Name________________________________

Algebra II

Lesson 7-1

Roots and Radical Expressions

Recall that the square root of 25 is written as $\sqrt{25}$; it has two possible roots ± 5, because $25 = (+5)^2 = (-5)^2$. Now the cube root of 27 (what times itself 3 times equals 27) is written as $\sqrt[3]{27} = 3$. The fourth root of 16 is written as $\sqrt[4]{16} = \pm 2$ because $16 = (+2)^4 = (-2)^4$. If we keep increasing the **index, n,** in $\sqrt[n]{a}$, we will see a pattern emerge.

Even roots (2, 4, 6, …)	**2 solutions (roots): form of** $\pm$ **Calculator:** the answer shown will only be one root on the calculator. REMEMBER: the solution will be the **conjugates ,** $\pm$.
Odd roots (3, 5, 7, …)	**1 solution (root): either** $+ \ \boldsymbol{or}\ -$

This comes from the fact that a positive or negative number multiplied by itself an ***even*** number of times yields a ***positive*** number. A negative number multiplied by itself an ***odd*** number of times yields a ***negative*** number. However, when the radical is presented to simplify, we focus on the **principal** square root which is given by the symbol $\sqrt{\ }$. The radical by itself, implies only one root. $\pm\sqrt{\ }$ therefore implies **two** roots. *When we have only the square root sign with no* $\pm$*, we will give the solution as a single +solution. (This is in contrast to Ch. 5&6 where we always found both solutions).*

Some roots should be straight forward to solve by virtue of your knowledge of the multiplication tables. When you get to a root that you may be uncertain of, use your calculator to check if the radicand is a perfect root:

Example: find each real root number — **Calculator:** type the nth root on the view screen,

$\sqrt[3]{1728}$ — **3 MATH 5:$\sqrt[x]{\ }$ ENTER 1728 ENTER 12 =** answer

$\sqrt[4]{81}$ — **4 MATH 5:$\sqrt[x]{\ }$ ENTER 81 ENTER 3** answer∴ $\pm\mathbf{3}$ with an even index we must remember that we will also have the conjugate answer: -3

$\sqrt[3]{-8}$ — **3 MATH 5:$\sqrt[x]{\ }$ ENTER -8 ENTER -2** answer

$\sqrt[6]{729}$ — **6 MATH 5:$\sqrt[x]{\ }$ ENTER 729 ENTER 3** answer∴ $\pm\mathbf{3}$

Evaluate: $\sqrt{x^6} = x^3 \ \ for\ x = -2$ — substitute -2 in for x

$\sqrt{(-2)^6} = (-2)^3$

$\sqrt{64} = \pm 8 \ \ but\ (-2)^3 = -8$ — this is a false statement

Even though 64 has two square roots, -8 and 8, as stated earlier, the $\sqrt{\ }$ indicates only the positive root. Because of this we get:

For any negative real number, we have the nth root rule: a, $\sqrt[n]{a^n} = |a|$, when n is even.

Simplify:

$\sqrt{9x^6} =$ rewrite radicand such that you have perfect squares where possible

$\sqrt{3^2(x^3)^2} =$ the exponent of x is 6/even so we use our nth root rule to solve this radical expression

$3|x^2|$

Now try:

$\sqrt[3]{8b^3} =$

$\sqrt[3]{2^3b^3} =$ rewrite the radicand such that you have the exponent 3 where possible

$2b$ while the exponent of our variable is even the index (cube root) is odd therefore if b is negative then the root must also be negative.

$\sqrt{4x^2y^4}$

rewrite the radicand such that you have exponents equal to the index
note the even exponents for the variables, remember the nth root rule

$\sqrt[3]{\frac{8}{216}}$

$\sqrt[2n]{x^{2n}}$