,a)) = \sum_{i=1}^{n} d_i(r_i^*(n,a)) + Q^*(n,a).$$

Now the assertion follows from the observation that the summation in the equation displayed above is bounded by $\sum_{i=1}^{n} d_i(0)$. ◁

Theorem 3. *Let the number of firms be n. There exists a value $a(n)$ for which $r_{lf}^*(n,a(n)) = r_{hf}^*(n,a(n))$. For values of a larger than $a(n)$ the market structure in equilibrium is* MS I. *On the other hand, for values of a below $a(n)$ the market structure will not be* MS I.

Proof. Let r_* be the crossover level of our model. By theorem 2 we know that

$$a - Q^*(n,a) > d_{lf}(r_*) + f_{lf}(r_*)$$

for sufficiently large values of a. Hence, for these values of a we will have that $r_{hf}^* > r_{lf}^*$ and by propositions 3 and 4 we automatically have MS I in equilibrium.

Now notice that, since $a - Q^*(n,a)$ is non-negative and continuous and increasing in a, lowering a continuously (starting sufficiently high) will at some point yield a value $a(n)$ for which

$$a - Q^*(n,a(n)) = d_{lf}(r_*) + f_{lf}(r_*).$$

And since r_* is the crossover level of our model we automatically have $r_{lf}^*(n,a(n)) = r_{hf}^*(n,a(n))$.

Finally, suppose that $a < a(n)$. Then, again since $a - Q^*(n,a)$ is increasing in a, we know that

$$a - Q^*(n,a) < d_{lf}(r_*) + f_{lf}(r_*).$$

Hence, $r_{lf}^*(n,a) > r_{hf}^*(n,a)$, and the market structure will not be MS I. ◁

Proposition 9. *For all n, $a(n) < a(n+1)$. Furthermore, $a(n) \to \infty$ as $n \to \infty$.*

Proof. Let $F^{n,a}$ and $F^{n+1,a}$ denote the respective functions when the price intercept equals a and there are n and $n+1$ firms, respectively. Then, since $K(n) \leq K(n+1)$ and $L(n) \leq L(n+1)$, we know that the situation with $n+1$ firms can be viewed as if there were n firms in the beginning and one additional firm, say firm $n+1$, entered the market. So,

$$F^{n+1,a}(Q) = f_1(R_1(Q)) + \cdots + f_{n+1}(R_{n+1}(Q)) = F^{n,a}(Q) + f_{n+1}(R_{n+1}(Q)) > F^{n,a}(Q).$$

This implies that $Q^*(n,a) < Q^*(n+1,a)$. In particular we get that

$$a - Q^*(n,a) > a - Q^*(n+1,a)$$

and hence $a(n) < a(n+1)$.

Secondly, notice that in equilibrium we have

$$a - Q^*(n,a) = d_i(r^*(n,a)) + q_i^*(n,a).$$

Summing up on both sides and rearranging yields

$$a - Q^*(n,a) = \frac{a}{n+1} + \frac{K(n)}{n+1} d_{lf}(r_{lf}^*(n,a)) + \frac{L(n)}{n+1} d_{hf}(r_{hf}^*(n,a)).$$

However, since in equilibrium we also have

$$a - Q^*(n,a) = d_i(r_i^*(n,a)) + f_i(r_i^*(n,a))$$

we get that

$$\frac{a}{n+1} = \frac{K(n)}{n+1} f_{lf}(r_{old}^*(n,a)) + \frac{L(n)}{n+1} f_{hf}(r_{new}^*(n,a)) + \frac{a - Q^*(n,a)}{n+1}.$$

In particular, for $a(n)$ we have that

$$\frac{a(n)}{n+1} = \frac{K(n)}{n+1} f_{lf}(r_*) + \frac{L(n)}{n+1} f_{hf}(r_*) + \frac{a(n) - Q^*(n,a(n))}{n+1} \geq \frac{1}{2} \min\{f_{lf}(r_*), f_{hf}(r_*)\},$$

where r_* is the crossover level. Now notice that the right-hand side of this expression does not depend on n. Hence, $a(n) \to \infty$ as $n \to \infty$. ◁

Appendix C. Analysis of the windowsizes

In this appendix we provide some of the proofs of statements from section6 concerning the occurrence of market structure MS II. As we explained in section 6,

$$r_{lf}^*(n,a) = r_* = r_{hf}^*(n,a).$$

is the equation that defines the boundary between the MS I and MS II regions, and

$$f_{lf}(r_{lf}^*(n,a)) = q_{**} = f_{hf}(r_{hf}^*(n,a)).$$

defines the boundary between the MS II and MS III/II regions. Let r_{**}^{lf} and r_{**}^{hf} be the (unique) solutions to

$$q_{**} = f_{lf}(r_{**}^{lf})$$
$$q_{**} = f_{hf}(r_{**}^{hf})$$

Now define price levels

$$P_* = d_{lf}(r_*) + f_{lf}(r_*)$$

and

$$P_{**} = d_{lf}(r_{**}^{lf}) + f_{lf}(r_{**}^{lf}).$$

Notice that these levels are independent of n and a. (3) Nevertheless, they govern the realization of market structure. This works as follows. First notice that $P_{**} < P_*$. Now, if the price level in equilibrium $a - Q^*(n,a)$ is higher than P_*, we are in MS I. If the equilibrium price level lies between P_* and P_{**} we are in MS II. Below P_{**} MS III/II will be realized.

Theorem 4. *The vertical windowsize tends to infinity as* $n \to \infty$ *and the horizontal windowsize tends to infinity as* $a \to \infty$.

Proof. In order to prove that the vertical windowsize tends to infinity as $n \to \infty$, take a fixed n. Let (n,a) be the point on the boundary between MS I and MS II and let (n,b) be the point on the boundary between MS II and MS III/II. Then we know that

$$a - K(n)f_{lf}(r_*) - L(n)f_{hf}(r_*) = P_*$$

and

$$b - nq_{**} = P_{**}.$$

So,

$$\begin{aligned} a - b &= P_* - P_{**} + K(n)f_{lf}(r_*) + L(n)f_{hf}(r_*) - nq_{**} \\ &\geq n\left(\frac{K(n)}{n}f_{lf}(r_*) + \frac{L(n)}{n}f_{hf}(r_*) - q_{**}\right) \\ &\geq n\left(\min\{f_{lf}(r_*), f_{hf}(r_*)\} - q_{**}\right) \end{aligned}$$

Hence, since $\min\{f_{lf}(r_*), f_{hf}(r_*)\} - q_{**}$ does not depend on n, we get that $a - b$ tends to infinity when $n \to \infty$.

In order to prove that the horizontal windowsize tends to infinity when $a \to \infty$, take a fixed price intercept a. Let (n,a) be the point on the boundary between MS I and MS II and let (k,a) be the point on the boundary between MS II and MS III/II. Then we know that

$$k = \frac{1}{q_{**}}a - \frac{P_{**}}{q_{**}}.$$

Similarly, writing $M = \max\{f_{lf}(r_*), f_{hf}(r_*)\}$ we get that

$$nM \geq K(n)f_{lf}(r_*) + L(n)f_{hf}(r_*) = a - P_*.$$

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(3) Also notice that P_{**} need not exist as example 4 shows. In that case MS III/II does not occur.

Hence,

$$n \geq \frac{1}{M}a - \frac{P_*}{M}.$$

However, since $P_* > P_{**}$, we know that $M > q_{**}$. This implies that $n - k$ tends to infinity as $a \to \infty$. $\triangleleft$

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