

Name _____

Algebra II
Lesson 7-7

Inverse Relations and Functions

When solving an equation like $x + 3 = 5$, your first step is to subtract 3 from both sides. We say that “subtracting 3” is the opposite or inverse of “adding 3.” Inverse operations can also be organized to create an “inverse” function. An inverse function is an exact opposite of what a function does and has a special symbol; $f^{-1}(x)$. There are 2 basic steps to formulating an inverse function.

- Step 1 Reverse the x and y
- Step 2 Solve for the new “y”, and replace y with $f^{-1}(x)$

Example: Find the inverse function for

$$\begin{aligned} y &= x + 3 \\ x &= y + 3 \\ x - 3 &= y \\ x - 3 &= f^{-1}(x) \end{aligned}$$

Step 1: reverse x and y
solve

Step 2: replace y with $f^{-1}(x)$

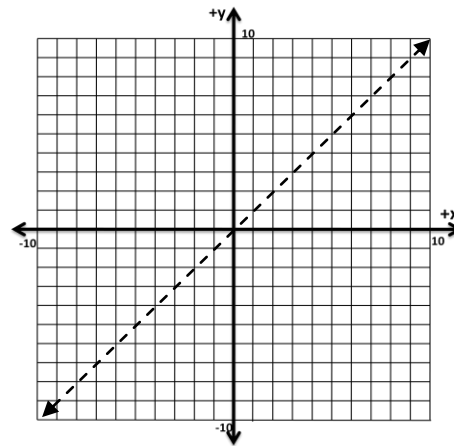
Sometimes you will be asked to plot a set of (x,y) coordinates and then graph the inverse. Understand, the inverse of a function has all the same points as the original function, except that the **x's and y's have been reversed**. For instance, supposing your function is made up of these points: $\{(1, 0), (-3, 5), (0, 4)\}$. Then the inverse is given by this set of point: $\{(0, 1), (5, -3), (4, 0)\}$. In this case simply switch the order of the x and y values.

Example: Plot the data given. Determine the inverse and plot the inverse.

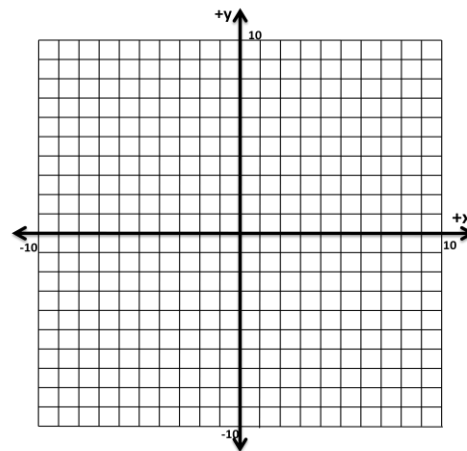
| | | | | |
|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 |
| y | 0 | 1 | 4 | 9 |

| | | | | |
|---|--|--|--|--|
| x | | | | |
| y | | | | |

Notice that a straight line passes through the center of both data sets. This line $y=x$ is called the **axis of symmetry**. In future lessons we will learn that not all functions have inverses. For now, assume that all given function have a unique inverse.



Example: Given $f(x) = 2(x - 3) + 1$ Find $f^{-1}(x)$, graph both $f(x)$ and $f^{-1}(x)$



Note: Watch out for domain restrictions and be able to state the restrictions. The domain of f^{-1} is the range of f , and the range of f^{-1} is the domain of f .

Now try: For the previous equation, $f(x) = 2(x - 3) + 1$
find $f(f^{-1}(4))$.

What happened? When we evaluated the function of the inverse for a given value of x , we got the same number. This is an important property that can be used to **determine if two equations are in fact the inverse of each other.**

| |
|---|
| <p>If f and f^{-1} are inverse functions, then: $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$</p> |
|---|