

Name _____

Lesson 8-5

Exponential and Logarithmic Equations

We have been looking at exponential equations and techniques to evaluate them, that is solve for x. We found that in a situation where for example: $3^x = 243$ we were able to get the right side into an exponential form and we found that $3^x = 3^5$. We also know a rule that says if the bases are equal the exponents must be equal; therefore $x = 5$.

By now we know that there is always a situation that will not have a nice and neat solution. For example, solve: $2^x = 30$. If the equation were $2^x = 32$, we would quickly find that $x=5$, but alas, we don't have a typo. In order to solve this equation, we will need x to be down on the ground floor. This can be done with logarithms. We now must remember the golden rule of algebra: I can do whatever I want to the equation, as long as I do the same thing to both sides. This combined with our **power property** of logarithms will allow us to solve for x. So let's look back at our equation:

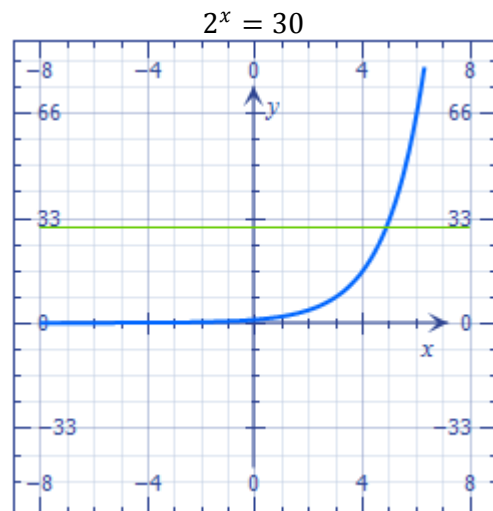
$2^x = 30$ $\log 2^x = \log 30$ $x \log 2 = \log 30$ $x = \frac{\log 30}{\log 2}$ $x =$	<ol style="list-style-type: none"> 1. Take the log of both sides 2. With the power property, take the exponent down in front of the log as a coefficient 3. Divide each side by the $\log 2$ 4. Use a calculator to simplify 	<p>Why not? $2^x = 30$ $2^x = 30$ $\log_3 2^x = \log_3 30$ <i>This does me little good, as I cannot use the calculator to evaluate a log base 3. Use log base 10</i></p>
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When I take the log of both sides, I can take the log to any base I choose, but by taking log base 10, I am able to use my calculator to evaluate for x. Later we will follow this process using the natural log, it works just as well.

We can also solve this equation by graphing. As in previous chapters, with a calculator we input 2 equations:

$Y_1=2^x$ ↓ $Y_2=30$ GRAPH

Find the point of intersection, and you find the solution of x. Note: you will most likely need to adjust your WINDOW to include the domain and range of each equation. Don't forget to set the scale so that it is easier to interpret. AND use the **2nd CALC INTERSECT** function to locate the exact value of x.



The table function may also be utilized to solve for x. As x is a zero, we can approximate the value where we see a sign change.

When x is part of the argument of a log function, a similar process may be used:

<p>Solve: $\log(7 - 2x) = -1$</p> $10^{-1} = 7 - 2x$ $2x = 7 - \frac{1}{10}$ $x = \frac{6.9}{2}$ $x = 3.45$	<ol style="list-style-type: none"> 1. Already in log form so make into an exponential function (base 10) 2. rearrange 3. solve for x by clearing coefficient
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CHANGE OF BASE

Now let's take a look at logs with bases other than 10. Evaluate $\log_3 29$, if this were $\log_3 27$ then we could figure out that $x=3$, but there is no neat solution for this equation, unless we come up with another method. A method that works for these types of equations is called the **Change of Base Formula**.

$$\log_b M = \frac{\log_c M}{\log_c b}$$

Letting c be our base 10, we may take a ratio of the log of the argument to the log of the original base. So our equation above becomes: $\log_3 29 = \frac{\log 29}{\log 3} = 3.065$

Solve: $5 - 3^x = -40$

Solve: $\log_3 54$

Solve by graphing: $6^x = 4565$

Solve: $\log_2 x = -1$