Quantum Effects in Relativistic Decays

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By using both explicit time-dependent and S-matrix formalisms of relativistic quantum theory we calculate the decay law of a moving unstable system and show that the classical Einstein time dilation formula is not rigorously applicable in this case and quantum corrections should be taken into account. The consistency of experimental data with Einstein time dilation formula and the absence of translation-induced decays indicate that the interactions responsible for decays belong to the Dirac instant form dynamics rather than to the point form dynamics. Our results suggest that even though the different forms of dynamics are scattering-equivalent, they are not exactly physically equivalent, as was thought before. Three different experiments with unstable particles are discussed which allow one in principle to determine the details of the interaction governing the decay.

1. INTRODUCTION

One of the most remarkable predictions of Einstein special relativity is the time dilation effect: in a system moving with velocity v all physical processes slow down by a universal factor of $1/(1-v^2/c^2)^{1/2}$. This prediction has been confirmed a number of times experimentally [for review see Newman et al. (1978) and MacArthur (1986)], in particular by measuring the increase of the lifetime of a moving particle τ_θ with respect to that for the particle at rest τ_0:

$$\tau_\theta = \frac{\tau_0}{\sqrt{1-v^2}} = \tau_0 \cosh\theta \tag{1}$$

(we denote $\theta := \tanh^{-1} v$, and, unless otherwise noted, use the system of units in which $\hbar = 1$, $c = 1$). In particular, Bailey, Farley, and co-workers (Bailey et al., 1977; Farley, 1992) confirmed equation (1) to be accurate within 0.1-0.2 % for decays of relativistic ($\cosh\theta \approx 29.3$) μ-mesons. However, the theoretical foundation for a universal formula (1) is far from being satisfactory (Nielsen and Picek, 1982). For instance, classical textbook derivations of formula (1) (see, for example, Schroder 1990) use several assumptions which go beyond the special principle of relativity (equivalence of all inertial observers) and have no proper justifications in the context of classical theory. These assumptions include, in particular, (i) the physical equivalence of systems "moving observer–unstable particle at rest" and observer at rest–moving unstable particle" and (ii) independence of the nondecay probability ω on "kinematic" transformations, such as translations, rotations, and boosts.

Due to the quantum nature of the decay process, any realistic description of the decay of a moving particle should be based on relativistic quantum theory. In particular, space-time symmetries should be realized as a unitary representation of the Poincare group in the Fock space which includes states of both unstable system and decay products. This is the basic assumption of our study, in which we follow general ideas suggested by Exner (1983) and de Dormale (1979). An advantage of our method is that it allows us to obtain exact results independent of the mass distribution of the unstable system.

The paper is organized as follows. In Sections 2 and 3 we collect some well-known facts about representations of the Poincare group (see, for instance, Polyzou, 1989; Lev, 1993; Weinberg, 1995) and the description of the decay process in quantum mechanics for future reference. In Section 4 we present time-dependent calculations of the decay law of a moving particle in both instant and point forms of Dirac relativistic dynamics (Dirac, 1949). These calculations indicate that there are important quantum corrections to the classical formula (1) depending on the relativistic form of the interaction governing the decay process. This result is confirmed by studying the time delay of scattering using the S-matrix formalism in Section 5. In Section 6 we discuss possibilities of experimental verification of the predicted quantum effects as well as their consistency with the S-matrix equivalence of different Dirac forms of relativistic dynamics (Sokolov, 1975; Sokolov and Shatnii, 1978).

2. POINCARE INVARIANCE

Dynamics and space-time symmetries of any quantum system with a variable number of particles are described by some reducible unitary representation U_g of the Poincaré group in the Fock space $\mathcal{H}$. If K and gK are two inertial frames of reference (observers) related to each other by a transformation g from the Poincaré group (g is a combination of time and space translations, rotations, and boosts), and the normalized state vector $|\Psi\rangle \in \mathcal{H}$ describes the state of the system with respect to the observer

K, then (in the Schrodinger picture) the state vector $U_g|\Psi\rangle$ describes the same state with respect to the observer gK. Unitary operators for pure time translations ($U_t = e^{-iHt}$), space translations ($U_a = e^{i\mathbf{Pa}}$), rotations $[U_\phi = exp(i\mathbf{J}\phi]$, and Lorentz boosts $[U_\theta = exp(i\mathbf{K}\theta)]$ are generated, respectively, by the Hamiltonian H and the vectors of total momentum $\mathbf{P}$, total angular momentum $\mathbf{J}$, and $\mathbf{K}$, which are self-adjoint operators satisfying the well-known commutation relations of the Poincaré group Lie algebra. In particular,

$$[K_i, P_j] = i\delta_{ij}H; \qquad i, j = x, y, z \tag{2}$$

$$[\mathbf{K}, H] = i\mathbf{P} \tag{3}$$

Operators of spin and Newton-Wigner position are defined as $\vec{\Sigma} := \mathbf{J} - \mathbf{X} \times \mathbf{P}$ and

$$\mathbf{X} := \frac{1}{2}(H^{-1}\mathbf{K} + \mathbf{K}H^{-1}) - \frac{\mathbf{P} \times (HJ - \mathbf{P} \times \mathbf{K})}{MH(M+H)} \tag{4}$$

respectively. The operators of mass $M := +\sqrt{H^2 - P^2}$ and spin squared Σ^2 commute with all generators of the Poincaré group. Therefore, operators $(P, \Sigma_x, \Sigma^2, M, \Xi)$ form a full set of mutually commuting operators in $\mathcal{H}$ with the corresponding basis of common eigenvectors $|\mathbf{p}, \sigma_x; \sigma, m, \zeta\rangle$, where the operator Ξ is defined in such a way that its eigenvalues ζ distinguish eigenvectors degenerate with respect to the set $(\mathbf{P}, \Sigma_x, \Sigma^2, M)$. When dealing with generalized state vectors $|\mathbf{p}, \sigma_x; \sigma, m, \zeta\rangle$ having definite momentum we always assume that they can be approximated with arbitrary accuracy by well-behaved wave packets.

Every state vector $|\Phi\rangle$ can be written as a linear superposition of basis vectors $|\mathbf{p}, \sigma_x; \sigma, m, \zeta\rangle$,

$$|\Phi\rangle = \sum_\zeta \sum_\sigma \sum_{\sigma_x=-\sigma}^{\sigma} \int \frac{d^3p}{\sqrt{p^2+m^2}} \int d[m]\phi(\mathbf{p}, \sigma_x; \sigma, m, \zeta)|\mathbf{p}, \sigma_x; \sigma, m, \zeta\rangle \tag{5}$$

where $\int d[m] \ldots$ denotes integration over the continuous spectrum of the mass operator and summation over the discrete spectrum of M. By using standard methods (Coester, 1965; de Dormale, 1979), we can decompose the representation U_g into a direct sum or integral,

$$U_g = \sum_{\zeta} \sum_{\sigma} \int^{\oplus} d[m] U_g[m, \sigma; \zeta] \tag{6}$$

$$\mathcal{H} = \sum_{\zeta} \sum_{\sigma} \int^{\oplus} d[m] \mathcal{H}[m, \sigma; \zeta] \tag{7}$$

of irreducible unitary representations $U_g[m, \sigma; \zeta]$ characterized by eigenvalues [m and $\sigma(\sigma+1)$, respectively] of operators M and Σ^2. Quantum numbers $m > 0$ and $\sigma = 0, 1/2, 1, \ldots$ are identified with the mass and spin, respectively.

3. DECAY OF THE PARTICLE AT REST

In this paper we will study the decay dynamics of a massive ($m_A > 0$) and spinless ($\sigma_A = 0$) system A (generalization of our approach to nonzero spin states does not present any difficulties). Let us first assume that interaction leading to the decay is "turned off." This situation is described by the "noninteracting" representation U_g^0 of the Poincaré group with generators H^0, $\mathbf{P}^0$, $\mathbf{J}^0$, and $\mathbf{K}^0$. Note that in our approach the representation U_g^0 is not the "free particle" representation, but includes all interactions except those leading to the decay process. Thus, system A may correspond either to one elementary particle or to a stable bound state of a composite system (for example, a particular energy level of an atom). In the decomposition (7), due to the action of U_g^0 in $\mathcal{H}$, there is a subspace $\{A\} := \mathcal{H}[m_A, \sigma_A; \zeta_A]$ corresponding to the system A with the value of m_A from the discrete part of the spectrum of the mass operator M^0. State vectors belonging to this subspace will be referred to as states of the unstable particle. The orthogonal complement $\mathcal{H}^{pr}$ to the subspace $\{A\}$ in $\mathcal{H}^{pr}$ ($\mathcal{H} = \mathcal{H}^{pr} \oplus \{A\}$) will be referred to as the subspace of states of the decay products. The probability of finding particle A (the nondecay probability) in each state $|\Phi\rangle$ is given by the mean value of the operator T projecting onto the subspace $\{A\}$ (Exner, 1983) $\omega_{|\Phi\rangle} = \langle \Phi | T | \Phi \rangle$. In the absence of interactions leading to the decay, operator T commutes with U_g^0, and the nondecay probability is equal for all inertial observers. In particular, the nondecay probability is time independent, and if $|\Psi\rangle \in \{A\}$ at $t = 0$, then for all times

$$\omega_{|\Psi\rangle}(t) := \omega_{U_t^0 |\Psi\rangle} = ||Te^{-iHt}|\Psi\rangle||^2 = 1 \tag{8}$$

Without loss of generality we can choose the operator Ξ so that the parameter ζ_A is equal to zero. In this case, basis vectors $|\mathbf{p}\rangle := |\mathbf{p}, 0; 0, m_A, 0\rangle_{free}$ corresponding to the representation U_g^0 form a full basis in the subspace $\{A\}$, so that

$$T = \int \frac{d^3p}{\sqrt{p^2 + m_A^2}} |\mathbf{p}\rangle\langle\mathbf{p}| \tag{9}$$

The vector $|0\rangle := |\mathbf{p} = 0\rangle$ can be interpreted as the state corresponding to the unstable particle at rest while the state vector

$$|\vec{\theta}\rangle := |\mathbf{p} = (m_A \sinh\theta, 0, 0)\rangle = e^{iK_x^0\theta|0\rangle} = e^{iX_x^0 m_A \sinh\theta|0\rangle} \tag{10}$$

corresponds to the unstable particle moving with velocity $v = \tanh\theta$ along the x-axis.

If the interaction leading to the decay is "turned on", the corresponding unitary representation $\overline{U}_g$ of the Poincaré group and its generators $\overline{H}$, $\overline{\mathbf{P}}$, $\overline{\mathbf{J}}$, and $\overline{\mathbf{K}}$ are generally different from their "noninteracting" counterparts. Since the non-decay probability is no longer time independent, the interacting Hamiltonian does not commute with T

$$[\overline{H}, T] \neq 0 \tag{11}$$

Our primary goal in this paper is to compare decay laws for the particle A at rest

$$\omega_0(t) := ||Te^{-i\overline{H}t}|0\rangle||^2 \tag{12}$$

with that for the moving particle A,

$$\omega_\theta(t) := ||Te^{-i\overline{H}t}|\theta\rangle||^2 \tag{13}$$

In both cases the state vector lies in the subspace $\{A\}$ at $t = 0$, so that $\omega_0(0) = \omega_\theta(0) = 1$.

The decay law for the particle at rest (12) can be calculated by expanding the state vector $|0\rangle$ in the basis set $|\mathbf{p}, \sigma_x; \sigma, m, \zeta\rangle_{int}$, constructed for the interacting representation $\overline{U}_g$. Assuming that

$$\overline{\mathbf{P}}|0\rangle = 0 \tag{14}$$

and

$$\overline{\mathbf{\Sigma}}|0\rangle = 0 \tag{15}$$

we obtain

$$|0\rangle = \sum_{\zeta} \int d[m] f(m, \zeta)|0, 0; 0, m, \zeta\rangle_{int} \tag{16}$$

so that function $c(m) := \sum_{\zeta} |f(m, \zeta)|^2$ describes the mass distribution of the unstable particle at rest. By using standard arguments (Brenig and Haag, 1959), one can show that, under rather general conditions, $c(m)$ can be approximated by the Breit-Wigner function

$$c(m) \approx \frac{\Gamma/(2\pi)}{\Gamma^2/4 + (m - m_A)^2} \tag{17}$$

centered at the value $m \approx m_A$ and having the width of $\Delta m \approx \Gamma$. Upon substitution into equations (16) and (12), this yields an *almost* exponential decay law for the particle at rest

$$\omega_0(t) = |\int d[m] c(m) e^{-imt}|^2 \approx e^{-\Gamma t} \tag{18}$$

with the lifetime of $\tau_0 = 1/\Gamma$. However, in contrast to the approach suggested in Alicki et al. (1986), where the exponential character of -the decay was used to justify the Poincaré semigroup property, the actual form of the function $c(m)$ is not important for the present study: our results will be valid for an arbitrary mass distribution $c(m)$.

In order to calculate the decay law for a moving particle (13) we need to define more carefully how interaction terms are present in the generators $\overline{\mathbf{P}}$, $\overline{\mathbf{J}}$, and $\overline{\mathbf{K}}$. A general classification of these interaction terms (forms of dynamics) was given by Dirac (1949). Up to now, no theoretical or experimental arguments have been found that make one form of dynamics more preferable than the others. In fact, on the basis of the S-matrix equivalence of different forms of dynamics it was suggested (Sokolov, 1975; Sokolov and Shatnii, 1978) that all of them are also physically equivalent. In this paper we will be concerned with two simple forms of Dirac relativistic dynamics much studied in the literature (for review see Polyzou, 1989; Lev, 1993): the point and instant forms. In the next section we show that these two forms of dynamics yield rather different decay laws for a moving unstable particle. Note also that existing quantum field theories, such as quantum electrodynamics, assume interactions in the instant form without proof (Weinberg, 1995).

4. DECAY OF A MOVING PARTICLE

4.1. Point Form Dynamics

In a general point form dynamics, generators of Lorentz transformations are interaction-free, $\overline{\mathbf{J}} = \mathbf{J^0}$, $\overline{\mathbf{K}} = \mathbf{K^0}$ while interaction terms are present in the total momentum operator $\overline{\mathbf{P}} \neq \mathbf{P^0}$. Let us construct a particular version of the point form dynamics using the Bakamjian prescription (Bakamjian, 1961; Coester and Polyzou, 1982). In this case, operators of the total momentum and energy have the form

$$\overline{\mathbf{P}} = \overline{M}\mathbf{Q} \tag{19}$$

$$\overline{H} = \overline{M}\sqrt{1+Q^2} \tag{20}$$

where $\mathbf{Q} := \mathbf{P}^0(M^0)^{-1}$ and the interacting mass operator $\overline{M}$ satisfies the following commutation relations.

$$[\overline{M}, \mathbf{J}^0] = [\overline{M}, \mathbf{K}^0] = [\overline{M}, \mathbf{Q}^0] = 0 \tag{21}$$

It is easy to show that both conditions (14) and (15) are valid in this case.

Using equation (10), commutators (2) and (3), and $[T, \overline{\mathbf{K}}] = [T, \overline{\mathbf{K}^0}] = 0$ we obtain from equation (13)

$$\omega_\theta(t) = \|Te^{i\overline{K}_x\theta}e^{-i\overline{K}_x\theta}e^{-i\overline{H}t}e^{i\overline{K}_x\theta}\, 0\rangle\|^2$$

$$= \|Te^{-it(\overline{H}\cosh\theta - \overline{P}_x \sinh\theta)}|0\rangle\|^2 \tag{22}$$

and finally, due to (14),

$$\omega_\theta(t) = \|Te^{-i\overline{H}\cosh\theta t}|0\rangle\|^2 = \omega_0(t\cosh\theta) \tag{23}$$

$$\tau_\theta = \frac{\tau_0}{\cosh\theta} \tag{24}$$

Thus, the decay law of a moving particle accelerates if the interaction has the Bakamjian point form. This contradicts both Einstein time dilation formula (1) and experimental observations.

In principle, there are other variants of the point form interaction in which the lifetime of a moving particle is not given by equation (24), so that better agreement with formula (1) can be achieved. However, even if such variants are found, they are not acceptable, for the following reason. Equations (2) and (11) imply, in particular, that $[T, \overline{P}_x] \neq 0$. Therefore, if $|\Psi\rangle \in \{A\}$, then the state $\exp(i\overline{P}_x a)|\Psi\rangle$ seen by the observer translated by the distance a generally does not belong to the subspace $\{A\}$ and thus contains nonzero contributions from the decay products. This state is not equivalent to the state $\exp(iP_x^0 a)|\Psi\rangle \in \{A\}$ corresponding to the particle shifted from the origin. Therefore we conclude that translations of the observer cause the decay of the unstable particle even at time $t = 0$. There are no experimental indications of such a dependence of the nondecay probability on the position of observer. This allows us to conclude that point form interactions cannot be responsible for particle decays and that

$$[T, \overline{\mathbf{P}}] = 0 \tag{25}$$

4.2. Instant Form Dynamics

Commutators (2), (11), and (25) imply that $[T, \overline{\mathbf{P}}] = 0$. These commutation relations are characteristic for the instant form dynamics (Coester and Polyzou, 1982). Similar to the discussion above, they imply that the state of the unstable particle seen by a moving observer has a nonzero contribution from decay products even at $t = 0$; thus boosts of the observer cause the decay of the unstable particle. Again, such a situation has been never observed experimentally. However, this is not surprising, taking into account the enormous difficulties associated with acceleration of macroscopic observers (or measuring devices) to relativistic velocities. Note also that due to equations (2) and (11) the operator T cannot commute with both $\overline{\mathbf{K}}$ and $\overline{\mathbf{P}}$. Therefore, one should expect decays caused by "kinematic" transformations $\overline{U}_a$ or/and $\overline{U}_\theta$ in all forms of relativistic dynamics; thus the classical assumption (ii) (see Introduction) is not valid.

A general instant form dynamics can be constructed by specifying the interacting "position" operator $\overline{\mathbf{X}}$ satisfying $[\overline{X}_i, J^0_j] = i\epsilon_{ijk}\overline{X}_k$, $[\overline{X}_i, \overline{X}_j] = 0$ $(i, j = x, y, z)$, and

$$[\overline{X}_i, P^0_j] = i\delta_{ij} \ (i, j = x, y, z) \tag{26}$$

and the mass operator $\overline{M}$ satisfying $[\overline{M}, \mathbf{J}^0] = [\overline{M}, \mathbf{P}^0] = 0$ and

$$[\overline{M}, \overline{\mathbf{X}}] = 0 \tag{27}$$

Then interacting generators of the Poincaré group have the form

$$\overline{H} = +\sqrt{(\mathbf{P}^0)^2 + \overline{M}^2} \tag{28}$$

$$\overline{\mathbf{K}} = \frac{1}{2}(\overline{\mathbf{X}}\overline{H} + \overline{H}\overline{\mathbf{X}}) + \frac{\mathbf{P}^0 \times \vec{\overline{\mathbf{\Sigma}}}}{\overline{M} + \overline{H}} \tag{29}$$

where $\vec{\overline{\mathbf{\Sigma}}} = \mathbf{J}^0 - \overline{\mathbf{X}} \times \mathbf{P}^0$ is the spin operator, and generators of rotations $\overline{\mathbf{J}} = \mathbf{J}^0$ and space translations $\overline{\mathbf{P}} = \mathbf{P}^0$ are interaction-free.

In this paper we will focus on a specific class of instant form interactions for which the position operator $\overline{\mathbf{X}}$ leaves the subspace $\{A\}$ invariant,

$$[\overline{\mathbf{X}}, T] = 0 \tag{30}$$

and the restriction of $\overline{\mathbf{X}}$ onto the subspace $\{A\}$ coincides with restriction of the noninteracting position operator $\mathbf{X}^0$ [see equation (4)] onto this subspace,

$$\overline{\mathbf{X}}|_{\{A\}} = \mathbf{X}^0|_{\{A\}} \tag{31}$$

For such interactions the spin operator in $\{A\}$ coincides with the noninteracting spin

$$\vec{\overline{\mathbf{\Sigma}}}|_{\{A\}} = \vec{\mathbf{\Sigma}}^0|_{\{A\}} \tag{32}$$

and conditions (14) and (15) are satisfied. Note that the Bakamjian-Thomas instant form dynamics (Bakamjian and Thomas, 1953) is a particular form of the interactions considered here.

By using equations (10) and (31) we can represent the state vector $|\theta\rangle$ as $|\theta\rangle = exp(i\overline{X}_x m_A \sinh\theta)|0\rangle$. Then, using equation (28) and commutators (26) and (27) and (30) in equation (13), we obtain

$$\begin{aligned}
\omega_\theta(t) &= \|T\exp(i\overline{X}_x m_A \sinh\theta)\exp(-i\overline{X}_x m_A \sinh\theta) \\
&\times \exp\{-it[(\mathbf{P}^0)^2 + \overline{M}^2]^{1/2}\}\exp(i\overline{X}_x m_A \sinh\theta)|0\rangle\|^2 \\
&= \|T\exp(-it((P_x^0 - m_A\sinh\theta)^2 + (P_y^0)^2 + (P_z^0)^2 + \overline{M}^2)^{1/2})|0\rangle\|^2 \\
&= \|T\exp(-it(m_A^2\sinh^2\theta + \overline{M}^2)^{1/2})|0\rangle\|^2
\end{aligned} \tag{33}$$

Since the operator $\overline{M}$ commutes with $\mathbf{P}^0$, the vector $\exp[-it(m_A^2\sinh^2\theta + \overline{M}^2)^{1/2}]|0\rangle$ is an eigenvector of the operator $\mathbf{P}^0$ with zero eigenvalue, so that effectively we can use the operator $|0\rangle\langle 0|$ instead of T in equation (33). This "effective one-dimensionality" of the decay was first realized by Exner (1983). Using equation (16) in formula (33), we obtain the final exact formula for the decay law of a moving particle

$$\omega_\theta(t) = |\int d[m]c(m)\exp[-it(m_A^2\sinh^2\theta + m^2)^{1/2}]|^2 \tag{34}$$

This result is different from the classical Einstein time dilation formula

$$\begin{aligned}
\omega_\theta^{class}(t) &= \omega_0^{class}(\frac{t}{\cosh\theta}) \\
&= |\int d[m]c(m)e^{-imt/\cosh\theta}|^2
\end{aligned} \tag{35}$$

and reduces to the latter only if the square root in the argument of the exponent in equation (34) is approximated by the first two (constant and linear with respect to $m - m_A$) terms in the Taylor expansion

$$\sqrt{m_A^2 \sinh^2\theta + m^2} = m_A \cosh\theta + \frac{m - m_A}{\cosh\theta} + \frac{\tanh^2\theta (m - m_A)^2}{2m_A \cosh\theta} + \ldots \quad (36)$$

Therefore, the relative magnitude of quantum corrections to Einstein formula (35) can be estimated as

$$\Delta\omega \approx \frac{\Delta m}{m_A} \frac{v^2}{2} \quad (37)$$

which is the ratio of the third and second terms in the expansion (36). According to (37), the largest quantum corrections are expected for particles with a small mass and large width of the mass distribution. The value of v^2 is always less than 1, and for exponential decays (17) $\Delta m \approx \Gamma$, so that the ratio $(\Delta m)/m_A$ is less than 10^{-5} for all known unstable systems (excluding strongly decaying resonances, for which measurements of the decay laws are beyond the experimental time resolution). Therefore, quantum corrections ($<$ 0.001 %) are much smaller than the accuracy of existing experimental techniques [which is about 0.1-0.2 % (Bailey *et al.*, 1977; Farley, 1992)], and Einstein's time dilation formula (1) holds with very high accuracy.

More precise values of quantum corrections can be obtained by numerical calculations using equation (34). Assume that the mass distribution $c(m)$ of the unstable particle has the Breit-Wigner form (17). In this case, it is convenient to measure time in units of the classical lifetime $\tau_0 \cosh\theta$. Denoting $\chi := t/(\tau_0 \cosh\theta)$, we then have the classical decay law (35) given by the universal exponential function

$$\omega_\theta^{class}(\chi) = e^{-\chi} \quad (38)$$

independent on the values of parameters θ, Γ, and m_A. This function is represented by a thick solid line in Fig. 1. Quantum corrections to the classical decay law

$$\Delta\omega_\theta(\chi) = \omega_\theta(\chi) - \omega_\theta^{class}(\chi) \quad (39)$$

do depend on parameters θ and Γ/m_A. They were calculated for three values of the parameter θ, namely 0.2, 1.4, and 10.0 (corresponding to velocities of 0.197c, 0.885c, and 0.999999995c) and plotted in Fig. 1 as circles, squares, and triangles, respectively. In our calculations we used the value of mass m_A= 1000 MeV/c^2 and the width of Γ= 20 MeV/c^2 (corresponding to the lifetime at rest $\tau_0 \approx 3.3 \times 10^{-13}$ sec), which are typical for strongly decaying baryons. At small values of θ ($\theta = 0.2$) the quantum correction $\Delta\omega_{0.2}(\chi)$ is small, as expected. For large values of θ ($\theta = 1.4$ and 10.0) the quantum corrections are almost independent of θ. The maximum relative quantum correction (about 0.25% of the total nondecay probability at $\chi \approx 1.6$) is comparable to the present experimental accuracy; however, decay laws for particles with such a short lifetime as in our example cannot be measured experimentally. Therefore, significant improvement of the experimental accuracy is required in order to measure deviations from the classical decay law predicted by equation (34).

5. SCATTERING

In scattering-type experiments the state of an unstable particle at $t = 0$

$$|\Psi(t=0)\rangle \in \{A\} \tag{40}$$

is prepared (Fonda et al., 1978) by colliding stable reactants having average energy E in a resonance with the energy $m_A \cosh\theta$ of the unstable particle so that they interact a short period of time about $t = 0$ and move freely before and after the scattering ($t \to \pm\infty$). Both asymptotic state vectors $|\Psi(t \to -\infty)\rangle$ and $|\Psi(t \to +\infty)\rangle$ lie in the subspace of products $\mathcal{H}^{pr}$. The time interval between preparation of the initial state and registration of outgoing particles is longer than the corresponding time interval for a system without interaction. In the S-matrix theory this time difference (the time delay of scattering τ^S) is calculated by using the Eisenbud-Wigner formula (Wigner, 1955; Amrein et al., 1977) as the energy derivative of the phase shift $\varphi(E)$

$$\tau^S(E) = 2\frac{d\varphi(E)}{dE} \tag{41}$$

and corresponds to the double lifetime of the resonant state at $t = 0$. Note that direct measurements of the time delay (41) are extremely difficult. Moreover, unlike measurements of the decay law that provide the most complete information about the decay dynamics, time-delay experiments can yield only the lifetime of the unstable particle, which is an average parameter of the decay process. Nevertheless, it is interesting to compare

S-matrix calculations for time delays of moving scattering systems in the point and instant forms dynamics with the results from the time-dependent approach presented in Section 4.

In Lorentz-invariant scattering theory both the S-matrix operator and the phase shift operator ($\Phi := (1/2i)\ln S$) commute with generators of the noninteracting representation U_g^0 (Fong and Sucher, 1964; Coester, 1965; Weinberg, 1995). Therefore Φ is a function of Kasimir invariants M^0, $(\Sigma^0)^2$, and other operators commuting with U_g^0 (Redei, 1965): $\Phi \equiv \Phi(M^0, (\Sigma^0)^2, \ldots)$. On the other hand, in order to use definition (41) for calculations of the time delay, the phase shift operator Φ and the free Hamiltonian H^0 must have common spectral decompositions (Amrein et al.. 1977) with the operator Φ acting as multiplication by a function of energy $\varphi(E)$ in the energy representation. Therefore, equation (41) is applicable only if the free mass operator M^0 is a function of the free Hamiltonian H^0 and acts as multiplication by a function of energy $m(E)$ in the energy representation. Obviously, this condition cannot be satisfied in the entire Hilbert space $\mathcal{H}$ of the system; however, it is still meaningful in certain subspaces of $\mathcal{H}$ described below. To determine what kind of subspaces should be used in calculations, it is of key importance to realize that results of time-delay and decay experiments can be compared only if the initial state of colliding particles $|\Psi(t \to -\infty)\rangle$ is prepared in such a way that the time evolution brings $|\Psi(t \to -\infty)\rangle$ to a vector lying in the subspace $\{A\}$ at t = 0 so that equation (40) is satisfied. In the case of a moving scattering system we need to ensure that the time evolution transforms the initial state $|\Psi(t \to -\infty)\rangle$ to the vector $|\theta\rangle$ at $t = 0$.

First, consider instant form dynamics. Since the interacting Hamiltonian $\overline{H}$ commutes with $\mathbf{P}^0$, and the vector $|\theta\rangle$ is an eigenvector of the operator $\mathbf{P}^0$ with eigenvalue $\mathbf{p} = (m_A \sinh\theta, 0, 0)$, then the vector $|\Psi(t \to -\infty)\rangle$ should also satisfy

$$\mathbf{P}^0|\Psi(t \to -\infty)\rangle = \mathbf{p}|\Psi(t \to -\infty)\rangle \tag{42}$$

Therefore the dynamics of the system is confined to the eigensubspace $\{\mathbf{p}\}$ of the total momentum $\mathbf{P}^0$ with eigenvalue $\mathbf{p}$. In this subspace, the mass operator M^0 acts as multiplication by a function $m(E) = (E^2 - m_A^2 \sinh^2\theta)^{1/2}$ in the energy representation. Now equation (39) can be used in the subspace $\{\mathbf{p}\}$, yielding

$$\tau_\theta^S \equiv 2\frac{d\varphi}{dE}|_{\{\mathbf{p}\}} = 2\frac{d\varphi(m)}{dm}\frac{dm(E)}{dE}|_{\{\mathbf{p}\}} \approx \tau_0^S \cosh\theta \tag{43}$$

where $\tau_0^S := d\varphi(m)/dm$ is the time delay for the scattering system at rest. The time delay increases with increasing velocity, which coincides with our result from Section 4.2.

Similarly, in the Bakamjian point form dynamics, $\overline{H}$ commutes with $\mathbf{Q}$ and the time evolution of the scattering state is limited to the eigensubspace $\{\mathbf{q}\}$ of the operator $\mathbf{Q}$ with eigenvalue $\mathbf{q} = (\sinh\theta, 0, 0)$. In particular, the incoming state must satisfy

$$\mathbf{Q}|\Psi(t \to -\infty)\rangle = \mathbf{q}|\Psi(t \to -\infty)\rangle \tag{44}$$

In the subspace $\{\mathbf{q}\}$ the mass operator acts as multiplication by a function $m(E) = E/\cosh\theta$ in the energy representation. Using equation (41) in this subspace, we obtain

$$\tau_\theta^S \equiv 2\frac{d\varphi}{dE}|_{\{\mathbf{q}\}} = 2\frac{d\varphi(m)}{dm}\frac{dm(E)}{dE}|_{\{\mathbf{q}\}} = \frac{\tau_0^S}{\cosh\theta} \tag{45}$$

Thus the time-delay decreases with increasing velocity, in agreement with our result (24) from Section 4.1.

6. DISCUSSION

From results presented in the last section it is clear that the time delay in scattering experiments depends on the initial condition $|\Psi(t \to -\infty)\rangle$ but not on the form of dynamics. For example, preparing $|\Psi(t \to -\infty)\rangle$ in the eigensubspace of the operator $\mathbf{Q}$ in the case of instant form interaction would yield the same time delay as for the point form interaction, i.e., equation (45), in full accord with the S-matrix equivalence of different forms of relativistic dynamics discovered by Sokolov (1975; Sokolov and Shatnii, 1978). However, this result has no relation to the lifetime of a moving unstable particle, because the time evolution of the scattering wave packet does not pass through the unstable particle's state $|\theta\rangle$. In order to compare results of time-delay and decay experiments, the initial state $|\Psi(t \to -\infty)\rangle$ should be prepared in such a way that condition (40) is satisfied. This choice of $|\Psi(t \to -\infty)\rangle$ is different for different forms of relativistic dynamics and leads to different relations between τ_θ^S and τ_0^S. In particular, in the instant form dynamics considered here, the state vector $|\Psi(t \to -\infty)\rangle$ must satisfy equation (42), so that equation (43) is valid for the time delay in a moving scattering system.

Assuming that the S-matrix (i.e., the correlation between incoming and outgoing asymptotic states) contains all relevant information about the physical system (Heisenberg, 1943), Sokolov concluded that different forms of dynamics are also physically equivalent. For instance, it was shown that the Bakamjian point form and Bakamjian-Thomas instant form dynamics are related by a unitary transformation conserving the S-matrix. However, our results show that these two forms of relativistic dynamics differ with respect to the behaviour of the decay law of moving particles compared to that for particles at rest; therefore, Sokolov's conclusion is not correct. More specifically, there exist experiments which in principle can provide information about the relativistic form of actual interactions. In contrast to scattering-type experiments, in which only noninteracting asymptotic states are registered, one must measure the dynamics of the system in the region of interaction in order to gain information about the relativistic form of the interaction. In addition, these measurements should include comparison between the dynamics observed from different frames of reference. Three kinds of experiments satisfying these conditions are described below.

First, our results suggest that classical assumption (i) (see Introduction) is not correct in the instant form dynamics, i.e., measurements of the nondecay probability give different results (even at $t = 0$) in the pairs "moving unstable particle–stationary observer" and "unstable particle at rest–moving observer." If interaction has point form, then, according to our discussion in Section 4.1, the nondecay probability is different for observers occupying different positions in space. Thus, experimental measurements of the noninvariance of the nondecay probability with respect to translations, rotations, and boosts of observers (or measuring devices) can provide information about the form of the dynamics. It would be interesting to estimate the magnitude of these effects and formulate experimental conditions under which such noninvariance can be measured.

The second possibility is to measure a statistical distribution (averaged over a large number of decay experiments with a given unstable particle) of momenta $\mathbf{p}_i$ of (stable) decay products with masses m_i. According to the discussion in Section 5, if the unstable system is prepared in the state $|\theta\rangle$ at $t = 0$, then the asymptotic state vector $|\Psi(t \to -\infty)\rangle$ satisfies equation (42) in the instant form dynamics and equation (44) in the Bakamjian point form dynamics, respectively. This means that in the instant form dynamics the total momentum of outgoing particles is constant:

$$\frac{\sum_i \mathbf{p}_i}{m_A} = (\sinh\theta, 0, 0) \tag{46}$$

In the Bakamjian point form dynamics, the total "velocity" of outgoing particles is constant:

$$\frac{\sum_i \mathbf{p}_i}{\{[\sum_i(\mathbf{p}_i^2 + m_i^2)]^2 - (\sum_i \mathbf{p}_i)^2\}^{1/2}} = (\sinh\theta, 0, 0) \tag{47}$$

Thus, different distributions of momenta of outgoing particles are characteristic for different types of interactions. Note, however, that there are two major problems in conducting such experiments. First, it is very difficult to prepare experimentally the state $|\theta\rangle$ of the unstable particle without an admixture of decay products at $t = 0$. Second, even if such a state is prepared, it is difficult to distinguish between situations (46) and (47), because the spread of the mass of unstable systems is normally too narrow to be detected.

Third, a careful analysis of the dependence of the decay law $\omega_\theta(t)$ on the velocities of unstable particles allows us to determine the form of relativistic dynamics and other details of the interaction responsible for the decay. Measurements of this kind are routinely performed in subnuclear experimental physics. As we noted earlier, existing experimental data allow us to rule out the possibility of the Bakamjian point form dynamics and are in excellent agreement with the instant form interaction satisfying conditions (30) and (31). Such experiments should in principle be able to detect deviations [predicted by equation (34)] from Einstein time dilation formula. However, a significant improvement of the experimental accuracy is required to observe these effects.

Our most important result is that relativistic kinematics alone does not determine the decay law of an unstable particle in the state of motion, as was thought before. The decay law of a moving particle is not given by universal formulas (1) and (35), but depends on the interaction governing the decay. Note that disagreement between our results and Einstein time dilation formulas does not mean any violation of relativistic invariance in our approach. In fact, both slowing down and acceleration (24) of the decay of a moving particle are consistent with special relativity and quantum mechanics. The slowing down of the decay observed in experiments is a consequence of interaction dynamics rather than simple relativistic kinematics.

7. CONCLUSIONS

In this paper we presented a rigorous relativistic quantum description of the decay of a moving particle using both explicit time-dependent and S-matrix formalisms. We avoided using controversial assumptions adopted in the classical approach. Instead we implemented relativistic invariance by explicit construction of a unitary representation of the Poincaré group, defined the nondecay probability as a mean value of the operator projecting onto the subspace of states of the unstable particle, and generalized the Eisenbud-Wigner formula (41) to define the delay time in a moving scattering system. This approach is consistent with the modern formalism of relativistic quantum theory. The following conclusions can be formulated.

1. The classical time dilation formula (1) is not rigorously applicable for calculating the decay laws of moving unstable systems.

2. The lifetime of a moving particle is determined by details of the interaction responsible for the decay. In particular, for Bakamjian point form dynamics the decay accelerates $\cosh\theta$ times; for instant form dynamics satisfying conditions (30) and (31) the decay slows down (approximately) by Einstein factor of $\cosh\theta$, as observed experimentally. This suggests that interactions governing decays have the latter form of dynamics or close to it.

3. The time delay in a moving scattering system depends on the initial state of the system and does not depend on the form of the dynamics. This is consistent with the scattering equivalence of different forms of dynamics established earlier.

4. Scattering equivalence of different forms of relativistic dynamics does not mean their exact physical equivalence, i.e., time evolution in the interaction region is different in different forms of dynamics, even if S-matrices are the same.

5. Additional experiments with unstable particles proposed in this work can provide further information about the form of dynamics and other details of interactions responsible for decays.

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8. FIGURE CAPTIONS

Fig. 1. Quantum corrections to the classical decay law [equation (38)] (thick full line) calculated by using equation (39) for $\theta = 0.2$ (circles and dashed line), $\theta = 1.4$ (squares and dotted line), and $\theta = 10.0$ (triangles and full line). The mass of the (hypothetical) unstable particle is $m_A = 1000 MeV/c^2$ and the lifetime at rest is 3.3×10^{-23} sec. The parameter χ measures time in units of particle lifetime.