Chapter 2
The Real Number System

2.1 Introduction

In this chapter we discuss the basic assumptions (axioms) about real numbers, together with certain definitions, and derive other properties of real numbers.

The set of *natural numbers*, denoted by $\mathbb{N}$, which consists of the ordinary counting numbers 1, 2, 3, . . . may be considered as the simplest real numbers. The counting numbers are also called *positive integers*. The natural numbers together with 0 and their negatives: –1, –2, –3, . . . comprise the *integers* and the set consisting these numbers is denoted by $\mathbb{Z}$. The set of *rational numbers*, denoted by $\mathbb{Q}$, consists of numbers which can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Observe that any integer m can be written as $\frac{m}{1}$, thus the set of rational numbers include the set of integers as a subset. Numbers that are not rationals are called *irrationals*. The set of irrationals is denoted by $\overline{\mathbb{Q}}$. Examples of irrationals are

$$\sqrt{2}\,,\ \pi\,,\ \sqrt[3]{6}\,.$$

The rational numbers together with the irrational numbers form the *set of real numbers*, denoted by $\mathbb{R}$.

Notations: One may visualize the set of real numbers using a *number line*, as shown in the Figure 2.1 below.

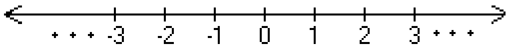

Figure 2.1 Real number line

Select a point as the *origin* to correspond to 0 and another point to the right of 0 to correspond to 1. A scale is now determined. To the left of zero the negative real numbers will be located while to the right are the positive real numbers. Using the concept of fractions we can easily locate the point corresponding to a rational number. For example, to locate the point corresponding to $\frac{3}{4}$, divide the line segment from 0 to 1 into 4 equal parts, then the right endpoint of the 3rd division would be the desired point. A point corresponding to an irrational number is more difficult to locate than that of a rational number. The irrationals, however, can always be approximated by rationals. For example, $\sqrt{2}$ is approximately 1.414.

2.2 **Decimal and Fractional Representation of Rational Numbers**

A rational number $\frac{a}{b}$, $b \neq 0$, can be expressed as a decimal by dividing a by b. The rational numbers have two forms when expressed as decimal expansions: (1) terminating such as 0.75 and (2) repeating such as 0.3333… = $0.\overline{3}$, where the bar above 3 means that 3 is the repeating digit.

Illustration 1.

a) $\frac{4}{5} = .8$

Solution:

$$\begin{array}{r} .8 \\ 5\,\overline{)4.0} \\ \underline{4.0} \\ 0 \end{array}$$

b) $\frac{2}{9} = 0.\overline{2}$

Solution:

$$\begin{array}{l} 0.22.. \\ 9\,\overline{)2.000} \quad \text{or} \quad 0.\overline{2} \\ \underline{1.8} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

On the other hand, a rational number expressed as a decimal expansion can be expressed as a fraction $\frac{a}{b}$, $b \neq 0$: if the decimal is a terminating decimal, we multiply it by $\frac{10^k}{10^k}$, where k is the number of digits after the decimal. The expression 10^k means that we multiply the number 10 k times.

Illustration 2.

a) $0.75 = 0.75\left(\frac{10^2}{10^2}\right) = \frac{75}{100}$

b) $0.6 = 0.6\left(\frac{10}{10}\right) = \frac{6}{10}$

Before we discuss the case where the decimal is a repeating decimal, we pause for a moment to discuss properties of equality. In mathematics, the symbol A = B means A and B are two names for the same thing. Thus, we may replace B by A or A by B in any expression involving either symbol. Thus, equality means ‘is identical to’. The following are the properties of equality.

1. *Reflexive Property*. If $a \in \mathbb{R}$, then $a = a$.
2. *Symmetric Property*. If $a, b \in \mathbb{R}$ and $a = b$, then $b = a$.
3. *Transitive Property*. If $a, b, c \in \mathbb{R}$ and $a = b$, $b = c$, then $a = c$.
4. *Addition Property*. If $a = b$, then $a + c = b + c$.
5. *Multiplication Property*. If $a = b$, then $ac = bc$.
6. *Cancellation Laws*. (i) If $ac = bc$ and $c \neq 0$, then $a = b$.
   (ii) If $a + c = b + c$, then $a = b$.

Properties 4 and 5 are our assurance that if the same number is added or multiplied to both sides of equality, the equality is maintained.

To express a repeating decimal to a fraction we suggest the following procedure.

Step 1. Let x = repeating decimal.
Step 2. Multiply both sides of the equality in (1) by 10^m, where m is the number of digits after the decimal before the repeating block.
Step 3. Multiply the resulting equality in Step 2 by 10^n, where n is the number of digits in the repeating block.
Step 4. Subtract the equality obtained in Step 2 from that obtained in Step 3.
Step 5. Divide both sides of the equality obtained in Step 4 by the coefficient of x to obtain the desired fraction.

Example.

a) Express $0.\overline{24}$ as a fraction.

Solution: $x = 0.\overline{24}$; $m = 0$, $n = 2$.

$$x = 0.\overline{24} = 0.24\overline{24}$$

$$10^2(x = 0.24\overline{24})$$

$$100x = 24.\overline{24}$$

$$-\quad x = 0.\overline{24}$$

$$99x = 24$$

$$x = \frac{24}{99}.$$

b) Express $0.1\overline{23}$ as a fraction.

Solution: $x = 0.1\overline{23}$; $m = 1$, $n = 2$.

$$10x = 1.\overline{23} = 1.23\overline{23}$$

$$10^2(10x = 1.23\overline{23})$$

$$\begin{aligned} 1000x &= 123.\overline{23} \\ -\ \ 10x &= \ \ \ 1.\overline{23} \\ \hline 990\,x &= 122 \\ x &= \frac{122}{990}. \end{aligned}$$

c) Express $2.13\overline{4}$ as a fraction.

Solution: $x = 0.\overline{24}$; $m = 2,\ n = 1.$

$$\begin{aligned} 100x &= 213.\overline{4} = 213.4\overline{4} \\ 10(100x &= 213.4\overline{4}\,) \\ 1000x &= 2134.\overline{4} \\ -\ \ 100x &= 213.\overline{4} \\ 900x &= 1921 \\ x &= \frac{1921}{900}. \end{aligned}$$

Exercises 2.2

A. Change the following fraction to decimal.

1. $\frac{2}{15}$
2. $\frac{3}{8}$
3. $\frac{2}{37}$
4. $\frac{8}{15}$
5. $\frac{27}{6}$
6. $\frac{5}{11}$
7. $\frac{3}{24}$
8. $\frac{6}{13}$
9. $\frac{13}{15}$
10. $\frac{18}{21}$
11. $\frac{6}{7}$
12. $\frac{5}{17}$
13. $\frac{26}{5}$
14. $\frac{9}{7}$
15. $\frac{1}{9}$
16. $\frac{8}{9}$
17. $\frac{19}{36}$
18. $\frac{33}{4}$
19. $\frac{16}{3}$
20. $\frac{15}{14}$

B. Express each terminating decimal number as a fraction.

1. 0.25
2. 3.128
3. 1.38
4. 7.24
5. 2.6
6. 3.125
7. 20.4
8. 1.295
9. 2.33
10. 2.2580
11. 1.82
12. 0.44
13. 25.64
14. 0.284
15. 8.125
16. 0.3750
17. 2.65
18. 2.495
19. 0.12
20. 3.24

C. Express each repeating decimal number as a fraction.

1. $0.\overline{8}$
2. $2.53\overline{9}$
6. $2.01\overline{32}$
7. $0.\overline{54}$
11. $4.\overline{13}$
12. $2.01\overline{3}$
16. $0.1\overline{25}$
17. $0.2\overline{5}$

3. $4.\overline{15}$
4. $2.15\overline{3}$
5. $1.\overline{25}$
8. $0.\overline{2}$
9. $0.\overline{83}$
10. $3.\overline{012}$
13. $1.3\overline{051}$
14. $2.\overline{113}$
15. $0.10\overline{23}$
18. $3.\overline{41}$
19. $1.\overline{24}$
20. $3.15\overline{1}$

2.3 Basic Properties of Real Numbers

The following are the basic properties of the real numbers involving the four fundamental operations: addition, subtraction, multiplication, and division. These properties are called *field properties* of $\mathbb{R}$.

1. *Closure Property*: The set $\mathbb{R}$ is *closed* with respect to addition and multiplication. That is, *both* $a + b$ *and* $a \cdot b$ *are in* $\mathbb{R}$.

2. *Commutative Property:* Addition and multiplication in $\mathbb{R}$ are *commutative.* That is, $a + b = b + a$ *and* $a \cdot b = b \cdot a$.

3. *Associative Property*: Addition and multiplication in $\mathbb{R}$ are *associative.* That is, $a + (b + c) = (a + b) + c$ *and* $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.

4. *Distributive Property of Multiplication over Addition* (*DPMA*): Multiplication is *distributive* over addition. That is, $a(b + c) = ab + ac$.

5. *Existence of Identity Element*: The number 0 *is the additive identity* in $\mathbb{R}$, and the number 1 *is the multiplicative identity* in $\mathbb{R}$. That is, $a + 0 = 0 + a = a$ *and* $a \cdot 1 = 1 \cdot a = a$.

6. *Existence of Inverse Element*: For each element a in $\mathbb{R}$ there is a unique *additive inverse* in $\mathbb{R}$, designated by $-a$, and for each $a \neq 0$, there is a unique *multiplicative inverse (or reciprocal)* in $\mathbb{R}$, designated by $\frac{1}{a}$. Thus, $a + (-a) = (-a) + a = 0$ *and* $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

Other Properties of $\mathbb{R}$:

1. The sum of two positive real numbers is positive.
2. The product of two positive real numbers is positive.

Using Property 6, the difference and quotient of two real numbers are defined as follows:

$$a - b = a + (-b) \quad ; \quad \frac{a}{b} = a \cdot \frac{1}{b} \quad , \; b \neq 0.$$

It should be noted that $\frac{a}{0}$ is not defined because 0 has no multiplicative inverse. This is why the denominator of a fraction cannot be 0; that is, we cannot divide by 0.

Further Properties of $\mathbb{R}$:

1. $a \cdot 0 = 0$
2. $-a = (-1)\,a$
3. $(-a)(b) = -(ab)$
4. $(-a)((-b) = ab$
5. If $ab = 0$, then either $a = 0$ or $b = 0$.

The preceding properties can be derived from the basic properties of $\mathbb{R}$.

Example 1. Show that $a \cdot 0 = 0$.

Solution:

$$\begin{aligned} a \cdot 0 &= a \cdot (0 + 0) \\ a \cdot 0 &= a \cdot 0 + a \cdot 0 \text{ , by DPMA} \\ (a \cdot 0) - (a \cdot 0) &= a \cdot 0 \\ 0 &= a \cdot 0. \end{aligned}$$

Example 2. Show that $(-a)(b) = -(ab)$.

Solution:

$$\begin{aligned} a + (-a) &= 0 \\ [a + (-a)] \cdot b &= 0 \cdot b \\ ab + (-a)b &= 0 \end{aligned}$$

The last equation says that $(-a)b$ is the additive inverse of ab. Thus, $(-a)b = -(ab)$.

Exercise 2.3

Identify the property which justifies the given statement. Let the variables represent real numbers.

1. $xy + 0 = xy$.
2. $a + b = a + b$.
3. $3\,(x + 5) = 3x + 15$.
4. $\left(-\frac{4}{5}\right)\left(-\frac{5}{4}\right) = 1$.
5. $(wy)\,z + 5 = w\,(yz) + 5$.
6. $(wy)\,z + 5 = (yw)\,z + 5$.
7. $-\sqrt{3} + \sqrt{3} = 0$.
8. If $x = 2$ and $2 = y$, then $x = y$.
9. If $ac + d = t$, then $t = ac + d$.
10. $uv + 7 = 7 + uv$.
11. $(2 + 4) \cdot 5 = 5 \cdot (2 + 4)$
12. $3 + \sqrt{3}$ is a real number.
13. $-\frac{1}{2} + \frac{1}{2} = 0$.
14. $2(3x + 4) = 6x + 8$.
15. $3 + (6 + 5) = (3 + 6) + 5$.
16. $2\left(\frac{1}{2}\right) = 1$.
17. $\frac{1}{3} = \frac{1}{3}$.
18. If $x = a$ and $a = 4$, then $x = 4$.
19. If $5x = 20$, then $x = 4$.
20. $6 \cdot (2 \cdot 5) = (6 \cdot 2) \cdot 5$.

2.4 Order Properties, the Law of Trichotomy, Absolute Value and Intervals on the Real Line

Definition 2.4.1 Given two real numbers a and b, we say that *a is less than b*, written $a < b$, if $b - a$ is positive. Alternatively, we may say *b is greater than a*, and write $b > a$. If *a is either less than or equal to b*, we write $a \leq b$. Alternatively, we say *b is greater than or equal to a*, and write $b \geq a$.

Now, if c is positive, $c - 0 = c$ is positive. Thus, $c > 0$ is equivalent to saying that c is positive.

Theorem 2.4.1 Let $a, b, c \in \mathbb{R}$.

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$, then $ac < bc$ provided $c > 0$.
3. If $a < b$, then $ac > bc$ provided $c < 0$.

Proof:

1. Since $a < b$, $b - a$ is positive. Then
$$\begin{aligned}(b-a) + 0 &= (b-a) + (c-c)\\ &= (b+c) - (a+c)\end{aligned}$$
is positive. Thus, $a + c < b + c$.

2. Since $a < b$, $b - a$ is positive. If c is positive, that is $c > 0$, $(b-a)c$ is positive. That is, $bc - ac$ is positive. By definition, $ac < bc$.

3. Since $a < b$, $b - a$ is positive. If $c < 0$, then $(b-a)c = (b-a)\,0$, by Property (2). Thus, $bc - ac < 0$, by DPMA or $bc < ac$, by Property (1).

The Law of Trichotomy

If $a \in \mathbb{R}$, then one and only one of the following is true:

(i) a is positive;
(ii) a is zero;
(iii) a is negative.

This law subdivides $\mathbb{R}$ into 3 mutually disjoint subsets: $\mathbb{R}^-$ the set of negative real numbers
$\mathbb{R}^+$ the set of positive real numbers
$\{0\}$

and that $\mathbb{R} = \mathbb{R}^- \cup \{0\} \cup \mathbb{R}^+$.

The absolute value of a real number is defined as follows:

Definition 2.4.2 Let $a \in \mathbb{R}$. The *absolute value of a* is defined as

$$|a| = \begin{cases} a & \text{if } a \text{ is positive} \\ 0 & \text{if } a = 0 \\ -a & \text{if } a \text{ is negative} \end{cases}$$

Illustration 1. $|9| = 9$, $|0| = 0$, $|-2| = 2$, $\left|-\frac{1}{5}\right| = \frac{1}{5}$.

Interval Notations

The symbols $a < b$, $a \leq b$ defined in Definition 2.1 are called inequalities. Intervals on the real line are defined using inequalities. For example, the set of all real numbers x that satisfy the inequality $a < x < b$ is called an *open interval*, denoted by (a, b). That is,

$$(a, b) = \{x : a < x < b\}.$$

The endpoints a and b are not included in the set. This inequality, $a < x < b$, is called a *compound inequality*. This is read as " a is less than x and x is less than b." Graphically, this interval is shown in Figure 2.1 below:

Figure 2.1

The table below summarizes the various kinds of intervals and their graphs on the number line.

Name	Symbol	Definition	Graph
Open interval	(a, b)	$\{x : a < x < b\}$	
Closed interval	$[a, b]$	$\{x : a \leq x \leq b\}$	
Half – open intervals	$(a, b]$	$\{x : a < x \leq b\}$	
	$[a, b)$	$\{x : a \leq x < b\}$	
Infinite intervals	$[a, \infty)$	$\{x : x \geq a\}$	
	(a, ∞)	$\{x : x > a\}$	
	$(-\infty, b)$	$\{x : x < b\}$	
	$(-\infty, b]$	$\{x : x \leq b\}$	
	$(-\infty, \infty)$	$\{x : -\infty < x < \infty\}$	

In an interval of the form $(a\ ,\ b]$, the parenthesis means that the endpoint a is not included in the set, while the bracket means that the endpoint b is included. The symbols ∞ and $-\infty$ are read "infinity" and "negative infinity," respectively. These symbols do not represent real numbers. Their use is simply shorthand for writing intervals that extend indefinitely.

Illustration 2. a) $[1\ ,3] = \{x : 1 \le x \le 3\}$:

b) $(-6\ ,\infty) = \{x : x > -6\}$:

c) $(-\infty\ , 5] = \{x : x \le 5\}$:

Exercises 2.4

Write the following sets in interval form and sketch the graph.

1. $\{x : x > 9\}$
2. $\{x:\ -6 < x < -2\}$
3. $\{x : 3\ \le\ x\ \le\ 5\ \}$
4. $\{x:\ -1\ \le\ x\ \ \le\ 5\}$
5. $\{x: x\ \ge\ -1\}$
6. $\{x:\ x > 8\}$
7. $\{x: x\ < 0\ \}$
8. $\{x:\ -7 < x\ \le\ 2\}$
9. $\{x:\ x < -4\ \}$
10. $\{x:\ x < 10\}$
11. $\{\ x : x \ge\ 6\ \}$
12. $\{\ x : x > 5/2\}$
13. $\left\{x : x < -\dfrac{3}{4}\right\}$
14. $\left\{x : x \le -\dfrac{5}{6}\right\}$
15. $\{x : x \le\ 8.2\}$
16. $\left\{x : x > \dfrac{3}{5}\right\}$
17. $\left\{x : x \ge \dfrac{15}{4}\right\}$
18. $\left\{x : x < \dfrac{23}{5}\right\}$
19. $\{x : -3\ \le\ x < 4\}$
20. $\{x :\ 0.5 < x < 5\}$

2.5 Factorization of positive Integers and the Concepts of GCF and LCM

Definition 2.5.1 If n, m, s are positive integers and $m \cdot s = n$, then m and s are said to be *factors* or *divisors* of n and n is said to be a multiple of m and s.

Illustration 1. $21 = 7 \cdot 3$ so that 7 and 3 are factors of 21 and 21 is a multiple of 3 and 7.

Since $n = n \cdot 1$ for any integer n, 1 is a factor of any integer n. The integer 0 is a multiple of any integer n since $0 = n \cdot 0$.

All positive integers except 1 may be classified as either *composite* or *prime*. A positive integer is called composite if it is different from 1 and has factors other than 1 and itself. A positive integer is called prime if it is not 1 and its factors are only 1 and itself.

Illustration 2. Since $6 = 3 \cdot 2$, $9 = 3 \cdot 3$, $12 = 4 \cdot 3$, 6, 9, and 12 are composite; while 2, 3, 5, and 7 are prime numbers.

The *Fundamental Theorem of Arithmetic* which states 'Any positive integer greater than 1 can be written as a product of primes in a unique way except for the order of the factors,' and which is proved in higher mathematics, establishes the relationship between primes and composites.

Illustration 3. a) $20 = 10(2) = 5(2)(2) = 5 \cdot 2^2$

b) $72 = 8(9) = 2(2)(2)(3)(3) = 2^3 \cdot 3^2$

c) $120 = 6(20) = 2(3)(10)(2) = 2(3)(5)(2)(2) = 2^3 \cdot 3 \cdot 5$

A positive integer may also be classified as *even* or *odd*. A positive integer is said to be *even* if it is a multiple of 2. That is, if x is an even integer,

$$x = 2k, \text{ where } k \text{ is an integer.}$$

For example 2, 4, 6, 8 are even integers. 2 is the only even integer that is prime. All other even integers are composite. A positive integer that is not even is said to be *odd*. In general, an odd integer can be written as

$$y = 2k + 1, \text{ where } k \text{ is an integer.}$$

For example 1, 3, 5, 7, 9 are odd integers. Many odd integers are prime like 3, 5, 7, 11.

Two integers are said to be *relatively prime* if they have no common prime factors. For example 15 and 32 are relatively prime, but 15 and 18 are not relatively prime because 3 is a factor of both. We may also say that two integers are relatively prime if their *greatest common factor* (gcf) is 1. To obtain the greatest common factor of two integers express each integer as a product of primes. The gcf is the product of all their common factors.

Example 1. Find a) gcf (8 , 24) b) gcf (42 , 48)

Solution:

a) $8 = 2(2)(2)$

$24 = 2(2)(2)(3)$

$\text{gcf}(8, 24) = 2(2)(2) = 8.$

b) $42 = 2(3)(7)$

$48 = 2(2)(2)(2)(3)$

$\text{gcf}(42, 48) = 2(3) = 6.$

The *least common multiple* (*lcm*) of two integers is the least integer which is a multiple of both integers. The concept of *lcm* is important in addition of fraction which is discussed in the next section. To find the *lcm* (k , n) where k and n are integers, express k and n as a product of prime factors. The *lcm* is the product of distinct primes with the highest power appearing in the prime factorization of k and n.

Example 2. Find a) *lcm* (15 , 20) b) *lcm* (8 , 12) c) *lcm* (45 , 72)

Solution:

a) $15 = 5(3)$
$20 = 5(2)(2) = 5(2^2)$
$lcm\,(15 , 20) = 5(3)(2^2) = 60$

b) $8 = 4(2) = 2^3$
$12 = 4(3) = 2^2\,(3)$
$lcm\,(8 , 12) = 2^3\,(3) = 24$

c) $45 = 9(5) = 3^2\,(5)$
$72 = 9(8) = 3^2\,(2^3)$
$lcm\,(45 , 72) = 3^2\,(5)(2^3) = 360$

Exercises 2.5

A. Find the prime factorization of the following.

1. 121
2. 360
3. 1,001
4. 2,185
5. 1,000,000
6. 136
7. 340
8. 230
9. 624
10. 410
11. 275
12. 570
13. 890
14. 160
15. 720
16. 250
17. 625
18. 415
19. 218
20. 116

B. For each pair of numbers find a) gcf b.) *lcm*.

1. (15 , 45)
2. (9 , 14)
3. (21 , 49)
4. (48 , 56)
5. (15 , 21)
6. (20 , 90)
7. (15 , 25)
8. (24 , 32)
9. (35 , 49)
10. (18 , 64)
11. (8 , 14)
12. (15 , 24)
13. (12 , 30)
14. (11 , 20)
15. (6 , 13)
16. (16 , 21 , 25)
17. (13 , 18 , 21)
18. (9 , 12 , 15)
19. (3 , 11 , 26)
20. (9 , 14 , 20)

2.6 The Arithmetic of Fractions

The primary goal in this section is to derive the rules for adding and multiplying fractions. Let m and n designate arbitrary positive integers. Then $-m$ and $-n$ are negative integers, and the rules listed below apply.

1. $$m + (-n) = \begin{cases} +(m-n) & \text{if } m \geq n \\ -(n-m) & \text{if } n > m \end{cases}$$

2. $m + (-n) = \begin{cases} +(m-n) & \text{if } m \geq n \\ -(n-m) & \text{if } n > m \end{cases}$
3. $(-m) + (-n) = -(m+n)$
4. $m(-n) = -(mn)$
5. $(-m)(-n) = mn$

Now let a, b, c, d denote integers with b and d not equal to 0. Then $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers. A fraction $\frac{a}{b}$ is said to be in *lowest terms* if a and b are relatively prime. To reduce fractions to lowest terms, express both the numerator and denominator as product of its prime factors, then cancel common factors.

Illustration 1. a) $\frac{16}{12} = \frac{2^4}{2^2 \cdot 3} = \frac{\cancel{2}^2 \cdot 2^2}{\cancel{2}^2 \cdot 3} = \frac{4}{3}$

b) $\frac{72}{108} = \frac{9 \cdot 8}{2 \cdot 54} = \frac{9 \cdot 8}{2 \cdot 9 \cdot 6} = \frac{\cancel{3}^2 \cdot 2^{\cancel{3}^1}}{\cancel{2}^{\cancel{2}} \cdot \cancel{3}^2 \cdot 3} = \frac{2}{3}$

The product of two fractions is obtained using the rule:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad .$$

That is, the product of two fractions is the product of their numerators over the product of their denominators and reducing answer to lowest terms.

Illustration 2. a) $\frac{3}{4} \cdot \frac{8}{15} = \frac{\cancel{3}(\cancel{8})^2}{\cancel{4}(1\cancel{5})_5} = \frac{2}{5}$

b) $\frac{24}{45} \cdot \frac{12}{40} = \frac{2\cancel{4}^3(1\cancel{2})^4}{4\cancel{5}_{15}(4\cancel{0})_5} = \frac{1\cancel{2}^4}{7\cancel{5}_{25}} = \frac{4}{25}$

Division of fraction follows the rule:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

That is, to divide $\frac{a}{b}$ by $\frac{c}{d}$, change the division sign to multiplication and invert the divisor.

Illustration 3. a) $\frac{5}{6} \div \frac{25}{8} = \frac{\cancel{5}}{\cancel{6}_3} \cdot \frac{\cancel{8}^4}{2\cancel{5}_5} = \frac{4}{15}$

b) $\frac{26}{42} \div \frac{13}{14} = \frac{2\cancel{6}^2}{4\cancel{2}_6} \cdot \frac{1\cancel{4}^2}{1\cancel{3}} = \frac{\cancel{4}^2}{\cancel{6}_3} = \frac{2}{3}$

Addition of fractions with the same denominator follows the rule:

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$$

When the fractions have different denominators, addition is done using the rule:

$$\frac{a}{b}+\frac{c}{d}=\frac{a\left[\frac{lcm(b,d)}{b}\right]+c\left[\frac{lcm(b,d)}{b}\right]}{lcm(b,d)}$$

The *lcm* (*b* , *d*) is also known as the *least common denominator* (*lcd*).

Illustration 4. a) $\frac{1}{3}+\frac{4}{3}=\frac{1+4}{3}=\frac{5}{3}$

b) $\frac{3}{4}+\frac{5}{6}=\frac{3}{2^2}+\frac{5}{2(3)}=\frac{3(3)+5(2)}{2^2\cdot 3}=\frac{19}{12}$

c) $\frac{5}{8}-\frac{3}{20}=\frac{5}{8}+\frac{-3}{20}=\frac{5(5)-3(2)}{4\cdot 2\cdot 5}=\frac{25-6}{40}=\frac{19}{40}$

Exercises 2.6

Perform the indicated operations.

1. $5 + 4$
2. $9 + 3$
3. $7 + 6$
4. $8 + 7$
5. $11 + 15$
6. $(-5) + (-4)$
7. $(-9) + (-3)$
8. $(-7) + (-6)$
9. $(-8) + (-7)$

21. $-15 - (-5)$
22. $-8 - (-4)$
23. $10 - (-3)$
24. $14 - (-17)$
25. $-9 + 5 - (-4)$
26. $3\,(8)$
27. $9\,(-4)$
28. $-7\,(6)$
29. $(-4)(-7)$

10. $(-11)+(-15)$

11. $9+(-5)$

12. $11+(-8)$

13. $16+(-9)$

14. $12+(-4)$

15. $\dfrac{28}{4}$

16. $\dfrac{36}{-9}$

17. $\dfrac{5}{6} - \dfrac{3}{7}$

18. $\dfrac{3}{5}+\dfrac{1}{4}$

19. $\dfrac{5}{9}\cdot\dfrac{2}{3}$

20. $\dfrac{2}{7}\cdot\dfrac{3}{5}$

30. $\dfrac{3}{5}+\dfrac{7}{15}$

31. $\dfrac{5}{8}+\dfrac{3}{20}$

32. $\dfrac{-4+5}{-7+7}$

33. $\dfrac{5}{4}+\dfrac{4}{3}-\dfrac{3}{2}$

34. $\dfrac{3(-2)(1)(-2)}{-4+7}$

35. $\dfrac{4(-5)(-2)(-1)}{2(-1)(3)(-2)}$

36. $\dfrac{4(-2)(0)(-3)}{-4-2}$

37. $\dfrac{-24+(-16)}{-5-3} + \dfrac{8-(-6)}{-1-6}$

38. $\dfrac{2+3}{3(-2)+4} \div \dfrac{10+3(-4-1)}{2(5-3)}$

39. $\dfrac{25-(-3)}{3+(-2)} \cdot \dfrac{14-2(2+4)}{3-2(5-2)}$

40. $\dfrac{21(33-30)}{3(8-15)} \div \dfrac{-5-(-7)}{-7+5}$