

Name _____

Algebra 2

Lesson 5-6

Complex Numbers

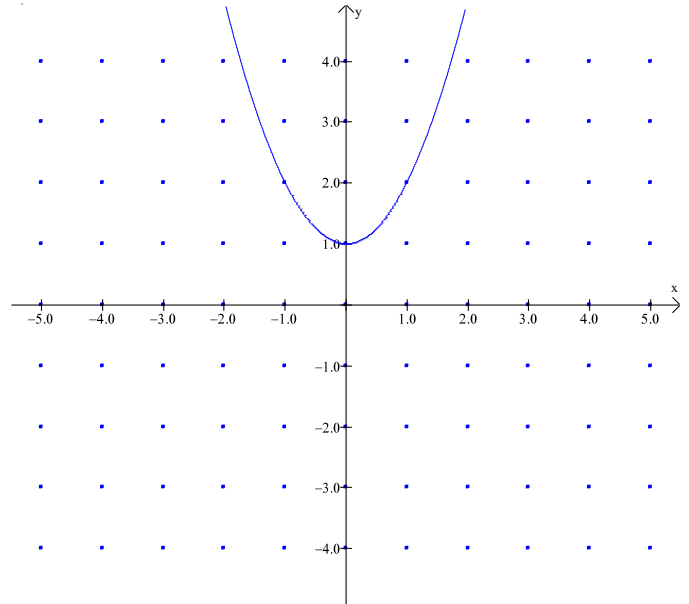
In order to solve equations and describe situations and the associated data, people invented numbers. Many of these number ideas were rejected or were met with resistance and were regarded as being imaginary or unreal. In fact, the number 0 was not invented at the same time as the natural numbers, and for that matter, the Romans had no numeral for 0. There were probably skeptics who wondered why it necessary to have a number to represent nothing. Then of course negative numbers met resistance as well. How could someone possibly need to count -6 oxen? The same was true for complex numbers. The catch is that complex numbers are no more imaginary than any other number created by mathematicians.

If you take a look at the graph of $y = x^2 + 1$, you will see that it has no x-intercepts.

Therefore, the equation $x^2 + 1 = 0$ has no real solutions.

Attempting to solve this equation symbolically, we get $x^2 = -1$, thus $x = \sqrt{-1}$ up to this point we have regarded this as having no solution, well this is true in part. It has no *real* solution, but it does have an *imaginary solution* as defined by:

$$i = \sqrt{-1}$$
$$i^2 = -1$$



Now, by inventing i , the solutions to the equation $x^2 + 1 = 0$, are i and $-i$.

Complex numbers may be defined by combining real numbers and the imaginary unit i . A **complex number** is written in standard form as $a + bi$, where a and b are real numbers. The **real part** is a and the **imaginary part** is bi .

Can a real number be a complex number? Yes! Because a real number may be written as $a + 0i$. When $b \neq 0$, we get an imaginary number: **bi**: **examples**: $3i$, $-2i$, $\frac{1}{2}i$, and so on. .

Example: Simplify: $\sqrt{-12} =$

$$\begin{aligned} & \sqrt{-1} \cdot \sqrt{12} = \\ & \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{3} = \\ & i \cdot 2 \cdot \sqrt{3} = 2i\sqrt{3} \end{aligned}$$

1. Using the product rule, separate real and imaginary parts (negative)
2. Now by definition, simplify (remembering, $i = \sqrt{-1}$)
3. And rearrange

Likewise we can simplify complex numbers into the form $a + bi$

Example: Write the complex number in the form $a + bi$

$$\begin{aligned} & \sqrt{-18} + 7 \\ & = \sqrt{-1}\sqrt{18} + 7 = \sqrt{-1}\sqrt{9 \cdot 2} + 7 = 3i\sqrt{2} + 7 \\ & = 7 + 3i\sqrt{2} \end{aligned}$$

1. simplify the radical expression
2. Now write in the form $a + bi$

Graphing complex numbers

Graphing a complex number is very much like graphing points on an x-y coordinate system. Here the horizontal axis is the real axis and the vertical axis is the imaginary axis:

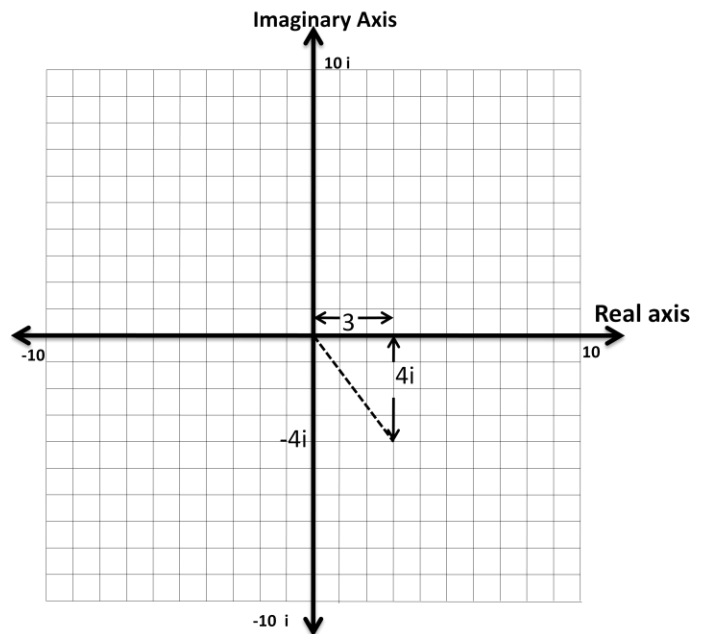
The **absolute value of a complex number** is the distance from the origin on the complex number plane.

It is found by using the Pythagorean Theorem:

$$|a + bi| = \sqrt{a^2 + b^2}$$

Example: Graph and find the absolute value of $|3 + 4i|$

$$|3 + 4i| = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$



- Other concepts from real numbers may be applied to complex numbers: **Additive inverse** is the sum of two numbers that equals 0.

Example: Find the additive inverse of $4 + 2i = -(4 + 2i) = -4 - 2i$

- Adding Complex numbers:** Add like terms, that is, add the real parts together and add the imaginary parts

Example: $(8 + 3i) - (2 + 4i) = (8 - 2) + (3i - 4i) = 6 - i$

- Multiplying Complex numbers:** Multiply the real terms together. Multiply the i 's getting i^2

Example: $(6 - 5i)(4 - 3i) = 24 - 18i - 20i + 15i^2 = 24 - 38i + (15)(-1) = 9 - 38i$

- AND** we can solve equations with complex solutions:

Example: $3x^2 + 48 = 0$

$$3x^2 = -48$$

$$x^2 = \frac{-48}{3} = -16$$

$$x = \sqrt{-16} = 4i$$

Simplify: $\sqrt{-81}$	Simplify: $(4 - 3i)(5 + 2i)$
Write in terms of $a+bi$: $\sqrt{-1} + 2$	Solve: $x^2 + 36 = 0$
Find the absolute value: $ 2i $	Find the first 3 out puts: $z^2 - i$ start with $z=0$