Name_________________________________

Algebra 2

Lesson 4-7

Inverse Matrices and Systems

Let's pick up with the question left unanswered from yesterday's lesson: what good is an inverse matrix?

You can use the inverse of a matrix to solve a system of equations. This process is in fact quite similar to solving an equation such as 5x=20. Multiply each side by 1/5 (the inverse of 5) in order to solve for x.

Before we can solve the systems of equations with the inverse, we must *write the* ***matrix equation:*** AX=B, where A is the **coefficient matrix**, X is the **variable matrix**, and B is the **constant matrix.**

Example: Represent the system of equations as a matrix equation and identify each part:

SYSTEM

$$\begin{cases} x + 2y = 5 \\ 3x + 5y = 14 \end{cases}$$

Matrix Equation:

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

Coefficient Matrix A	**Variable Matrix X**	**Constant Matrix C**
$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 5 \\ 14 \end{bmatrix}$

To solve systems follow the steps below:

1. Write the system as a matrix equation.
2. Find A^{-1}
3. Multiply $A^{-1}B$
4. Solve for the variable matrix.

Example: Solve the system of equations by using matrices.

$$\begin{cases} x + 2y = 5 \\ 3x + 5y = 14 \end{cases}$$

1.

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

2.

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

3.

$$\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 14 \end{bmatrix} = \begin{bmatrix} -53 \\ 29 \end{bmatrix}$$

4.

Answer $X = \begin{bmatrix} -53 \\ 29 \end{bmatrix}$

Can we solve systems of equations in 3 variables? 4 variables? Yes, *with the assistance of a calculator, we will!*

Example:

$$\begin{cases} 2x + y + 3z = 1 \\ 5x + y - 2z = 8 \\ x - y - 9z = 5 \end{cases}$$

1. Write the system as a matrix equation
2. Enter the coefficient matrix as matrix A and the constant matrix as matrix B on the calculator.
3. Enter $[A]^{-1}B$ into the calculator to solve for (x, y, z)

THINK The solution to a system of equations (x, y) is a point where the two lines intersect. It is a unique solution as there are infinite solutions to each linear equation but only one solution where the two particular lines intersect. In Ch. 3 we found that we could have situations where the lines were either the same line or parallel, and we had methods to determine this. Here we also have a method that will indicate no unique solution, parallel or same lines. The method is the determinant. **If the determinant equals 0 then either the lines are parallel or the same.**

PRACTICE

1. Solve each system of equations $x+5y=-4$ $x+6y=-5$	3. $9y+2z=18$ $3x+2y+z=5$ $x-y=-1$
2. $x+2y=5$ $2x+4y=8$	4. $-2w+x+y=-2$ $-w+2x-y+z=-4$ $-2x+3x+3y+2z=2$ $w+x+2y+z=6$