Name________________________________

Algebra 2

Lesson 4-8

Cramer's Rule

Suppose you have a system of equations and are only interested in the solution of one variable. With the matrix methods learned so far, this is not possible. Cramer's Rule is a theorem in linear algebra, which may be used to solve systems for only the desired variables. This theorem was derived by a Swiss mathematician, Gabriel Cramer, (1704-1752) and uses determinant values to solve for the variables.

So, given a system of equations: $2x + y + z = 3$

$$x - 2y - z = 0$$
$$x + 2y + z = 0$$

when made into a matrix equation, we get: $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$

We can now make several determinants out of this system. First, let **D** be the determinant of the coefficient matrix. Second, let $\mathbf{D_x}$ be the determinant formed by replacing the x- coefficient column values with the constant matrix. Next, let $\mathbf{D_y}$ be a determinant formed when you replace the y-column values with the constant matrix. Lastly, when you replace the z-column values with the constant matrix you get the $\mathbf{D_z}$ determinant. So we have:

$$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{vmatrix}, \quad D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix}, \quad D_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -2 & 0 \\ 1 & 2 & 0 \end{vmatrix}, \quad \text{and } D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

Now we can get our x value. All we need to do is solve the determinants D and D_x. We need to remember with a 3x3 matrix the determinant is solved by expanding the matrix by rewriting column 1 and 2, summing the products of the down-to-the-rights and subtracting the sum of the down-to-the-left products.

Here is the answer: $\boldsymbol{x = \frac{Dx}{D}}$

Likewise, $\boldsymbol{y = \frac{Dy}{D}}$ and $\boldsymbol{z = \frac{Dz}{D}}$

Example: Solve using Cramer's Rule

1. Set up determinants and solve
2. The fractions D_x /D and D_y/D solve
3. Check your answers for (x,y)

2x+y=8 x- y=-2	D= $\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$	D_x= $\begin{vmatrix} 8 & 1 \\ -2 & -1 \end{vmatrix} = -6$	D_y= $\begin{vmatrix} 2 & 8 \\ 1 & -2 \end{vmatrix} = -12$	$x = \frac{-6}{-3}$= 2 , $y = \frac{-12}{-3}$= 4