Name____________________________________

Algebra II

Lesson 8-3

Logarithmic Functions as Inverses

Recall that each algebraic operation has an opposite or an inverse:

Operation	**Inverse**
multiplication	division
addition	subtraction
square: x^2	square root: $\sqrt{\ }$
cube: x^3	cube root: $\sqrt[3]{\ }$

Logarithms are the "opposite" of exponents, just as subtraction is the opposite of addition and division is the opposite of multiplication. Logs "undo" exponentials. Technically speaking, logs are the inverses of exponentials. More practically speaking, you can think of logs in terms of the relationship:

$$\underset{\uparrow\text{ base}}{y = b^x} \xleftrightarrow{\text{inverse}} \underset{\uparrow\text{ base}}{log_b(y) = x}$$

On the left-hand side above is the exponential statement $y = b^x$ On the right-hand side, is $log_b(y) = x$, the equivalent logarithmic statement, we say this as the "log-base-b of *y* equals *x*";

Exponential	**Logarithm**	**To change forms**
b is the base	b is the base	
$b > 0$	$b > 0\ and\ b \neq 1$	$log_b x = y \quad so \quad b^y = x$
	inside of the log is the **argument**	

Since exponential and logs are inverses, when we have equation in one form, we can write it in the other form. The **common logarithm** is base 10 and unless otherwise written, log is understood to be base 10. So: $log_{10}y = \log y$.

Write 100=10^2 in logarithmic form

$y = b^x$ and $log_b(y) = x$ 1. Write the definition

$100 = 10^2\ and\ \boldsymbol{log_{10}100 = 2}$ 2. Substitute

Write 49=7^2 in logarithmic form

Now try: $\frac{1}{10} = 10^{-1}$

$y = b^x$ and $log_b(y) = x$

$49 = 7^2\ \therefore\ \boldsymbol{log_7 49 = 2}$

We can also go from log to exponent thus solving for **x**:

Evaluate $log_8 16 = x$

$y = b^x \quad 16 = 8^x$ 1. convert to exponential form

$2^4 = (2^3)^x$ 2. write side using base 2

$4 = 2^{3x}$ 3. power property of exponents

$4 = 3x$ 4. set the exponents equal to each other

$\frac{4}{3} = x$ 5. solve for x

Evaluate $log_4 2$

$y = b^x$

Refer to textbook page 454: The strength of an earthquake is a measurement of the amount of energy released at the epicenter (source). The Richter scale is an exponential measure of earthquake magnitude that is related to the energy released through this equation: $E \times 30^{magnit\ ude}$. So when given two earthquake magnitudes, we can compare the energy released by forming a ratio:

Compare energy released between these two earthquakes: 1997 Alabama earthquake of magnitude 4.9 and the 1999 California of magnitude 7.0: $\frac{E\times30^{7.0}}{E\times30^{4.9}} = \frac{30^{7.0}}{30^{4.9}}$ =1265 thus the earthquake of California released about 1265 times as much energy as the Alabama earthquake.

Compare: 1812 New Madrid Earthquake of 7.9 to 1995 Kobe Japan Earthquake of 6.9

pH of a substance is also a logarithmic equivalent based on hydrogen ion concentrations: **pH=-log[h$^+$]**

Find the concentration of hydrogen ions, [H$^+$]

lime juice pH = 2.2

$2.2 = -logH^+$

$log_{10}H^+ = -2.2$

$H^+ = 10^{-2.2}$

$.0063 = 6.3 \times 10^{-3}$

- log is understood to be base 10
- rewrite with base 10 written and move negative to other side.
- convert into exponential form
- solve

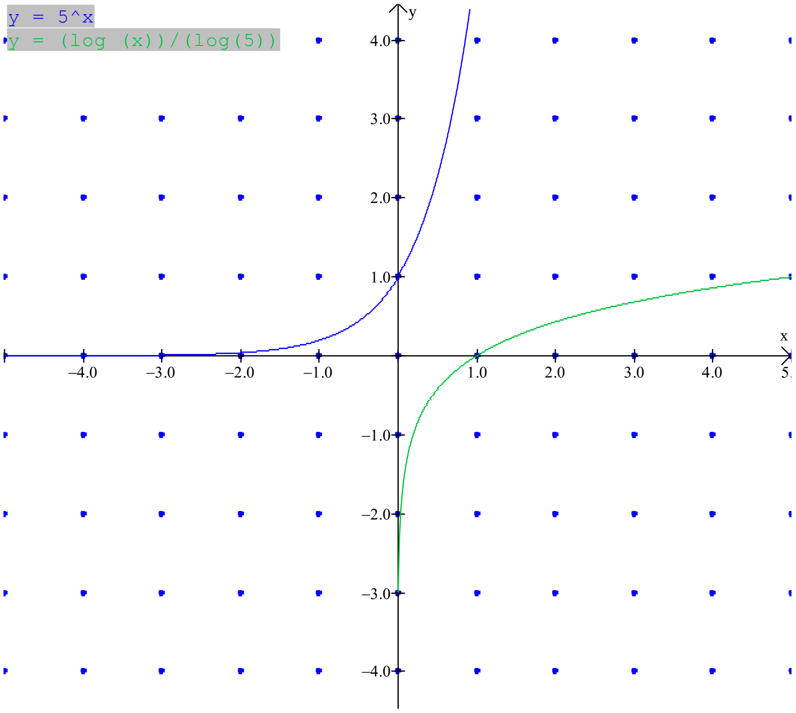
Graph: $log_5 x = y \quad (rewrite: 5^y = x)$

By the definition of logarithm, the log is the inverse of $y = 5^x$ (x is y and y is x)

1. graph the exponential function
2. reverse the coordinates for the x and y values and plot $y = log_5 x$

exponential:

x	y
-1.5	.09
-1	.2
0	5
.5	2.23
1	5



When the log function gets more complicated, you will need to use translations.

Parent function	$y = log_b x$
stretch a>1	$y = alog_b x$ **big a** the curve stretches or gets larger, vs. **fractional a** curve gets very thin
shrink 0<a<1	
shift up/down	$y = log_b(x-h) + k$ where the h is a horizontal shift and the k is our vertical shift
shift left/right	

Graph:

$log_7(x-2)$ -- $log_7 x = y \therefore 7^y = x$

1. make a table of x, exponential equivalent for the parent function and y
2. graph the function by shifting the points from the table 2 units to the right

x	$7^y = x$	y
7	$7^y = 7 \therefore y = 1$	1
1	$7^y = 1. y = 0$	0
1/7	$7^y = \frac{1}{7}, y = -1$	-1
1/49	$7^y = \frac{1}{49}, y = -2$	-2
1/343	$7^y = \frac{1}{343}, y = -3$	-3