

Name _____

Algebra II

Lesson Rational Functions and Their Graphs

9-3

The functions studied in the previous sections are members of a bigger group of functions called **rational functions**. A rational function is a ratio of two polynomial functions: $f(x) = \frac{P(x)}{Q(x)}$. As seen with the reciprocal function, we will also have both vertical and horizontal asymptotes with rational functions.

Vertical Asymptotes

When we look at a rational function realize there is a polynomial in the denominator. As we learned several chapters ago, the solutions to a polynomial are the values where $x = 0$. Now the moment we put a polynomial in a denominator of a fraction, we are going to have a set of numbers that will be restrictions on the domain of the rational function, because the denominator cannot be equal to zero. The restriction on the domain is described by the following definitions:

discontinuous: a break in the line or curve on a graph

point of discontinuity: the point at $x=a$ where the function is undefined (point where the denominator =0).

vertical asymptote: the line formed by $x=a$, the curve will get closer and closer but never touch or cross this line.

How do we find the points of discontinuity and thus the vertical asymptotes? Set the denominator equal to zero and solve for x . These values are the points of discontinuity and hence the vertical asymptotes.

Find the vertical asymptotes for:

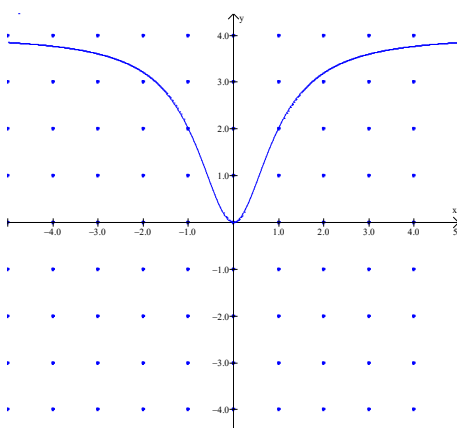
$$y = \frac{x+3}{x-4}$$

$$y = \frac{x}{x^2-9}$$

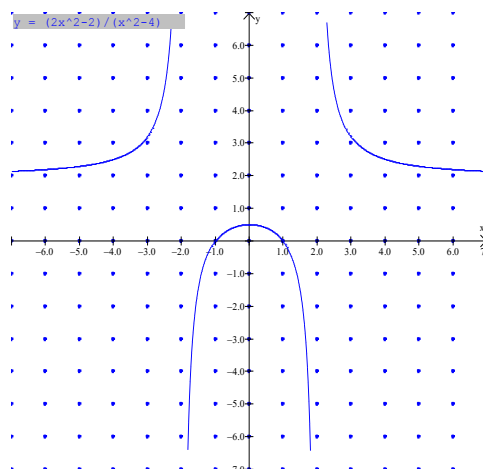
$$y = \frac{2}{x^2-4}$$

Some rational functions may not have any restrictions while others may have one or more, depending on the denominator.

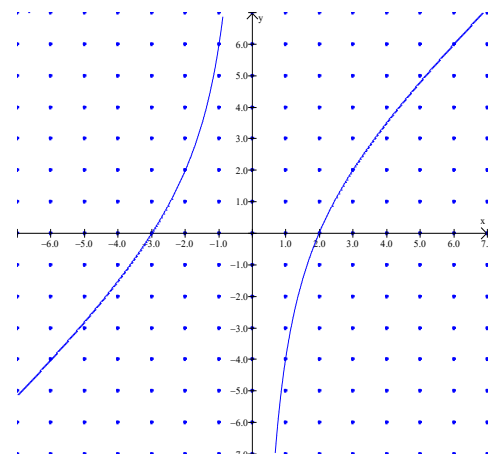
$4x^2/(x^2 + 1)$; no restrictions on x



$y = \frac{2x^2-2}{x^2-4}$ where $x \neq 2, -2$



$y = \frac{x^2+x-6}{x}$ where $x \neq 0$

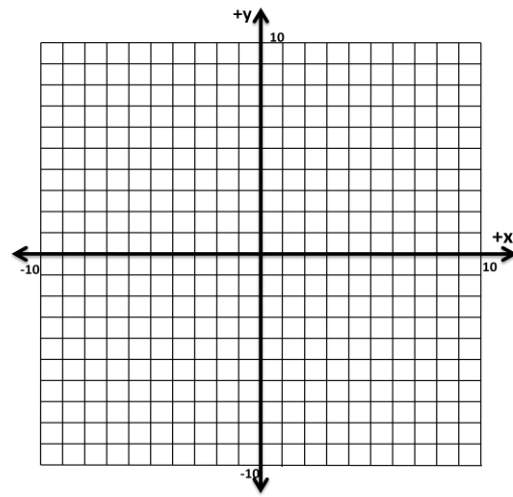


Graph: $y = \frac{(x-2)(x+1)}{(x-2)}$

What does the graph look like?

Note that the $(x-2)$ cancels out in the numerator and denominator. Check the table generated by the graph what is the value of y for $x=2$?

When $P(x)$ and $Q(x)$ have a common real zero at a , then there is a hole in the graph. (the graphing calculator will not show the hole)



Horizontal asymptotes

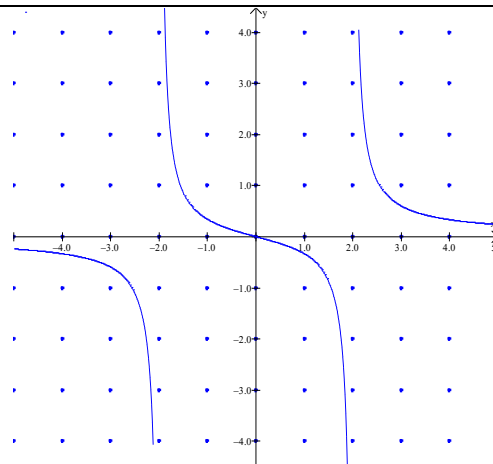
The middle graph above shows a horizontal asymptote. Why is it at $y=2$? Note the following rules for horizontal asymptotes:

1. A rational function will have no more than one horizontal asymptote.
2. Degree of numerator < degree of denominator: HA at $y = 0$
3. Degree of numerator = degree of denominator: HA at $y = \text{ratio of leading coefficients}$
4. Degree of numerator > degree of denominator: No HA.

Graph: $\frac{x}{x^2-4}$

VA: $x^2 - 4 = 0$
 $(x + 2)(x - 2) = 0$
 $x = 2 \text{ or } -2$

HA: degree of num. < degree of denom. so
 $y = 0$

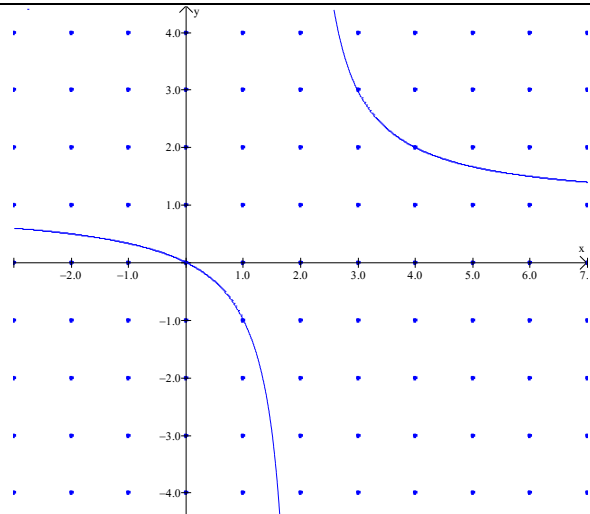


Graph: $\frac{x}{x-2}$

VA: $x - 2 = 0$
 $x = 2$

HA: degrees of num. and denom. are equal so HA is
 ratio leading coefficients:

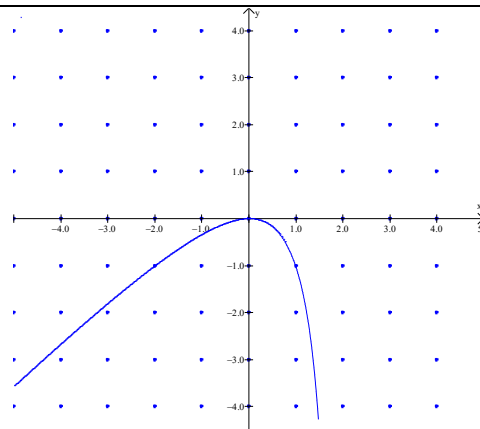
$y = \frac{1}{1} = 1$



Graph: $\frac{x^2}{x-2}$

VA:

HA:



Find the points of discontinuity:

$$y = \frac{2x^2+5}{x^2-2x}$$

$$y = \frac{6-3x}{x^2-5x+6}$$

Describe vertical asymptotes and holes:

$$y = \frac{3}{x+2}$$

$$y = \frac{9-x^2}{x^2-9}$$

Describe the horizontal asymptotes:

$$y = \frac{3x-4}{4x+1}$$