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Algebra 2
Lesson Square Roots and Radicals

Geometrically, a square has four equal sides and the area of a square is the product of any two of its sides. Thus, a square with sides of 5 cm. would have an area of $5 \cdot 5\ or\ 25\ cm^2$. Imagine being given the area of a square, 81 cm^2. What are the lengths of its sides? The answer can be found by going backwards from 81; that is, what number times itself is 81? In this case, the sides would be 9 cm long because $9 \cdot 9 = 81$.

In mathematical terms, we use a square root to find the length of a square's sides and use the operation "$\sqrt{\ }$"(a radical sign) to symbolize "square root." The number inside a square root is sometimes called a **radicand** and the positive square root is called the **principal root.**

principal root

$$\sqrt{\mathbf{81}} = \mathbf{9}$$

radicand

A square root of a number in fact has two possible square roots. For example $\sqrt{5} = -5\ or\ 5$ because $5 \cdot 5 = -5 \cdot -5 = 25$. To indicate both positive and negative square roots, we can use the symbol "$\pm$". Thus, $\sqrt{25} = \pm 5$.

There are two basic square root properties:

1. **Product Property** of Square Roots, which states that the square root of a number is equal to the product of the square roots of the factors

   example: $\sqrt{\mathbf{20}} = \sqrt{\mathbf{4} \cdot \mathbf{5}} = \sqrt{\mathbf{4}} \cdot \sqrt{\mathbf{5}} = \mathbf{2}\sqrt{\mathbf{5}}$ ⟶ $\sqrt{ab} = \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

   $\sqrt{\mathbf{5}} \cdot \sqrt{\mathbf{20}} = \sqrt{\mathbf{5} \cdot \mathbf{20}} = \sqrt{\mathbf{100}} = \pm\mathbf{10}$ $\qquad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} = \sqrt{ab}$

2. **Quotient Property** of Square Roots, states that the square root of a quotient is equal to the quotient of the square roots of the dividend and the divisor.

   example: $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$ ⟶ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

   $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{\mathbf{4}} = \pm\mathbf{2}$ ⟶ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

   Square roots that have the same radicand are called like radical terms.
   Example: $4\sqrt{5}\ and\ 7\sqrt{5}$ are *like radicals*.

   To "rationalize" a denominator simply means to make the denominator into an integer by multiplying it with an identical square root. For example: $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$