Name________________________________

Algebra 2
Lesson 4-5/4-5
Inverses 2x2 and 3x3

Recall that for any real number *a* that the number 1 is the multiplicative identity. That is $a \cdot 1 = 1 \cdot a$. Well, our square matrices also have multiplicative identities too. The matrix identity is called, the **multiplicative identity matrix.** and is equivalent to "1" in matrix terminology.
The identity matrix of a 2x2 and a 3x3 square matrix are:

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note: the identity matrix is Identified with a capital **I** and a subscript indicating the dimensions.

Example: Show that B is the inverse of A when:

$A=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ If B is the inverse then AB should equal the identity matrix, does it?

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2\cdot 2 + 3\cdot -1 & 2\cdot -3 + 3\cdot 2 \\ 1\cdot 2 + 2\cdot -1 & 1\cdot -3 + 2\cdot 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrices also have inverses just as real numbers. Remember that an inverse is a number that when multiplied by another number the product is one. Real numbers' inverses are their reciprocals. OK, for matrices the notation for the inverse of a matrix, let's say, matrix A is $\mathbf{A^{-1}}$. In other words:

$$\mathbf{A\,A^{-1}=A^{-1}A=I} \quad \text{(remember I, identity, is the "1" for matrices)}$$

Well, how do we find the inverse?

The inverse of a 2x2 matrix $\mathbf{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **is** $\mathbf{A^{-1}}=\frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ since we have det A in the denominator, the determinant **cannot** be equal to 0. So, a matrix with a determinant of 0 has no inverse and is called a **singular matrix**.

Example: Find the inverse of the matrix, if it is defined: A= $\begin{bmatrix} -2 & 2 \\ 3 & -4 \end{bmatrix}$

1. check to see det ≠0: det A= 8-6=2
2. set up inverse equation (note: switch *a* and *d*, and make *c* and *b* opposite sign)

$$\frac{1}{2}\begin{bmatrix} -4 & -2 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -3/2 & -1 \end{bmatrix}$$

Well, what good is an inverse matrix? You can use the inverse of a matrix to solve a system of equations. This process is in fact quite similar to solving an equation such as 5x=20. Multiply each side by 1/5 (the inverse of 5) in order to solve for x.

To solve a **matrix equation, AX=B**:

1. Multiply each side by A^{-1}
2. $A\,A^{-1}$=1, so we are left with X on the left side which is what we are solving for. So: <u>**X= A^{-1}B**</u>
3. **WARNING!!!** You must always keep the order of the matrices uniform! $A^{-1}B$ is NOT the same as $B\,A^{-1}$!

Example: Solve the matrix equation for X

$$\begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Find A^{-1} $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-2\cdot 3-(-5)1}\cdot\begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix} = -1\begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix}$

Find **$X = A^{-1}B$** **=** $\begin{bmatrix} -3 & -5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -3(-2) \; + & (-5)(2) \\ (1)(-2) & (2)(2) \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$

This method is used to solve systems of equations with the inverse; you first write the **matrix equation** AX=B, where A is the **coefficient matrix**, X is the **variable matrix**, and B is the **constant matrix.** *This we will do in section 4-7!!*

As with 2x2 matrices, when we multiply a 3x3 matrix with its inverse, we will get the identity matrix, I_3. So we can also show that a 3x3 matrix is the inverse of another 3x3 matrix in the same fashion as the 2x2 example on the first page.

The methods for finding 2x2 inverses also holds true for 3x3 inverses, BUT we use a calculator:

1. Using a calculator, enter the data for a 3x3 matrix and the matrix located on the right side of the equal sign
2. Now to calculate the inverse hit **MATRIX** select the matrix you want the inverse for and hit **ENTER**
3. **x^{-1}** (for example: $[A]^{-1}$) **ENTER** the view screen will show the inverse of the 3x3 matrix.
4. With the matrix inverse on the screen hit *** Matrix [B] ENTER** (will show Ans *[B], that is our inverse times the B matrix). The resulting matrix will be our answer, the matrix that equals X.

Below shows how matrix equations may be solved by using the inverse.

Example: Solve the matrix equation: $\begin{bmatrix} 0 & 0 & 2 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} 6 \\ -11 \\ 8 \end{bmatrix}$

$A^{-1}AX = A^{-1}B$
$X = A^{-1}B$

1. Find A^{-1}using the calculator $\begin{bmatrix} .1 & .4 & .6 \\ .3 & .2 & -.2 \\ .5 & 0 & 0 \end{bmatrix} = A^{-1}$
2. solve for X X= $\underset{A^{-1}}{\begin{bmatrix} .1 & .4 & .6 \\ .3 & .2 & -.2 \\ .5 & 0 & 0 \end{bmatrix}} \underset{B}{\begin{bmatrix} 6 \\ -11 \\ 8 \end{bmatrix}} = \underset{X}{\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}}$

PRACTICE

1. Solve:

$$\begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} -9 \\ -4 \end{bmatrix}$$

4. Solve: $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 3 & -2 \\ 1 & -2 & 1 \end{pmatrix} X = \begin{pmatrix} 0 \\ -6 \\ 19 \end{pmatrix}$

2. Determine if the matrix has an inverse:

$$\begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix}$$

3. Are the matrices multiplicative inverses?

$$\begin{bmatrix} 2 & .5 \\ 5 & 1 \end{bmatrix}, \quad \begin{bmatrix} -2 & 1 \\ 10 & -4 \end{bmatrix}$$