

Algebra 2
Lesson 3-6
Systems with Three Variables

In Sec. 3.2 we studied systems made up of 2 equations. We discovered that when we solved both equations for a point (x,y) , that this point is not only the a solution to both equations, but also the point where the two lines intersect. Thus, it is a solution to the system of equations. What if we are given a system of 3 equations containing 3 variables, x , y , and z ? Note that this system could not be graphed on the Cartesian plane; it may be graphed in a 3-dimension system. At this time we will not concern ourselves with graphing the system to determine a solution; rather we will look at the algebraic methods studied in 3.2 and apply them to 3 equations.

<p>Example: Solve the system of equations using the elimination method.</p> <ol style="list-style-type: none"> 1. $x + y + z = 6$ 2. $2x - y + 3z = 9$ 3. $-x + 2y + 2z = 9$ <p>Step 1 Add eq. 1 and 2 to eliminate y Step 2: Multiply 2 times eq 2 then add eq 2 and 3 to eliminate y. Step 3: With 2 equations in terms of x and z, follow the process outlined in 3.2 to solve for x and z. Step 4: Substitute x and z into one of the equations to solve for y.</p>	<p>Step 1</p> <ol style="list-style-type: none"> 1. $x + y + z = 6$ 2. $\underline{2x - y + 3z = 9}$ $3x + 4z = 15$ <p>Step 2</p> <ol style="list-style-type: none"> 2. $2(2x - y + 3z = 9)$ $4x - 2y + 6z = 18$ 3. $\underline{-x + 2y + 2z = 9}$ $3x + 8z = 27$ 	<p>Step 3</p> $\begin{array}{r} 3x + 4z = 15 \\ \underline{-(3x + 8z = 27)} \\ -4z = -12 \\ z = 3 \\ x = 1 \end{array}$ <p>Step 4 subbing x and z into:</p> $\begin{array}{r} x + y + z = 6 \\ 1 + y + 3 = 6 \\ y = 2 \end{array}$ <p>ANS: (1, 2, 3)</p>
----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

The key is to analyze the equations to see which may be combined to eliminate a variable with little or no alteration.

<p>Example: Solve the system of equations using the substitution method.</p> $\begin{array}{r} x + y + z = -8 \\ x - y - z = 6 \\ 2x - 3y + 2z = -1 \end{array}$ <p>Step 1 Solve one equation in terms of the other variables. Step 2 Substitute this term into both remaining equations for the variable. Step 3 Write the 2 equations as a system and solve as outlined in 3.2 Step 4 Once the two variable are solved, the first variable may then be calculated.</p>	<p>Step 1</p> $x = y + z + 6$ <p>Step 2</p> $\begin{array}{r} y + z + 6 + y + z = -8 \\ 2y + 2z = -14 \end{array}$ $\begin{array}{r} 2(y + z + 6) - 3y + 2z = -1 \\ 2y + 2z + 12 - 3y + 2z = -1 \\ -y + 4z = -13 \end{array}$ <p>Step 3 multiply second by 2 and add to first:</p> $\begin{array}{r} 2y + 2z = -14 \\ \underline{-2y + 8z = -26} \\ 10z = -40 \\ z = -4 \\ y = -3 \end{array}$	<p>Step 4</p> $\begin{array}{r} x - 3 - 4 = -8 \\ x = -1 \end{array}$ <p>Ans (-1, -3, -4)</p>
--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------