Name___________________________________

Algebra 2

Lesson 6-4

Solving Polynomial Equations

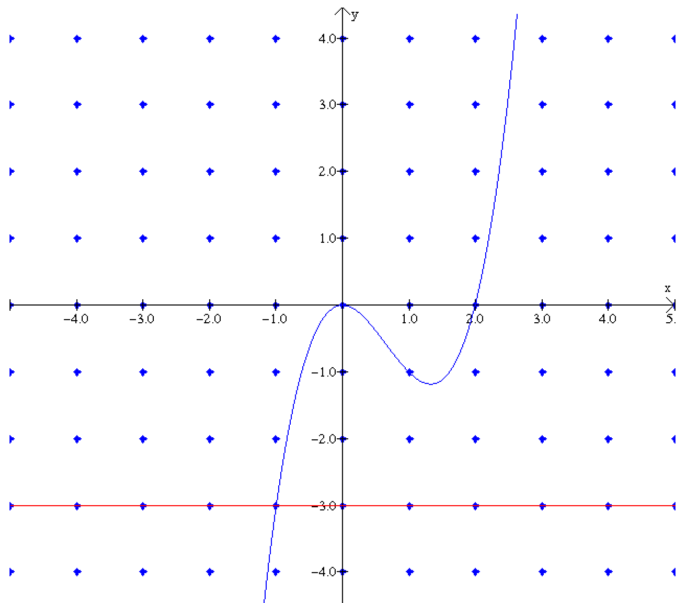
When an equation has a polynomial on each side, you can solve the equation by graphing each side separately and finding the x values at the points of intersection.

Example: Graph and solve: $x^3 - 2x^2 = -3$

1. Graph $y = x^3 - 2x^2$
2. Graph $y = -3$
3. To find the intersection point use: **2nd TRACE – 5:intersect**

Where do the two equations intersect?

This is very similar to what we did in Ch.3; we found the solution of systems of linear equations graphically by finding intersection of two lines



Cubic equations also have special factoring patterns as seen with **the Sum and Differences in Cubes:**

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example: Factor $8x^3 - 1$

$$= (2x)^3 - 1^3$$
$$= (2x - 1)((2x)^2 + (2x)(1) + 1^2)$$
$$= \mathbf{(2x - 1)(4x^2 + 2x + 1)}$$

To solve a polynomial equation: f(x)=0, we can use the same techniques we used in Ch. 5.

Solve:

$x^3 + 8 = 0$

$x^3 + 2^3 = 0$ — identify the **Sum of cubes**

$(x + 2)(x^2 - 2x + 4) = 0$ — **Zero-Product Property**

$(x + 2) = 0 \; or (x^2 - 2x + 4) = 0$ — use the quadratic formula to solve

$\boldsymbol{x = -2} \quad or \quad x = \frac{-(-2) \pm \sqrt{4 - 4(4)}}{2}$

$x = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = \mathbf{1 \pm i\sqrt{3}}$

When solving higher-degree polynomials, consider using substitution to make factoring easier:

Solve:

$x^4 + 11x^2 + 18 = 0, \quad let \; u = x^2 \; \therefore u^2 = x^4$ — substitute $u = x^2$ and rewrite the equation

$u^2 + 11u + 18 = 0$

$(u + 9)(u + 2) = 0$ — factor

$(x^2 + 9)(x^2 + 2) = 0$ — now put back into the $x \; form$ and solve for x

$(x^2 + 9) = 0 \quad or \; (x^2 + 2) = 0$

$x^2 = -9 \quad or \; x^2 = -2$

$\boldsymbol{x = \pm 3i \quad or \quad x = \pm i\sqrt{2}}$

Factor: $x^3 + 64$	Solve by graphing: $2x^3 + 5x^2 = 7x$
Solve: $2x^3 + 2 = 0$	Solve: $125x^3 + 216 = 0$
Factor and then solve: $x^4 - 8x^2 + 16 = 0$ (use substitution)	The product of three consecutive intergers: $n - 1,\ n,\ n + 1$ is 210. Write and solve an equation to find the numbers.