Name____________________________

Lesson 8-4

Properties of Logarithms

Last section we saw that pH=-log [H+] can also be expressed in exponential form: $10^{-pH} = [H^+]$. Since logarithms are related to exponents, you can derive the properties of logarithms from the properties of exponents:

Operation	**Exponents**	**Logarithms**
product	$n^a \cdot n^b = n^{a+b}$	$log_b m + log_b n = lob_b(m \cdot n)$
quotient	$n^a / n^b = n^{a-b}$	$log_b\, {}^{m}/{}_{n} = log_b m - log_b n$
power	$(n^a)^b = n^{ab}$	$log_b m^x = x log_b m$

Example: express as a single logarithm and simplify

$log_4 2 + log_4 32$

$log_4(2 \cdot 32)$

$log_4 64$

to evaluate: $4^x = 64$

$4^x = 4^3$

$x = 3$

1. To add logarithms, multiply the arguments
2. simplify
3. If the directions say to evaluate then we go further and solve the log by converting to its exponential form and solving for x.

Now try:

$log_6 625 + log_6 25 =$

$log_{\frac{1}{3}} 27 + log_{\frac{1}{3}} \frac{1}{9} =$

Example combining operations:

$log(5) + log(15) - log(3)$

$log(5 \cdot 15) - log(3)$

$log\left(\frac{(5 \cdot 15)}{3}\right)$

$\log 25$

to evaluate via calculator,

$log 25 = 1.3979$

1. product rule
2. quotient rule
3. simplify
4. *You must read and understand the directions. Depending on what is required, you will either stop at this point or continue on to evaluate, that is x=….*

Now try:

$5 log_3 x + log_3 2x - log_3 x$

Evaluate: $log_2 4 - log_2 16$

Not only can we combine logarithms, we can go the other direction and expand a combined logarithm:

Example: expand

$log \frac{2xy^2}{3z^3}$

$\log 2xy^2 - log 3z^3$ Quotient rule

$[log\, 2 + log\, x + log\, y^2 - [log 3 + log z^3]$ Product rule

$[log\, 2 + log\, x + 2\, log\, y] - [log 3 + 3 log z]$ Power rule

Your turn!

$log \frac{\sqrt{x+1}}{(x-2)^3}$

Logarithms are uses to model sound. The intensity of a sound is measure of the energy carried by the sound wave. The greater the intensity of a sound, the louder it seems. Loudness is measured in decibels with the formula: $L = 10log\frac{I}{I_o}$.

Earplugs are advertized to block a certain amount of noise. One earplug brand claims to block the sound of noise as loud as 822 dB. A second brand claims to block 8 times that amount. If this claim is true, how many more decibels are blocked?

First off this is a subtraction problem as we are looking at "how many more." So let L_2= brand 2 loudness and L_1= brand 1 loudness. Identify the relationship between the two brands: $I_2 = 8I_1$, so using our equation for loudness:

$L_2 - L_1 = 10log\frac{8I_1}{I_0} - 10log\frac{I_1}{I_0} =$

$10log8 + 10log\frac{I_1}{I_0} - 10log\frac{I_1}{I_0} =$

$10log8 \approx 9dB$