

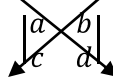
Name _____

Algebra 2
Lesson 4-5/4-6

Determinants: 2x2 and 3x3 Matrices

Recall a square matrix has the same number of rows and columns. Just like fractions have a common denominator, the elements in a square matrix also have a common “denominator”, called a **determinant**.

The determinant, abbreviated **det** and symbolized with $\begin{vmatrix} \end{vmatrix}$, it is a nonzero quantity (when the det=0 we have another situation that we will look at in the next lesson). For a 2x2 matrix, its determinant is found by subtracting the products of its diagonals:

Given a matrix $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, where a, b, c, and d are real numbers 

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

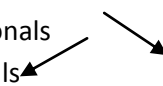
Example: Compute the determinant of $A = \begin{vmatrix} -3 & 4 \\ 2 & -5 \end{vmatrix}$

$$\begin{aligned} \det A &= \det \begin{vmatrix} -3 & 4 \\ 2 & -5 \end{vmatrix} \\ &= (-3)(-5) - (4)(2) \\ &= 15 - 8 \\ &= 7 \end{aligned}$$

One can also compute a determinant using a graphing calculator:

- Press **MATRIX** >> to **EDIT**. Down to **1:[A]**. **ENTER**
- Enter the matrix dimensions: # rows **ENTER** # columns **ENTER**. Enter the data for the 2x2 matrix in the matrix.
- Press **2nd MODE (QUIT)**
- Press **MATRIX** again. Go right once to **MATH**. Down to **1:det**.
- Press **MATRIX** again. Down to **1:[A]**. **ENTER**. Answer is displayed.

The computations for 3x3 determinants are messier than for 2x2's. Various methods can be used, but the simplest method is probably the following:

- Write down the determinant
- Expand the determinant by rewriting the first two columns of numbers
- Then multiply along the down-to-the-right-diagonals
- and multiply along the down-to-the-left-diagonals 
- Add the down-right-diagonals and subtract the down-left-diagonals

Example:

$$A = \begin{pmatrix} 4 & -2 & 0 \\ -3 & 10 & 1 \\ 2 & 6 & -1 \end{pmatrix} \quad \det A = \begin{vmatrix} 4 & -2 & 0 \\ -3 & 10 & 1 \\ 2 & 6 & -1 \end{vmatrix} \quad \text{expand: } \begin{matrix} 4 & -2 & 0 & 4 & -2 \\ -3 & 10 & 1 & -3 & 10 \\ 2 & 6 & -1 & 2 & 6 \end{matrix}$$

down to the right	minus	down to the left
(4)(10)(-1) + (-2)(1)(2) + (0)(-3)(6)	—	(0)(10)(2) + (4)(1)(6) + (-2)(-3)(-1)
-40 - 4 + 0	—	0 + 24 - 6
-44 - 18 = - 62		

A 3x3 determinant may be calculated on a calculator using the same steps as those for a 2x2