Name________________________________

Algebra 2
Lesson 5-7
Completing the Square

When we had a binomial such as $4x^2 - 64 = 0$ we were able to either factor by the **difference in squares property** or by **square rooting.** Unfortunately, most quadratics don't come neatly squared like this. For your average everyday quadratic, you first have to use the technique of **completing the square** to rearrange the quadratic into the neat "(squared part) equals (a number)" format demonstrated above.

First off, you will need to be comfortable using the factoring formula of the **Perfect square trinomials:**

$$\boldsymbol{a^2 + 2ab + b^2 = (a+b)^2 \;\; OR \;\; a^2 - 2ab + b^2 = (a-b)^2}$$

For example, factor: :$\boldsymbol{9x^2 - 42x + 49}$;
where a=3x, b=7

$(3x)^2 - 42x + (7^2) =$ — 1. rewrite the first and third terms as perfect squares

$(3x)^2 - 2(3x)(7) + (7^2) =$ — 2. rewrite the linear term to verify that it is in fact in the form of 2ab

$(3x - 7)^2$ — 3. rewrite in terms $(a - b)^2$

Now you try: Factor the equation $\boldsymbol{4x^2 + 12x + 9}$ using the perfect square trinomial formula

So to complete the square, let's take a look at example below and follow the steps:

$4x^2 - 2x - 5 = 0$

$4x^2 - 2x = 5$ — 1. Move the constant (c) over to the right side

$x^2 - \frac{1}{2}x = \frac{5}{4}$ — 2. clear out the quadratic term's coefficient (a)

$\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = -\frac{1}{4}; \left(-\frac{1}{4}\right)^2 = \frac{1}{16}$ — 3. Take the coefficient of the linear term **halve it and square** it. *Remember the sign of the linear term, will need it in a couple steps!*

$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{5}{4} + \frac{1}{16}$ — 4. add this square to both sides of the equation (both sides so to keep the equation balanced).

$\left(x - \frac{1}{4}\right)^2 = \frac{20}{16} + \frac{1}{16}$ — 5. Since we have a negative, the square root of the constant will be negative
6. and simplify the right hand side of the equation.

$\left(x - \frac{1}{4}\right)^2 = \frac{21}{16}$

7. Now we can square root both sides of the equation and get

$\left(x - \frac{1}{4}\right) = \pm\sqrt{\frac{21}{16}} = \pm\frac{\sqrt{21}}{4}$ — 8. Solve for x and simplify the radical as needed

$x = \frac{1}{4} \pm \frac{\sqrt{21}}{4}$

Now you try: Factor the equation $\boldsymbol{x^2 - 10x = -11}$ using the completing the square method.

Complete the squares

$x^2 + 8x\_\_\_\_\_ = 0$ ******missing term = $\left(\frac{b}{2}\right)^2$

$x^2 + 5x\_\_\_\_\_ = 0$

$x^2 = 4x + 5$

solve by completing the square

$3x^2 = 2x - 4$

Rewrite the quadratic in Vertex Form using the *completing the square technique:*

$y = x^2 + 6x + 2$ (here keep all numbers on the right side of the equation)

Factor the perfect square trinomial

$9x^2 + 48x + 64;\quad a = \qquad , b = \qquad , c =$

Solve by finding square roots:

$3x^2 + 81 = 6$

The height of a punted football can be modeled with the quadratic function: $h = -0.01x^2 + 1.18x + 2$. The horizontal distance in feet from the point of impact with the kicker's foot is *x*, and *h* is the height of the ball in feet.

a) Find the vertex of the graph of the function by completing the square

b) What is the maximum height of the punt?

c) The nearest defensive player is 5 ft. horizontally from the point of impact. How high must the player reach to block the punt?