

Name _____

Algebra II

Lesson 7.4

Rational Exponents

We use the radical sign to indicate taking a square root of a number. There is another way to indicate a radical expression it is called a **rational exponent**, here we take the index and make it into a fraction in terms of: $\frac{1}{index}$

so for example: $\sqrt{4} = 4^{\frac{1}{2}}$.

Rewrite using a rational exponent: $\sqrt{18} =$

$\sqrt[3]{27} =$

$\sqrt[5]{89} =$

Simplify: $512^{\frac{1}{3}}$

$4096^{\frac{1}{6}}$

The property rules that we learned in the previous sections still hold true. We can also use the rules of exponents (summarized below), which include this key formatting principal for exponents, so: $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = \sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$, but superimposing rules for exponents we get: $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2} + \frac{1}{2}} = 5$. *Understanding this technique of converting the radicals into exponential fractions can really help speed up the simplification process.*

Simplify: $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$

$7^{\frac{2}{3}} \cdot 7^{\frac{1}{3}}$

We now come to a point where we need to look at a list of rules for rational exponents. If you study and understand these rules, you will find many short cuts for solving problems that have rational exponents.

$a^{\frac{1}{n}} = \sqrt[n]{a}$,	$27^{\frac{1}{3}} = \sqrt[3]{27}$,
$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$5^{\frac{2}{3}} = \sqrt[3]{5^2} = (\sqrt[3]{5})^2$
$a^m \cdot a^n = a^{m+n}$	$7^{\frac{1}{3}} \cdot 7^{\frac{2}{3}} = 7^{\frac{1}{3} + \frac{2}{3}} = 7^{\frac{3}{3}} = 7$
$(a^m)^n = a^{mn}$	$(5^{\frac{1}{2}})^4 = 5^{\frac{1}{2} \cdot 4} = 5^2 = 25$
$(ab)^m = a^m b^m$	$(4 \cdot 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 2 \cdot 5^{\frac{1}{2}}$
$a^{-m} = \frac{1}{a^m}$	$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{\pi^{\frac{3}{2}}}{\pi^{\frac{1}{2}}} = \pi^{\frac{3}{2} - \frac{1}{2}} = \pi^1 = \pi$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{27}\right)^{\frac{1}{3}} = \frac{5^{\frac{1}{3}}}{27^{\frac{1}{3}}} = \frac{5^{\frac{1}{3}}}{3}$

Now try: $25^{-\frac{3}{2}}$

Simplest form:

We have learned that simplest form for a radical means that we are to have no radicals in the denominator. Now, for expressions with rational exponents, we need to make every exponent a positive number.

Try: Write in simplest form $(8x^{15})^{-\frac{1}{3}} =$