Lesson 9-1
Inverse Variation

In chapter 2 we learned that when two variable quantities have a constant (unchanged) ratio, their relationship is called a **direct variation**. We say that y varies directly as x. The constant ratio, **k**, is called the **constant of variation**. The formula for direct variation is: $\frac{y}{x} = k,\ or\ y = kx$. Remember in a linear situation our constant of variation is our slope, $\frac{rise}{run} = \frac{y}{x}$.

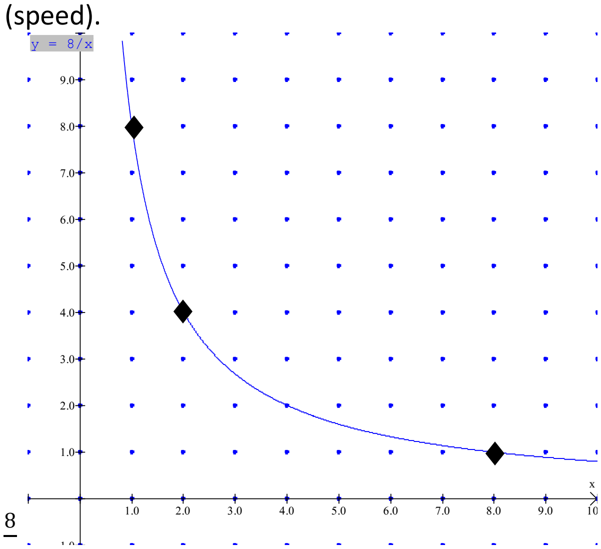
The opposite of direct variation is known as **Inverse Variation**. In an inverse variation, the values of the two variables change in an opposite manner, that is, as one value increases, the other decreases. Our equation for inverse variation is $xy = k,\ so\ y = \frac{k}{x},\ or\ x = \frac{k}{y}$. As with direct variation the variable *k* is called the *constant of variation.*

Let's look at an example. How long will it take a cycler to bike 8 miles? Well that depends on his speed. A biker traveling at 8 mph can cover 8 miles in 1 hour. If the biker's speed decreases to 4 mph, it will take the biker 2 hours to cover the same distance.

Rate (mi/hr)	Time (hr)
8	1
4	2
1	8

Notice that as the rate decreases, the time increases. Cut the rate in half, the time doubles. Our equation to calculate distance is: $rt = d$, at distance equal to 8 miles we get: $rt = 8,\ or\ t = \frac{8}{r}$, as our question asked for how long – time.

Graphically we see can see the relation between time and rate (speed).



$y = \frac{8}{x}$

Example: Given y varies inversely as x. Write a variation function when $y = 1.4\ and\ x = 0.3$

1. What is our equation for inverse variation? $xy = k???$
2. Substitute x and y into our equation and solve for our dependent variable y or f(x).

When given a set of ordered pairs, we can determine if x and y vary directly or inversely, or neither:

1. Does y increase WRT x? Yes: consider direct variation

   $\frac{y}{x} = k$ Is the ratio constant? Yes: Direct Variation

2. Does y decrease WRT x? Yes: consider indirect variation

   $y = \frac{k}{x} \therefore xy = k$ Is the product of xy constant? Yes: Indirect Variation

x	2	4	6
y	3.2	1.6	1.1
xy			

1. Does y increase (direct variation) or decrease (inverse variation) WRT x?
2. Since **xy=k**, multiply each ordered pair to calculate the constant of variation.
3. Are the products equal? Yes, variation. No, no variation.

Now Try:

x	0. 8	0.6	0.4
y	0.9	1.2	1.8
xy			

Variation is not only for linear relationships. We can just as easily have a situation where y varies inversely with x^2, such that: $y = \frac{k}{x^2}$. Also, we can have situations where we have what is called a **joint variation**. Here a variable will vary jointly with two other variables: $z = kxy$. Let's put some of these combinations into table form:

Combined Variation	**Equation form**
z varies jointly with *x* and *y*	$z = kxy$
z varies jointly with *x* and *y* and inversely with *w*	$z = \frac{kxy}{w}$
z varies directly with *x* and inversely with the product *wy*	$z = \frac{kx}{wy}$

Example: The volume **V** of a tetrahedron varies jointly with its altitude **h** and base of area **b**. Find the formula that models this joint variation. Given that the tetrahedron has an altitude of 5 cm., a base area of 6 in^2, and a volume of 10 cm^3

1. Identify the type of variation (jointly with ___and ___).
2. Substitute the given values of **V, b,m and h.**
3. Solve for **k**.
4. Write general equation with calculated value of **k**.

Now try:

Given a direct variation, find the missing variable for the pair of values: (4,6), (x,3)

Given an inverse variation, find the missing variable for the pair of values: (4,6), (x,3)