Name_____

Algebra 2

Lesson 6-3

Dividing Polynomials

When we have a number such as 74, we can use division to find the factors. Since 74 is even we know that 2 will be a factor, and we can find the factor pair by dividing: $\begin{array}{r} \mathbf{37}, \\ \mathbf{2}\,)\,\overline{\mathbf{74}} \end{array}$ hence, 2 and 37 are factor pairs.

Well, we can also divide a linear factor into a polynomial to find the factor:

Example:

Divide $x^2 + 3x - 12$ *by* $x - 3$

$$\begin{array}{r} x + 6 \\ x - 3\,)\overline{x^2 + 3x - 12} \\ -(x^2 - 3x) \\ \hline 6x - 12 \\ -(6x - 18) \\ \hline 6 \end{array}$$

So: $(x - 3)(x + 6) + 6 = x^2 + 3x - 12$

When there is a remainder, the proper form for the factor is: **(divisor)(quotient) + remainder**

1. look at the first term in each polynomial. Here, ask, *x goes into x^2 how many times?*
2. As with long division, multiply quotient by the divisor. and simplify; drop the next term from the dividend
3. Repeat the process of bringing down the next term followed by dividing, multiplying, and subtracting

How does this dividing help us? First, given a factor, we can simplify by dividing to find the factor pair. Secondly, we can verify if a polynomial is a factor of another polynomial. Is $x + 6$ a factor of $x^2 + 3x - 12$? NO. Why? Because there is a remainder!

A second type of division we can use which is quicker than long division is known as synthetic division. This technique works only when we have a **linear binomial in the form of $x - a$.**

Example:

Use synthetic division to divide

$3x^3 - 4x^2 + 2x - 1 \quad by \quad x + 1 = x - (-1)$

$$\begin{array}{r|rrrr} -1 & 3 & -4 & 2 & -1 \\ \times & & -3 & 7 & -9 \\ \hline & 3 & -7 & 9 & -10 \end{array}$$

Ans: $3x^2 - 7x + 9,\ remainder - 10$

If we write the quotient as the factorization of our polynomial, then we have:

$$(x + 1)(3x^2 - 7x + 9) - 10$$

1. Identify **a**. Note the minus sign.
2. Set up an "up-side-down" division symbol, with **a** in the box and the coefficients to the right. *Remember that the constant is a coefficient of x^0.*
3. Bring down the first term, 3, this is the start of the quotient.
4. Multiply – 1 and 3, locate under the second term, add terms, multiply – 1 by – 7, bring up and repeat until finished.
5. The numbers we now have are coefficients for the quotient. **Keep the degree of the variables straight!** Here the first term will be $3x^2$

The **Factor Theorem** says that if we have a polynomial P(x) and it is divided by $x - a$, the remainder is P(a)

Example:

Given $4x^4 - 5x^2 + 4x + 12$, find $P(-4)$ using synthetic division:

$$\begin{array}{r|rrrr} -4 & 4 & -5 & 4 & 12 \\ & & 16 & -44 & 160 \\ \hline & 4 & 11 & -40 & \mathbf{172} \end{array}$$

$P(-4) = 172$

1. Using the remainder theorem, we can divide by – 4 and find the remainder. *The remainder is equal to $P(-4)$*