Name________________________________

Algebra II

Lesson 8.1

Exploring Exponential Models

When a function increases or decreases by a constant rate that is expressed as a percent, we get what is known as an **exponential function. Exponential functions** are used to predict population growth patterns, nuclear power reactions, ozone and rain forest depletion. There are two types of exponential functions: **growth** function, which means the graph will get larger by an exponent value, and **decay** function, which means the graph will get smaller by an exponent value. An exponential function has the form

$$f(x) = b^x, where\ b\ (base) > 0, b \neq 1, and\ x\ is\ an\ exponent.$$

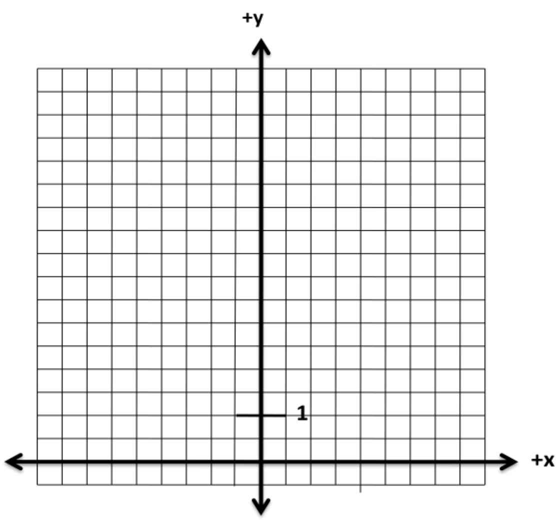
- If $0 < b < 1$, then you will have a decay function
- If $b > 1$, then you will have a growth function. Study the examples below:

GROWTH

- Base number >1
- Exponent is positive
- graph rises as x-values get larger

x	-2	-1	0	1	2	3
y						

Graph $y = 2^x$, here b>1 so growth

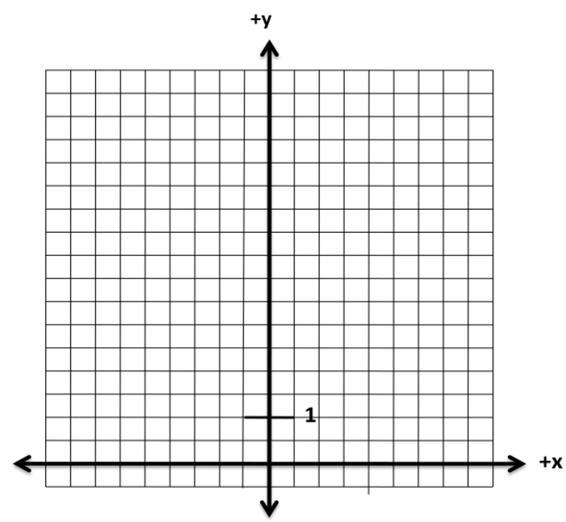


DECAY

- 0<base number <1
- exponent is negative
- graph falls as x-values get larger

x	-2	-1	0	1	2	3
y						

Graph $y = \left(\frac{1}{4}\right)^x$, 0<b<1 so decay



Notice that the graphs of both types of exponentials get infinitely close to but **do not touch or cross** the x-axis. The x-axis or y=0 is called an **asymptote**, which means a boundary that a graph cannot cross or touch.

The exponential form above can be revised to calculate growth or decay patterns, given a fixed rate of increase (growth) or decrease (decay).

$A(t) = a(1 \pm r)^t$,

where **A(T)** is the final amount

a is the initial amount

r is the rate or increase (1+r) or decrease (1-r)

t is the number of time periods

The revised formulation can be used to calculate compound interest, bacteria colony growth, human population patterns, appreciation (increased value) and depreciation (decreased value) of cars, truck homes or office buildings, and many other similar applications.

So let's take a look an example. A new costs about $24,500. It is estimated that the car will depreciate (lose value) by 15% each year. What will the car be worth in 4 years?

Since the car is losing value, we will need to use the formula, $A(t) = a(1-r)^t$, where a=$24,500, r=15%, and t=4 years. Thus, A$(t) = 24500(1-0.15)^4$

$= 24500(0.85)^4$

$[24500 \times 0.85^4 ENTER]$

$\approx \$12{,}789.15$ This means that the car will lose about half if its value in the first 4 years!

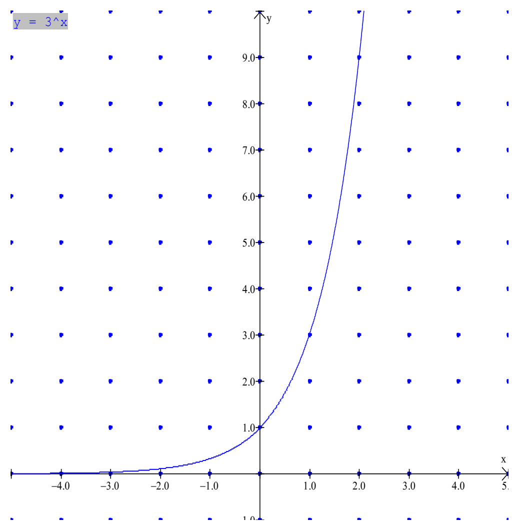
As with other problems we have worked with, it may be necessary to readjust the WINDOW, as some (x,y) values may be too large or too small for the default ± 10 unit grid. Try $y = 20(0.5)^x$ We need to adjust the WINDOWS to x=0 to 5 and y=0 to 20

Now we need to look at some basic equation set-ups and solution methods.

Graph $y = 3^x$

1. Make a table of values, and plot. You will need several points to verify the shape of the curve. 0 ± 3 is a good amount, recognizing that you may not be able to graph all points.

x	3^x	y
-3	3^{-3}	.037
-2	3^{-2}	.111
-1	3^{-1}	.333
0	3^0	1
1	3^1	3
2	3^2	9
3	3^3	27



It is important to bring up here again the term asymptote. Note that as x gets smaller, y approaches 0, **y=0** is the equation for the horizontal asymptote.. *No matter how small x becomes, y will never be less than or equal to 0.*

Given points (2,4) and (3,16), from an exponential graph, determine the exponential function.

$y = ab^x$

$4 = ab^2$

$\frac{4}{b^2} = a$

$16 = \frac{4}{b^2} b^3$

$16 = 4b$

$\frac{1}{4} \cdot 16 = b = 4$

$\frac{4}{4^2} = \frac{4}{16} = \frac{1}{4} = .25 = a$

$y = .25(4)^x$

1. Using the general form, substitute (2,4) for x and y.
2. Solve for **a**
3. Substitute $^4/_{b^2}$ for **a** and (3,16) for x and y.
4. With the division property of exponents, simplify and solve for b
5. With a solution for **b** go back to our **a=** equation and solve for a
6. With **a and b** we can now write the exponential equation for a curve that contains the given points.

What is the horizontal asymptote of this function?

Is this an exponential increase or decrease function?