Name________________________________

Algebra II

Lesson 7.5

Solving Square Root and Other Radical Equations

In previous chapters we were solving polynomials: $x^2 - 18 = 0$; $x^2 = 18$; $\sqrt{x^2} = \sqrt{18}$; $x = 3\sqrt{2}$. This is what is called solving a square root equation, because we need to take the square root of x. Now, when we have the variable as a radicand or the variable has a fraction for an exponent (indicates that it is a root), we have what is called a radical equation. So to solve for our variable we would have to what???

Solve: $\sqrt{5x+1} - 6 = 0$

Start by isolating the variable. First add/subtract numbers

$$\sqrt{5x+1} = 6$$

To get rid of the radical square both sides of the equation and simplify

$$(\sqrt{5x+1})^2 = 6^2$$
$$5x + 1 = 36$$
$$5x = 35$$

solve for x

$$x = 7$$

We can also solve equation where instead of a radical there is a fractional exponent. How? **Raise each side of the equation to the reciprocal power of the exponent.** With the following rule resulting: $(\boldsymbol{x^{\frac{m}{n}}})^{\frac{n}{m}} = |\boldsymbol{x}|$

- With the absolute values, we will need to fall back on our knowledge them: that is our Case 1 and Case 2 solutions.
- When both sides of an equation are raised to a power, a chance for extraneous solutions is introduced so **check solutions to verify presence of an extraneous solution.**

Let's see how this works, Solve:

$$2(x+3)^{\frac{3}{2}} = 54$$

get rid of coefficient 2 by multiplying by reciprocal

$$(x+3)^{\frac{3}{2}} = 27$$
$$((x+3)^{\frac{3}{2}})^{\frac{2}{3}} = 27^{\frac{2}{3}}$$

the exponent is $\frac{3}{2}$, so we raise each side by the reciprocal of $\frac{2}{3}$ **rule of exponents:** exponent raised to an exponent, multiply the exponents.

now simplify the right side of the equation

$$|x+3|^1 = \sqrt[3]{27 \cdot 27}$$

the rational exponent means we take the cube root of 27^2

Case one: "as is" and Case two: bring down the expression and equal to the negative.

Case I		*Case 2*
$x + 3 = 9$	*or*	$x + 3 = -9$
$x = 9 - 3$	*or*	$-9 - 3$
$x = 6$	*or*	-12

$(6+3)^1 = 27^{\frac{2}{3}}$

$9 = 9$

$(-12+3)^1 = 27^{\frac{2}{3}}$

$\_9 \neq 9$

when we check we realize that the -12 solution is an extraneous solution as we don't get a true equation

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$\therefore\ \boldsymbol{x = 6}$

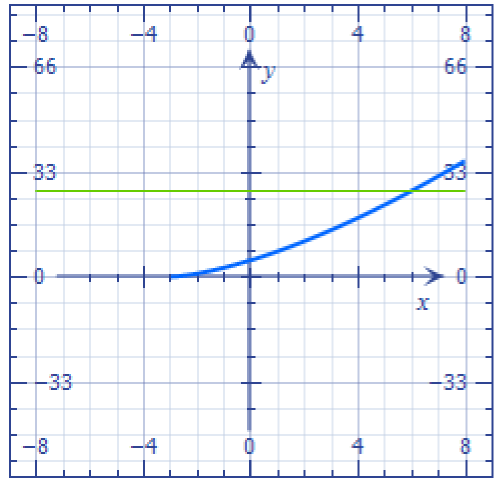
Using a calculator to check your work will allow you to confirm both solutions as either true or confirm an extraneous solution. Enter each side of the equation as a separate entry in your calculator's **Y=** function. Locate the intersection of the two lines to verify the solutions.

For the above example we find that there is only one solution for Y=27 as shown on the following graph:

Graph the equation $(x+3)^{\frac{3}{2}} = 27$

on calculator enter

Y1=$(x+3)^{\frac{3}{2}}$
Y2= 27
WINDOW Xmin= -15
Xmax= 15
Scl= 5
Ymin= -10
Ymax= 40
Scl=5
set windows to include y=27 on the graph
we know that $x = 6\ or -12$ so set windows accordingly.



Equations can contain more than one radical expression. In which case focus on isolating one of the radicals. IF it contains an expression with a variable under a radical and a variable outside the radical, focus on isolating the variable under the radical by getting rid of the radical and then solving for .

Now Try:

Solve: $\sqrt{3x+2} - \sqrt{2x+7} = 0$

Solve: $(x+1)^{\frac{2}{3}} - (9x+1)^{\frac{1}{3}} = 0$