Is Minkowski Space-Time Compatible with Quantum Mechanics?

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Abstract: In quantum relativistic Hamiltonian dynamics, the time evolution of interacting particles is described by the Hamiltonian with an interaction-dependent term (potential energy). Boost operators are responsible for (Lorentz) transformations of observables between different moving inertial frames of reference. Relativistic invariance requires that interaction-dependent terms (potential boosts) are present also in the boost operators and therefore Lorentz transformations depend on the interaction acting in the system. This fact is ignored in special relativity which postulates universality of Lorentz transformations and their independence of interactions. Taking into account potential boosts in Lorentz transformations allows us to resolve the "no-interaction" paradox formulated by Currie, Jordan, and Sudarshan [Rev. Mod. Phys. **35** (1963), 350] and to predict a number of potentially observable effects contradicting special relativity. In particular, we demonstrate that longitudinal electric field (Coulomb potential) of a moving charge propagates instantaneously. We show that this effect as well as superluminal spreading of localized particle states is in full agreement with causality in all inertial frames of reference. Formulas relating time and position of events in interacting systems reduce to the usual Lorentz transformations only in the classical limit ($\hbar \to 0$) and for weak interactions. Therefore, the concept of Minkowski space-time is just an approximation which should be avoided in rigorous theoretical constructions.

Keywords': special relativity; quantum mechanics; quantum field theory; Lorentz transformations; action-at-a-distance; superluminal propagation

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1 INTRODUCTION

One of the greatest challenges faced by theoretical physics in the 20th century was formulation of a consistent theory combining relativity and quantum mechanics. Such a combination was most closely approached by the quantum field theory (QFT) which successfully predicts majority of observable effects in high energy physics, e.g., scattering. Recently suggested "clothed particle" form of the QFT Hamiltonian[1] allows us to remove notorious ultraviolet divergences from the theory and to describe the time evolution of wave functions and observables. This approach will be referred to as relativistic quantum dynamics (RQD).

The present study was motivated by an intention to understand the relationship between RQD and the "no-interaction theorem" first proven by Currie, Jordan, and Sudarshan.[2] This theorem states that in relativistically invariant Hamiltonian dynamics (an example of which is RQD) trajectories of interacting particles do not transform according to Lorentz transformations known from special theory of relativity (STR). The Lorentz transformations were found to be valid only when the interaction was absent and particle trajectories were straight lines, indicating a clear contradiction between special relativity and relativistic Hamiltonian dynamics. Remarkable simplicity and beauty of the 4-dimensional space-time concept is possibly the reason why majority of theoretical physicists still trust in the validity of special relativity and why several attempts were undertaken to formulate relativistic classical and quantum mechanics using non-Hamiltonian formalisms.[3] However, these attempts have not yet achieved the degree of completeness and agreement with experiment, as compared to RQD.

In this paper we adopted a different approach. Since RQD is a full dynamical theory, it permits direct calculations of different physical phenomena such as the time dependence of observables in systems of interacting particles and the time dependence of decay processes in different frames of reference. So, major predictions of STR, in particular, the time-position Lorentz transformations and time dilation can be directly verified. The paper is organized as follows. In Sections 2 and 3, we present a brief description of RQD and STR, respectively. These sections mostly repeat results of previous works and establish notation and terminology needed for the rest of the paper. The comparisons of the two theories with each other and with experiment are discussed in Sections 4-6 and in Section 7, respectively. Sections 8 and 9 are devoted to discussion of our results and to conclusions.

We use the system of units in which $c = \hbar = 1$, although sometimes, for clarity, these constants will be written explicitly in equations.

2 RELATIVISTIC QUANTUM DYNAMICS

2.1 Experimental setup

Any experimental setup has two principal components: a *preparation device* (P) which prepares a *system* (S) in a certain *state* $|\Psi\rangle$ and an *observer* (O). The observer has a measuring apparatus which interacts with the system and yields a value of an *observable* F depending on the kind of the system being measured and on the state of this system. We will assume that the system S is a quantum object, so that results of individual measurements are likely to be unpredictable. The system should be prepared in the same state $|\Psi\rangle$ many times and repetitive measurements of the observable F should be made in order to obtain the probability distribution for different values of F and the expectation value of F in the state $|\Psi\rangle$. This statistical nature of measurements has its mathematical expression in the formalism of quantum mechanics:

- Proposition (A) [postulates of quantum mechanics] *Any physical system is described by a complex Hilbert space $\mathcal{H}$ with scalar product $\langle\Phi|\Psi\rangle$. Pure states are represented by rays (unit vectors $|\Psi\rangle$ defined up to a phase factor) in $\mathcal{H}$. Observables are represented by Hermitian operators in $\mathcal{H}$. Expectation value of an observable F in a state $|\Psi\rangle$ is given by $f = \langle\Psi|F|\Psi\rangle$.*

In addition to the measuring apparatus, each observer has a *clock* which is a device of a special kind. Clock readings do not depend on the kind of the system being measured and on the state the system is in. Thus time is just a c-number label attached to each measurement according to the reading of the clock at the instant of the measurement. There is no quantum uncertainty in the clock readings, so there is no time operator in quantum mechanics.

2.2 Principle of relativity

We suppose that, unlike the system S, the preparation device P and the observer O are classical (non-quantum) objects both having well-defined positions, orientations, and velocities. We will also assume that they move with a constant speed, without acceleration or rotation, so that we can associate inertial frames of reference with P and O. A pair of an inertial preparation device and an inertial observer constitutes an *inertial laboratory*.

There are ten classes of transformations g that can be applied to an observer without changing its status as an inertial frame of reference: the observer can be shifted in space, its orientation and velocity can be changed, and the time at which the measurement is done can be changed as well. In quantum mechanics, continuous inertial transformations g applied to observers are represented by unitary operators U_g acting on the operators of observables in $\mathcal{H}$

$$F \to F' \quad = \quad U_g F U_g^{-1} \tag{1}$$

and, consequently, affecting the results of measurements

$$f' = \langle\Psi|(U_g F U_g^{-1})|\Psi\rangle \neq f$$

(the state vector $|\Psi\rangle$ is the same for all observers). This description is called the *Heisenberg picture.*

An alternative and equivalent *Schrödinger picture* is obtained when inertial transformations are applied to the preparation device while all measurements are done by the same observer. In this case the state of the system changes

$$|\Psi\rangle \to |\Psi'\rangle = U_g|\Psi\rangle, \tag{2}$$

while the operator of the observable remains the same.

When both the preparation device and observer are transformed simultaneously, the results of measurements do not change

$$f \to f' = (\langle\Psi|U_g^{-1})(U_g F U_g^{-1})(U_g|\Psi\rangle) = \langle\Psi|F|\Psi\rangle = f$$

in agreement with the special principle of relativity.

- Proposition (B) [the special principle of relativity]: *In all inertial laboratories, the laws of Nature are the same: they do not change with time, they do not depend on the position and orientation of the laboratory in space, and there is no experiment which can tell whether laboratory is moving or at rest.*

2.3 Poincaré group

Evidently, inertial transformations of observers form a 10-parameter differentiable group (a Lie group). There is only one choice of the group consistent with enormous amount of experimental data.

- Proposition (C) [Poincaré group] *Transformations between inertial observers form the Poincaré group (also known as the inhomogeneous Lorentz group).*

Proposition (A) and Eqs. (1) - (2) then imply that there is a unitary representation of the universal covering group of the Poincaré group in the Hilbert space $\mathcal{H}$.[4] Elementary transformations are represented by unitary operators: e^{-iHt} for time translations,

$e^{i\mathbf{Px}}$ for spatial translations, $e^{i\mathbf{J}\vec{\phi}}$ for rotations, and $e^{-i\mathbf{K}\vec{\theta}}$ for boosts ($\vec{\theta}$ is a boost parameter such that $\mathbf{v}/c = \vec{\theta}\theta^{-1}\tanh\theta$, where $\mathbf{v}$ is the velocity of the boost). Infinitesimal transformations form a Lie algebra with 10 Hermitian generators $(H, \mathbf{P}, \mathbf{J}, \mathbf{K})$ satisfying the well-known commutation relations:

$$[J_i, P_j] = i\hbar\epsilon_{ijk}P_k, \tag{3}$$

$$[J_i, J_j] = i\hbar\epsilon_{ijk}J_k, \tag{4}$$

$$[J_i, K_j] = i\hbar\epsilon_{ijk}K_k, \tag{5}$$

$$[P_i, P_j] = [J_i, H] = [P_i, H] = 0, \tag{6}$$

$$[K_i, K_j] = -i\hbar\epsilon_{ijk}J_k, \tag{7}$$

$$[K_i, P_j] = -i\hbar H\delta_{ij}, \tag{8}$$

$$[\mathbf{K}, H] = -i\hbar\mathbf{P}. \tag{9}$$

As usual, we assume summation over repeating indices $i, j, k = x, y, z; \epsilon_{xyz} = \epsilon_{zxy} = \epsilon_{yzx} = -\epsilon_{xzy} = -\epsilon_{yxz} = -\epsilon_{zyx} = 1$, and all other components of ϵ_{ijk} are zero.

2.4 Operators of observables

Major observables of the theory are given by the following operators:

- linear momentum $\mathbf{P}$;
- angular momentum $\mathbf{J}$;
- energy (Hamiltonian) H;
- mass $M = +\sqrt{H^2 - \mathbf{P}^2}$;
- velocity[5] $\mathbf{V} = \mathbf{P}H^{-1} = \mathbf{P}(M^2 + \mathbf{P}^2)^{-1/2}$.

For systems whose mass spectrum is strictly positive we can define additional observables of spin $\mathbf{S}$ and position $\mathbf{R}$ which are required to have the following properties consistent with their physical meaning

$$[J_i, S_j] = [S_i, S_j] = i\hbar\epsilon_{ijk}S_k,$$

$$[R_i, P_j] = i\hbar\delta_{ij}, \tag{10}$$

$$[J_i, R_j] = i\hbar\epsilon_{ijk}R_k, \tag{11}$$

$$[R_i, R_j] = [\mathbf{R}, \mathbf{S}] = [\mathbf{P}, \mathbf{S}] = 0. \tag{12}$$

It was shown[6] that, Hermitian functions of Poincaré generators satisfying the above properties are, essentially, unique

$$\begin{aligned} \mathbf{S} &= \mathbf{W}M^{-1} - W_0\mathbf{P}M^{-1}(M+H)^{-1}, \\ \mathbf{R} &= \frac{1}{2}(H^{-1}\mathbf{K} + \mathbf{K}H^{-1}) - [\mathbf{P} \times \mathbf{S}](M+H)^{-1}, \end{aligned} \tag{13}$$

where the Pauli-Lubanski 4-vector is defined as

$$\begin{aligned} W_0 &= (\mathbf{J} \cdot \mathbf{P}), \\ \mathbf{W} &= \mathbf{J}H - [\mathbf{K} \times \mathbf{P}], \end{aligned}$$

and operator (13) is called the *Newton-Wigner position.*

Without losing generality we will consider only boosts along the x-axis and denote O a reference observer and O' an observer moving with respect to O with velocity $v = c \tanh\theta$ along the x-axis, so that coordinate axes of both observers coincide when their clocks show the same time $t = t' = 0$. Let us denote $F(0,0) = F$ the operator of observable measured by the observer O at time $t = 0$. Then (in the Heisenberg picture)

$$F(0,t) = e^{-iHt} F e^{iHt} \tag{14}$$

is the operator of observable measured by O at time t. Similarly,

$$F(\theta, 0) = e^{-iK_x\theta} F e^{iK_x\theta}$$

is the operator of observable measured in the reference frame O' at its local time $t' = 0$. Time evolution in the moving reference frame O' is described by the transformed Hamiltonian $H' = e^{-iK_x\theta} H e^{iK_x\theta}$. Therefore the time dependence of F in O' is given by

$$\begin{aligned} F(\theta, t') &= e^{-iH't'} e^{-iK_x\theta} F e^{iK_x\theta} e^{iH't'} \\ &= e^{-iK_x\theta} e^{-iHt'} e^{iK_x\theta} e^{-iK_x\theta} F e^{iK_x\theta} e^{-iK_x\theta} e^{iHt'} e^{iK_x\theta} \\ &= e^{-iK_x\theta} e^{-iHt'} F e^{iHt'} e^{iK_x\theta}. \end{aligned} \tag{15}$$

Using Eq. (15), commutators (3) - (9), and operator identity

$$\exp(iA)B\exp(-iA) = B + i[A,B] - \frac{1}{2}[A,[A,B]] + \ldots$$

we can find expressions for various operators in the moving frame of reference O'

$$\begin{aligned}
H(\theta,t') &= H\cosh\theta - P_x\sinh\theta, && (16)\\
P_x(\theta,t') &= P_x\cosh\theta - H\sinh\theta, && (17)\\
P_y(\theta,t') &= P_y, && (18)\\
P_z(\theta,t') &= P_z, && (19)\\
V_x(\theta,t') &= \frac{V_x - \tanh\theta}{1 - V_x\tanh\theta}, && (20)\\
V_y(\theta,t') &= \frac{V_y}{(1 - V_x\tanh\theta)\cosh\theta}, && (21)\\
V_z(\theta,t') &= \frac{V_z}{(1 - V_x\tanh\theta)\cosh\theta}, && (22)\\
K_x(\theta,t') &= K_x + P_x(\theta,t')t', &&\\
K_y(\theta,t') &= K_y\cosh\theta - J_z\sinh\theta + P_y t', &&\\
K_z(\theta,t') &= K_z\cosh\theta + J_y\sinh\theta + P_z t'. &&
\end{aligned}$$

For systems with zero spin and nonzero mass, the Newton-Wigner position (13) and angular momentum operators reduce to

$$\begin{aligned}
\mathbf{R} &= 1/2\{\mathbf{K}, H^{-1}\},\\
\mathbf{J} &= [\mathbf{R}\times\mathbf{P}],
\end{aligned}$$

where $\{A,B\} = AB + BA$, and

$$\begin{aligned}
R_x(\theta,t') &= \frac{1}{2}\{K_x, (H\cosh\theta - P_x\sinh\theta)^{-1}\} + V_x(\theta,t')t', && (23)\\
R_y(\theta,t') &= \frac{1}{2}\{K_y\cosh\theta - J_z\sinh\theta, (H\cosh\theta - P_x\sinh\theta)^{-1}\} + V_y(\theta,t')t', &&\\
R_z(\theta,t') &= \frac{1}{2}\{K_z\cosh\theta + J_y\sinh\theta, (H\cosh\theta - P_x\sinh\theta)^{-1}\} + V_z(\theta,t')t'. &&
\end{aligned}$$

2.5 One-particle States

The Hilbert space of states of a stable elementary particle a is defined as a space $\mathcal{H}^a$ carrying an *irreducible* unitary representation of the Poincaré group. The classification of such spaces and representations according to their mass m and spin were given by Wigner.[7] In this paper we will focus on massive spinless particles for simplicity.

Let us introduce a basis of state vectors $|\mathbf{q}\rangle$ in $\mathcal{H}^a$ which are eigenvectors of the linear momentum and energy: $\mathbf{P}|\mathbf{q}\rangle = \mathbf{q}|\mathbf{q}\rangle$; $H|\mathbf{q}\rangle = \omega_{\mathbf{q}}|\mathbf{q}\rangle$, where $\omega_{\mathbf{q}} = \sqrt{m^2 + \mathbf{q}^2}$. We use the following definitions[4, 8] for the scalar product of two basis vectors

$$\langle \mathbf{q}|\mathbf{q}'\rangle = \delta(\mathbf{q} - \mathbf{q}')$$

and for the action of the Poincaré group in the Shrödinger picture

$$\begin{aligned}
e^{i\mathbf{Pa}}|\mathbf{q}\rangle &= e^{i(\mathbf{qa})}|\mathbf{q}\rangle, \\
e^{-iHt}|\mathbf{q}\rangle &= e^{-i\omega_{\mathbf{q}}t}|\mathbf{q}\rangle, \\
e^{-iK_x\theta}|\mathbf{q}\rangle &= \sqrt{\frac{\omega_{\Lambda\mathbf{q}}}{\omega_{\mathbf{q}}}}|\Lambda\mathbf{q}\rangle,
\end{aligned}$$

where

$$\begin{aligned}
\Lambda\mathbf{q} &= (q_x \cosh\theta + \omega_{\mathbf{q}} \sinh\theta, q_y, q_z), \\
\omega_{\Lambda\mathbf{q}} &= \sqrt{m^2 + (\Lambda\mathbf{q})^2} = \omega_{\mathbf{q}} \cosh\theta + q_x \sinh\theta.
\end{aligned}$$

We can introduce the decomposition of the identity operator I in $\mathcal{H}^a$

$$I = \int d\mathbf{q}|\mathbf{q}\rangle\langle\mathbf{q}|$$

which is, of course, invariant with respect to Poincaré transformations, e.g.,

$$\begin{aligned}
e^{-iK_x\theta} I e^{iK_x\theta} &= e^{-iK_x\theta}(\int d\mathbf{q}|\mathbf{q}\rangle\langle\mathbf{q}|)e^{iK_x\theta} = \int d\mathbf{q}\frac{\omega_{\Lambda\mathbf{q}}}{\omega_{\mathbf{q}}}|\Lambda\mathbf{q}\rangle\langle\Lambda\mathbf{q}| \\
&= \int d(\Lambda\mathbf{q})\frac{\omega_{\Lambda\mathbf{q}}}{\omega_{\Lambda\mathbf{q}}}|\Lambda\mathbf{q}\rangle\langle\Lambda\mathbf{q}| = \int d\mathbf{q}|\mathbf{q}\rangle\langle\mathbf{q}| = I,
\end{aligned}$$

where we used the equality

$$\frac{d(\Lambda\mathbf{q})}{d\mathbf{q}} = \frac{\omega_{\Lambda\mathbf{q}}}{\omega_{\mathbf{q}}}.$$

Then any state $|\Psi\rangle$ can be represented by its wave function $\psi(\mathbf{q}) = \langle \mathbf{q}|\Psi\rangle$ in the momentum representation

$$|\Psi\rangle = \int d\mathbf{q}|\mathbf{q}\rangle\langle\mathbf{q}|\Psi\rangle = \int d\mathbf{q}\psi(\mathbf{q})|\mathbf{q}\rangle,$$

and Poincaré transformations of the state vector $|\Psi\rangle$ can be viewed as transformations of its wave function, e.g.,

$$e^{-iK_x\theta}\psi(\mathbf{q}) = \sqrt{\frac{\omega_{\Lambda\mathbf{p}}}{\omega_{\mathbf{p}}}}\psi(\Lambda\mathbf{q}).$$

The scalar product of two arbitrary vectors $|\Phi\rangle$ and $|\Psi\rangle$

$$\langle\Phi|\Psi\rangle = \int d\mathbf{q}\phi^*(\mathbf{q})\psi(\mathbf{q})$$

is Lorentz invariant

$$\langle\Phi'|\Psi'\rangle = \int d\mathbf{q}\frac{\omega_{\Lambda\mathbf{q}}}{\omega_{\mathbf{q}}}\phi^*(\Lambda\mathbf{q})\psi(\Lambda\mathbf{q}) = \int d\mathbf{q}\phi^*(\mathbf{q})\psi(\mathbf{q}) = \langle\Phi|\Psi\rangle.$$

Action of the x-components of boost and position operators on wave functions can be derived as

$$\begin{aligned} K_x\psi(\mathbf{q}) &= i\lim_{\theta\to 0}\frac{d}{d\theta}e^{-iK_x\theta}\psi(\mathbf{q}) = i(\omega_{\mathbf{q}}\frac{d}{dq_x} + \frac{q_x}{2\omega_{\mathbf{q}}^2})\psi(\mathbf{q}), \\ R_x\psi(\mathbf{q}) &= \frac{1}{2}(H^{-1}K_x + K_xH^{-1})\psi(\mathbf{q}) = i\frac{d}{dq_x}\psi(\mathbf{q}). \end{aligned}$$

We can introduce eigenvectors of the position operator

$$\begin{aligned} \mathbf{R}|\mathbf{r}\rangle &= \mathbf{r}|\mathbf{r}\rangle, \\ |\mathbf{r}\rangle &= (2\pi)^{-3/2}\int d\mathbf{q}e^{-i\mathbf{q}\mathbf{r}}|\mathbf{q}\rangle \end{aligned} \tag{24}$$

with normalization

$$\langle\mathbf{y}|\mathbf{r}\rangle = \delta(\mathbf{y}-\mathbf{r}),$$

so, the coordinate-space wave function of a position eigenstate is well localized, as expected. Using Eq. (24) we find the position-space wave function of the momentum eigenvector

$$\langle \mathbf{r}|\mathbf{q}\rangle = (2\pi)^{-3/2}\int d\mathbf{q}'\delta(\mathbf{q}-\mathbf{q}')e^{i\mathbf{q}'\mathbf{r}} = (2\pi)^{-3/2}e^{i\mathbf{q}\mathbf{r}} \tag{25}$$

which is a usual plane wave.

2.6 The classical limit of quantum mechanics

So far we have been discussing a quantum description of particle's states as vectors in the Hilbert space $\mathcal{H}^a$. However, for derivation of Lorentz transformations in RQD (see Sections 4.2 and 4.3), we will need a classical picture in which the states are described by points $(\mathbf{r}, \mathbf{p})$ in the 6-dimensional phase space P^a. There is a close link between these two seemingly very different approaches, and we would like to discuss it in this subsection.

In classical physics, we are dealing with very massive (by microscopic standards) objects whose positions and momenta are well defined at all times. This means that action ("momentum $\times$ position", "energy $\times$ time", etc.) associated with classical processes is much larger than $\hbar$. Therefore, in order to make the transition from the quantum to the classical description, we assume that the following three conditions are satisfied.

First, we must limit our discussion to the states whose wave functions are well localized in both position and momentum spaces. We will call such states *quasiclassical*. A quasiclassical state $|\Psi\rangle$ is normally described by the momentum-space wave function

$$\psi(\mathbf{q}) = \langle \mathbf{q}|\Psi\rangle = N z_{\mathbf{p}}(\mathbf{q})e^{-i\mathbf{q}\mathbf{r}}, \tag{26}$$

where N is a normalization factor and $z_{\mathbf{p}}(\mathbf{q})$ is a smooth (non-oscillating) function sharply localized around the value of momentum $\mathbf{p}$. For example, we can take $z_{\mathbf{p}}(\mathbf{q}) = e^{-\sigma(\mathbf{q}-\mathbf{p})^2}$. In the state (26), the expectation values of position and momentum are $\mathbf{r}$ and $\mathbf{p}$, respectively. The uncertainty of the momentum of the particle is $|\Delta\mathbf{p}| \approx \sigma^{-1}$ and the uncertainty of the position is $|\Delta\mathbf{r}| \approx \hbar/|\Delta\mathbf{p}| = \hbar\sigma$. In a wide range of values of σ, both these uncertainties are rather small and the particle is well localized in both momentum and position spaces. Moreover, the state (26) is (approximately) an eigenstate of both position and momentum operators simultaneously:

$$\mathbf{R}|\Psi\rangle \approx \mathbf{r}|\Psi\rangle, \tag{27}$$

$$\mathbf{P}|\Psi\rangle \approx \mathbf{p}|\Psi\rangle, \tag{28}$$

so, we can roughly characterize this state by a pair of vectors $(\mathbf{r}, \mathbf{p})$, or by a point in the 6-dimensional phase space P^a. For a spinless particle, any observable is some function of position and momentum $f(\mathbf{R}, \mathbf{P})$. Then according to (27) - (28), in the classical limit, all operators are some real functions $f(\mathbf{r}, \mathbf{p})$ on the phase space. The expectation value (or eigenvalue) of the classical observable f in the quasiclassical state $(\mathbf{r}, \mathbf{p})$ is just the value of the function $f(\mathbf{r}, \mathbf{p})$.

Second, we will assume that the mass m of the particle is sufficiently large, so that the spreading of the wave packet (26) in the position space is slow, and the localization is preserved over a long time interval.

Third, the Plank constant $\hbar$ should be effectively set to zero. According to (3) - (9), all commutators are proportional to $\hbar$, so in the limit $\hbar \to 0$ all operators of observables commute with each other.

2.7 The Fock Space

Physical processes may involve creation and destruction of particles, therefore, in the most general description of a system, the number of particles should not be fixed. The Hilbert space of a system with a variable number of particles is constructed as a Fock space which is a direct sum of a no-particle vacuum vector $|0\rangle$, one-particle Hilbert spaces $\mathcal{H}^a, \mathcal{H}^b$, etc. and properly (anti)symmetrized tensor products of two, three, etc. one-particle Hilbert spaces

$$\mathcal{H} = |0\rangle \oplus \mathcal{H}^a \oplus \mathcal{H}^b \oplus \ldots \oplus (\mathcal{H}^a \otimes \mathcal{H}^b) \oplus \ldots \tag{29}$$

The Fock space naturally carries a *non-interacting* representation U_g^0 of the Poincaré group which is constructed as a direct sum of tensor products (with proper (anti)symmetrization) of single particle irreducible representations U_g^a, U_g^b, etc.

$$U_g^0 = 1 \oplus U_g^a \oplus U_g^b \oplus \ldots \oplus (U_g^a \otimes U_g^b) \oplus \ldots \tag{30}$$

The generators of this representation will be denoted as $(H_0, \mathbf{P}_0, \mathbf{J}_0, \mathbf{K}_0)$. Operators H_0, $\mathbf{P}_0$, and $\mathbf{J}_0$ describe *total* energy, linear momentum, and angular momentum of the non-interacting system, respectively. For example, the Hamiltonian is

$$H_0 = 0 \oplus H^a \oplus H^b \ldots \oplus (H^a \otimes 1^b + 1^a \otimes H^b) \oplus \ldots$$

In the subspace $\mathcal{H}^a \otimes \mathcal{H}^b$, the energy of the particle a is given by $H^a \otimes 1^b$ and the energy of the particle b is given by $1^a \otimes H^b$, so that the total energy is simply a sum of energies of components. Clearly, representation (30) describes a non-interacting system: the two particles a and b evolve in time independent of each other

$$e^{-iH_0t}(|\psi^a\rangle \otimes |\psi^b\rangle) = (e^{-iH^at}|\psi^a\rangle \otimes e^{-iH^bt}|\psi^b\rangle).$$

In what follows we will simplify the notation, e.g., write H^a and H^b instead of $H^a \otimes 1^b$ and $1^a \otimes H^b$, respectively.

2.8 Quantum field theory

The interaction is introduced in the theory by specifying a unitary representation U_g of the Poincaré group in $\mathcal{H}$ with generators $(H, \mathbf{P}, \mathbf{J}, \mathbf{K})$ which are generally different from generators of U_g^0, i.e.,

$$\begin{aligned} H &= H_0 + V, \\ \mathbf{P} &= \mathbf{P_0} + \Delta\mathbf{P}, \\ \mathbf{J} &= \mathbf{J_0} + \Delta\mathbf{J}, \\ \mathbf{K} &= \mathbf{K_0} + \mathbf{W}. \end{aligned}$$

Various forms of interacting relativistic dynamics were classified by Dirac.[9] The *instant form dynamics* is more preferable than other forms (see Section 7.2). In this form of dynamics, operators of momentum and angular momentum are kinematical (they do not contain interaction terms: $\Delta\mathbf{P} = 0$, and $\Delta\mathbf{J} = 0$), but operators of energy and boosts depend on interaction: $V \neq 0$, $\mathbf{W} \neq 0$. The interaction corrections V and $\mathbf{W}$ must obey a set of non-trivial conditions following from commutation relations (3) - (9). Several interesting suggestions were proposed how to satisfy these conditions[10], however, it seems that *quantum field theory* is the only known relativistically invariant cluster separable approach which is capable to describe particle creation/destruction and agrees with experiment.[4]

The primary goal of quantum field theory is calculation of the S-operator. Currently such calculations can be performed only in the framework of perturbation theory. The simplest ("old fashioned") perturbation expansion for S can be written as

$$S = 1 - i\int_{-\infty}^{+\infty} V(t)dt - \int_{-\infty}^{+\infty} V(t) \int_{-\infty}^{t} V(t')dt'dt + \ldots,$$

where

$$V(t) = e^{iH_0t}Ve^{-iH_0t}.$$

In QFT, the interacting generators are build by the following formal construction. First, introduce three numerical parameters $-\infty < x_i < \infty$, $(i = 1, 2, 3)$. Then one can formally join 4 variables t and $\mathbf{x}$ in one 4-vector $x = (t, \mathbf{x})$ and introduce a pseudo-Euclidean metric and *interval*

$$s = \sqrt{c^2(\Delta t)^2 - (\Delta \mathbf{x})^2} \tag{31}$$

in the space M of these 4-vectors. Then, to each Lorentz transformation b (rotation and boost) there corresponds an interval-preserving "rotation" in this 4-dimensional space expressed by a 4×4 matrix L_b. For example, a boost along the x-axis is represented by the matrix

$$L_b = \begin{bmatrix} \cosh\theta & -\sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Now, assume that we can construct an operator function $V(x) = V(t, \mathbf{x})$ on M which transforms as a scalar with respect to the non-interacting representation of the Poincaré group

$$U_g^0 V(x)(U_g^0)^{-1} = V(L_b x + a)$$

(where g is a composition of a Lorentz transformation b and a space-time translation a) and commutes with itself at space-like separations

$$[V(x), V(x')] = 0, \ \textit{for} \ (x - x')^2 \leq 0.$$

For example, to satisfy these conditions, we can build $V(x)$ as a polynomial in *quantum fields* $\phi_i(x)$ which are linear superpositions of particle creation or destruction operators[4] and have manifestly covariant transformation properties

$$U_g^0 \phi_i(x)(U_g^0)^{-1} = \sum_i D_{ij}(L_b^{-1})\phi_j(L_b x + a),$$

where $D_{ij}(L)$ is a representation of the homogeneous Lorentz group. In addition, boson (fermion) fields at points x and x' are required to commute (anticommute) if $(x - x')$ is a space-like 4-vector. Then in QFT, the *potential energy* $V(t)$ and *potential boost* $\mathbf{W}(t) = e^{iH_0 t}\mathbf{W}e^{-iH_0 t}$ operators are built as integrals

$$V(t) = \int d^3x V(t, \mathbf{x}), \tag{32}$$

$$\mathbf{W}(t) = -\int d^3x \mathbf{x} V(t, \mathbf{x}). \tag{33}$$

and one can show that Poincaré commutation relations are satisfied.[4] The easiest way to obtain $V(t)$ and $\mathbf{W}(t)$ for realistic quantum field theories is to write the Lagrangian as a relativistically invariant functional of $\phi_i(x)$ and $\partial\phi_i(x)/\partial x^\mu$ and then to obtain the Hamiltonian following the canonical quantization procedure.[4]

We would like to emphasise that knowing 10 generators $(H, \mathbf{P}, \mathbf{J}, \mathbf{K})$ and their expressions through particle creation and destruction operators is sufficient to obtain all information about any physical process and to compare it with experiment. The way these generators were obtained, e.g., quantum fields, canonical quantization procedure, etc., is less important.

2.9 Clothed particle representation

There is a serious problem with the above construction of Poincaré generators. Unfortunately, in order to obtain a finite and accurate S-matrix, the interaction Lagrangian and Hamiltonian $V(t)$ in QFT must contain infinite counterterms. These ultraviolet infinities cancel out when transitions between free asymptotic states (S-matrix) are calculated. However, the infinite Hamiltonian $H = H_0 + V$ cannot be used to describe the time evolution, and the resulting formalism of renormalized QFT is not a true dynamical theory.

This problem was solved in Ref. [1] where the QFT Hamiltonian was redefined using the *clothed particle* representation,[11] so that the divergences disappeared, but the S-matrix remained unchanged. As a result of this construction one can obtain true finite operators $H^r = H_0 + V^r$ and $\mathbf{K}^r = \mathbf{K}_0 + \mathbf{W}^r$, while operators $\mathbf{P}$ and $\mathbf{J}$ keep their non-interacting form. The cluster separability of the theory and Poincaré commutation relations of operators $(H^r, \mathbf{P}_0, \mathbf{J}_0, \mathbf{K}^r)$ are preserved in this approach.[1, 12]

In the clothed particle picture, the interaction V^r is a perturbation series in powers of the coupling constant e. In each order, there is a finite number of terms each of which corresponds to a real physical process. For example, the clothed particle interaction in quantum electrodynamics (QED) can be written as[1]

$$V^r \sim a^\dagger a^\dagger aa + a^\dagger c^\dagger ac + a^\dagger a^\dagger c^\dagger aa + a^\dagger a^\dagger aac + \ldots, \tag{34}$$

where we used an abbreviated notation in which only the operator structure of terms is displayed ($a^\dagger, a$ and $c^\dagger, c$ are electron and photon creation and destruction operators, respectively).

The first term on the right hand side of (34) describes the Coulomb repulsion between electrons plus relativistic and radiative corrections to this potential. The second term describes the electron-photon interaction responsible for the Compton scattering. The third and fourth terms describe bremsstrahlung (emission and absorption of photons in electron-electron collisions).

The clothed particle approach provides a true relativistic quantum dynamical theory in which the Hamiltonian, S-operator, and other quantities are finite, and all calculations can be performed according to usual rules of quantum mechanics without renormalization corrections. Interaction (34) leads to exactly the same S-matrix as in the renormalized QED.[1] So, all predictions of QED (electron's magnetic moment, Lamb shifts, etc.) remain valid in the clothed particle approach.

2.10 Decay processes in RQD

The RQD formalism presented above can be also applied to decays of unstable particles.[13] Suppose, for example, that particle α decays via reaction $\alpha \to \beta + \gamma$. Then we can divide the total Fock space $\mathcal{H}$ into the subspace $\mathcal{H}^\alpha$ of states of the unstable particle and its orthogonal complement $\mathcal{H}_\perp$, such that $\mathcal{H} = \mathcal{H}^\alpha \oplus \mathcal{H}_\perp$ and $\mathcal{H}_\perp$ contains the subspace of decay products: $\mathcal{H}^\beta \otimes \mathcal{H}^\gamma \subset \mathcal{H}_\perp$. If Ω is the projection operator on the subspace $\mathcal{H}^\alpha$, then for each state $|\Psi\rangle$ the probability of finding a single unstable particle α is

$$\omega = \langle \Psi | \Omega | \Psi \rangle. \tag{35}$$

The operator Ω commutes with the non-interacting representation of the Poincaré group U_g^0 (30)

$$[\Omega, \mathbf{P}_0] = [\Omega, \mathbf{J}_0] = [\Omega, \mathbf{K}_0] = [\Omega, H_0] = 0, \tag{36}$$

so that in the absence of interactions the particle α is stable with respect to any Poincaré transformation. As discussed in Section 2.7, $\mathcal{H}^\alpha$ carries an irreducible representation of U_g^α characterized by the mass m^α, such that $m^\alpha > m^\beta + m^\gamma$.

The subspace $\mathcal{H}^\alpha$ is no longer invariant when interaction governing the decay is "turned on". If vector $|\Psi\rangle$ lies in $\mathcal{H}^\alpha$ at time $t = 0$ (so that the unstable particle is found with unit probability) then the state vector $e^{-iH^r t}|\Psi\rangle$ at $t \neq 0$ develops a component lying in $\mathcal{H}_\perp$ (more specifically, in $\mathcal{H}^\beta \otimes \mathcal{H}^\gamma$), and

$$[\Omega, H^r] \neq 0. \tag{37}$$

According to the general rule (14), the decay law seen by the observer at rest is given by

$$\omega_0(t) \;=\; \langle\Psi|e^{-iH^r t}\Omega e^{iH^r t}|\Psi\rangle. \tag{38}$$

It can be shown[13] that if the state $|\Psi\rangle$ corresponds to the particle at rest, i.e., $\mathbf{P}|\Psi\rangle = 0$, then quantum mechanical formula for the decay law takes the familiar form

$$\omega_0(t) \;=\; |\int dmc(m)\exp(-imt)|^2, \tag{39}$$

where $c(m)$ is the mass distribution of the unstable particle. If the decay interaction is weak then $c(m)$ has a Breit-Wigner form[14]

$$c(m) \approx \frac{\Gamma/(2\pi)}{\Gamma/4 + (m - m^A)^2},$$

where $m^A \approx m^\alpha$, so that $\omega_0(t)$ in (39) is almost exponential $\omega_0(t) \approx e^{-\Gamma|t|}$.

3 SPECIAL THEORY OF RELATIVITY

It is common to say that special relativity is derived as a consequence of the principle of relativity (proposition (B)) and of the following proposition which has been well tested experimentally

- Proposition (D) [the constancy of the speed of light] *The speed of light is the same for all inertial observers, and does not depend on the velocity of the source.*

The heart of special relativity is in *Lorentz transformations* for time and position of *events*. Generally, events are defined as physical processes localized in space ant time. In special relativity, one starts with more specific class of *photonic* events which are related to light pulses or photons. An example of a photonic event is when a light pulse is reflected from a mirror or is registered by a detector. As described in virtually all textbooks on special relativity, when propositions (B) and (D) are applied to thought experiments with such events, one obtains

- Proposition (E) [Lorentz transformations for time and position] *If observer O registers a photonic event localized at position (x_e, y_e, z_e) and time t_e, then space-time coordinates of this event measured by the observer O' are*

$$
\begin{aligned}
t'_e &= t_e \cosh\theta - x_e \sinh\theta, && (40)\\
x'_e &= x_e \cosh\theta - t_e \sinh\theta, && (41)\\
y'_e &= y_e, && (42)\\
z'_e &= z_e, && (43)
\end{aligned}
$$

In special relativity, equations (40) - (43) are derived for photonic events only, and say nothing about events involving other (e.g., massive) particles either free or interacting with each other and/or with an external potential. Nevertheless, this theory introduces a hypothesis:

- Proposition (F): [universality of Lorentz transformations] *Lorentz transformations (40) - (43) are exact and universal: they are valid for all kinds of events (not only photonic); they do not depend on the composition of the system, on the state of the system, and on the form of interaction acting in the system.*

This hypothesis is often omitted in derivations of special relativity, however, its importance in STR cannot be overestimated. Virtually all significant results of STR are based on the hypothesis (F) which does not have any theoretical or experimental support. The proposition (F) sets very strong limitations on possible physical processes. For example, it can be shown that in STR all time intervals between events increase $\cosh\theta$ times when measured by the observer O'. This, in particular, implies

- Proposition (G) [time dilation]: *Every time-dependent process which is at rest with respect to the observer O and has time dependence $\omega_0(t)$ will slow down according to formula*

$$\omega_\theta(t) = \omega_0(t/\cosh\theta) \tag{44}$$

  when seen by the observer O'. This slowing down does not depend on the nature of the process and on the type of interaction leading to the process.

Another important consequence of proposition (F) is the impossibility of superluminal propagation of particles and interactions.

- Proposition (H) [Einstein causality]: *Any signal emitted from point A arrives to point B not earlier than r_{AB}/c, where r_{AB} is the distance between A and B.*

In STR, Proposition (H) implies that velocities of particles are limited by the speed of light and all interactions are *retarded*. Suppose that a particle emitted from point A at time $t=0$ arrives to point B earlier than at time r_{AB}/c. This fact would not contradict any physical principle as long as this process is viewed from a fixed reference frame. However, assuming the validity of (F) and applying Lorentz transformations we will find a contradiction with the causality principle in moving reference frames: according to (40) - (43), there will exist a moving frame of reference in which the particle arrives to B *before* it was emitted from A, and "the effect precedes the cause", which is clearly unphysical.

Furthermore, STR sets very strict limitations on the structure of allowed physical theories. In particular, it requires that all observables transform in a manifestly covariant way, i.e., as 4-scalars, or 4-vectors, or 4-tensors, etc. Actually, it is common to claim that a theory is relativistically invariant only if all quantities and equations are written in the manifestly covariant form. All these restrictions find their ultimate expression in the proposition which is considered to be the essence of special relativity

- Proposition (I) [Minkowski space-time] *Material world is embedded in a 4-dimensional space-time continuum (Minkowski space-time) with pseudo-Euclidean metric. Transformations between different observers simply result in coordinate changes preserving the interval (31) between space-time points in this continuum.*

Thus, in STR, the Lorentz transformations become completely separated from the physical nature of events and obtain purely geometrical meaning.

4 LORENTZ TRANSFORMATIONS IN RQD

In previous sections we outlined two physical theories: relativistic quantum dynamics and special relativity. RQD, being a full dynamical theory, can, in principle, calculate and verify all predictions of STR. Our goal in this Section and in Sections 5 and 6 is to check the validity of special relativistic Propositions (D) - (I) by direct RQD calculations. Both theories are compared with experiment in Section 7.

4.1 Constancy of the speed of light

Let us start from the second postulate of special relativity (proposition (D)). According to the definition of velocity in RQD

$$|\mathbf{V}| = |\mathbf{P}|c^2(M^2c^4 + \mathbf{P}^2c^2)^{-1/2}, \tag{45}$$

massless particles (photons) have expectation value of velocity equal to the speed of light ($\sqrt{V_x^2 + V_y^2 + V_z^2} = c$). Using (20) - (22) it is easy to show that this value is invariant with respect to boosts

$$e^{-iK_x\theta}\sqrt{V_x^2+V_y^2+V_z^2}e^{iK_x\theta}=\sqrt{V_x^2+V_y^2+V_z^2}=c.$$

Thus, postulate (D) is valid in RQD. Since postulate (B) is also shared by both STR and RQD, one may erroneously conclude that all predictions of both theories must be the same. However, this is not true, as we shall see below, because the STR hypothesis (F) is not valid in RQD.

Since Lorentz transformation for time and position form a basis for all results in special relativity, our first goal is to obtain these transformations in the RQD formalism. (A similar discussion can be found in Refs. [2, 15]) However, before turning to Lorentz transformations, we need a definition of events in RQD.

4.2 Events in quantum mechanics

We start with the definition of event slightly different from that used in STR. Instead of photonic events, we will consider events which are intersections of trajectories of two massive spinless non-interacting particles. We saw in Sections 2.4 and 2.6 that for these particles we have a definition of the position operator and trajectory in the classical limit.

Consider, for example, a state $|\Psi\rangle$ of a two particle system

$$|\Psi\rangle=\int d\mathbf{p}^a d\mathbf{p}^b \psi(\mathbf{p}^a,\mathbf{p}^b)(|\mathbf{p}^a\rangle\otimes|\mathbf{p}^b\rangle)$$

with the momentum space wave function

$$\psi(\mathbf{p}^a,\mathbf{p}^b)=Ne^{-\sigma(p^a)^2}e^{-\sigma((p_x^b-q)^2+(p_y^b)^2+(p_z^b)^2)}e^{-ip_x^b d}. \tag{46}$$

Let us denote the expectations values of observables of individual particles in the state $|\Psi\rangle$ at time $t=0$ by small letters, e.g., $h^a=\langle\Psi|H^a|\Psi\rangle$, $\mathbf{p}^a=\langle\Psi|\mathbf{P}^a|\Psi\rangle$, $\mathbf{k}^a=\langle\Psi|\mathbf{K}^a|\Psi\rangle$, $\mathbf{v}^a=\langle\Psi|\mathbf{V}^a|\Psi\rangle$, $\mathbf{r}^a=\langle\Psi|\mathbf{R}^a|\Psi\rangle$. In the state $|\Psi\rangle$, the y- and z- components of vectors $\mathbf{r}$, $\mathbf{p}$, $\mathbf{k}$, and $\mathbf{v}$ for both particles are zero, whereas x-components are $p_x^a=q$, $v_x^a=q/\sqrt{q^2+(m^a)^2}$, $k_x^b=m^b d$, $r_x^b=d$, $v_x^b=r_x^a=p_x^b=k_x^a=0$. The energies of the two particles are $h^a=\sqrt{q^2+(m^a)^2}$ and $h^b=m^b$. Using Eq. (23) with $\theta=0$ we can calculate trajectories of particles a and b in the reference frame O (full lines in Fig. 1). Only x-components of position are non-zero

$$\begin{aligned} r_x^a(0,t) &= v_x^a t, \\ r_x^b(0,t) &= d. \end{aligned}$$

Then we can define the event in the reference frame O as intersection of particle trajectories at time t_e

$$\mathbf{r}^a(0, t_e) = \mathbf{r}^b(0, t_e) = (x_e, y_e, z_e).$$

Solving these equations yields the space-time coordinates of the event

$$\begin{aligned} t_e &= d/v_x^a, && (47) \\ x_e &= d, && (48) \\ y_e &= z_e = 0. && (49) \end{aligned}$$

4.3 Lorentz transformations for free particles

Now we want to find the time (t'_e) and position (x'_e, y'_e, z'_e) of the same event from the point of view of the observer O'. First we find the trajectories of particles a and b in the reference frame O' (dashed lines in Fig. 1). Using Eq. (23) and the fact that in the classical limit the expectation value of a product of observables is the product of expectation values, we obtain

$$\begin{aligned} r_x^a(\theta, t') &= \langle \Psi | e^{-i(K_x)_0\theta} e^{-iH_0 t'} R_x^a(0,0) e^{iH_0 t'} e^{i(K_x)_0\theta} | \Psi \rangle && (50) \\ &= \frac{k_x^a}{h^a \cosh\theta - p_x^a \sinh\theta} + \frac{v_x^a - \tanh\theta}{1 - v_x^a \tanh\theta} t' && (51) \\ &= \frac{v_x^a - \tanh\theta}{1 - v_x^a \tanh\theta} t' \end{aligned}$$

and, similarly,

$$r_x^b(\theta, t') = \frac{d}{\cosh\theta} - \tanh\theta t'.$$

Then the space-time coordinates of the event in the reference frame O' are determined by the solution of equation

$$\mathbf{r}^a(\theta, t'_e) = \mathbf{r}^b(\theta, t'_e) = (x'_e, y'_e, z'_e), \qquad (52)$$

which yields

$$
\begin{aligned}
t'_e &= \frac{d}{v_x^a}\cosh\theta - d\sinh\theta, &(53)\\
x'_e &= d\cosh\theta - \frac{d}{v_x^a}\sinh\theta, &(54)\\
y'_e &= z'_e = 0. &(55)
\end{aligned}
$$

Comparing this result with (47) - (49) we find the usual Lorentz transformations for time and position (40) - (43) in full agreement with STR. Although our derivation used a specific initial state (46) of the two particles, it is not difficult to show that one obtains the Lorentz transformations (40) - (43) in the general case as well. However, the validity of (53) - (55) rests on two assumptions. First, it is valid only for expectation values of the position observable in quasiclassical states. For general quantum states and for light particles with fast-spreading wave packets the notion of trajectory looses its clear meaning, and events (as intersections of trajectories) cannot be properly defined. Second, equations (53) - (55) have been derived for non-interacting particles a and b. In the next two sections we will consider Lorentz transformations for non-quasiclassical states and for interacting systems.

5 SUPERLUMINAL PROPAGATION OF PARTICLES

Proposition (H) forbids superluminal propagation of particles. However, the arguments supporting this proposition are valid only if there is a definite meaning for the statement "particle moves from A to B". Although this statement is always valid in classical physics, it is often problematic in the quantum case. One example of a situation where the direction of particle's movement has no meaning is a state $|\mathbf{q}\rangle$ with definite momentum. The wave function (25) of this state is spread uniformly over all position space, so that particle can be found anywhere at each time instant. Measurements of particle position are completely unpredictable, and velocity calculated using the usual "distance/time-of-travel" formula can take any value from 0 to $+\infty$.

Less trivial examples of the "superluminal spreading of wave packets" are provided by well-localized particle states.[16] Consider a state of the particle sharply localized at the origin (let us call it "point A") at time $t = 0$ (see Eq. (24))

$$
|\mathbf{r}=0\rangle = (2\pi)^{-3/2}\int d\mathbf{q}|\mathbf{q}\rangle. \qquad (56)
$$

The wave function in the position representation is

$$\psi(\mathbf{y}) \quad = \quad \langle \mathbf{y}|\mathbf{r}=0\rangle = \delta(\mathbf{y}).$$

At time $t \neq 0$ the position-space wave function transforms to

$$e^{-iHt}\psi(\mathbf{y}) \quad = \quad \langle \mathbf{y}|e^{-iHt}|\mathbf{r}=0\rangle = (2\pi)^{-3/2}\int d\mathbf{q} e^{i\mathbf{q}\mathbf{y}} e^{-i\sqrt{m^2+\mathbf{q}^2}t}.$$

The exact value of this integral[17] is not important to us at this point. The important fact is that due to the presence of a non-analytical square root in the exponent, the wave function $e^{-iHt}\psi(\mathbf{y})$ is nonzero for *all* values of $\mathbf{y}$ when $t \neq 0$.[16] At time $t > 0$ the probability of finding the particle anywhere in the space is non-zero, although this probability is exponentially small outside the "light cone" $|\mathbf{y}| > ct$. In spite of the smallness of this probability, there is still a chance that the particle will be registered at point B earlier than at $t = r_{AB}/c$ in obvious disagreement with proposition (H).

The solution of this paradox is rather simple. As was said above, the mere fact that a particle moves faster than light in a fixed reference frame is not troublesome by itself. The important thing is to make sure that there are no acausal effects in all frames of reference. Let us find the wave function at time $t' = 0$ (the time of preparation of the particle at "point A") from the point of view of the moving observer O'

$$\begin{aligned} e^{-iK_x\theta}\psi(\mathbf{y}) &= \langle \mathbf{y}|e^{-iK_x\theta}|\mathbf{r}=0\rangle \\ &= (2\pi)^{-3/2}\int d\mathbf{q} e^{i\mathbf{q}\mathbf{y}}\sqrt{\frac{\sqrt{\mathbf{q}^2+m^2}\cosh\theta + q_x\sinh\theta}{\sqrt{\mathbf{q}^2+m^2}}}. \end{aligned}$$

Again, the non-analyticity of the square roots in the integrand implies that $e^{-iK_x\theta}\psi(\mathbf{y})$ is non-zero for all values of $\mathbf{y}$ if $\theta \neq 0$. Therefore, observer O' would not agree with O that at $t' = 0$ the particle was prepared at point A. At $t' = 0$ observer O' can find the particle anywhere in space, even at point B, although with a low probability. Therefore, the statement "particle travels from A to B" has no definite meaning for O'. For well-localized states (56) (just as for eigenstates of momentum discussed above) we cannot rigorously speak about the cause-effect relationship in all reference frames.

Eigenvalues of the velocity operator (45) are always less than c. This statement seems to be in agreement with proposition (H) and at odds with just discussed superluminal propagation. However, operators of velocity and position do not commute, and in quantum mechanics velocity cannot be defined simply as "distance/time-of-travel". Therefore condition $|\mathbf{V}| < c$ does not preclude a particle which was prepared at point A at time $t = 0$ to be found (with a low probability) at point B at time $t < r_{AB}/c$.

6 LORENTZ TRANSFORMATIONS FOR INTERACTING PARTICLES

6.1 Action-at-a-distance

In order to obtain Lorentz transformations (53) - (55), it was assumed that the Hamiltonian and the generators of boosts do not contain interaction terms (see Eq. (50)). If these operators do contain interaction terms, so that $H^r = H_0 + V^r$ and $K_x^r = (K_x)_0 + W_x^r$ are used instead of H_0 and $(K_x)_0$, then the formula for the position of a particle in the reference frame O' will differ from the simple result (51), and solution of Eq. (52) will depend on the interaction terms V^r and W_x^r. It has been proven[2] that Lorentz transformations will retain their usual form (40) - (43) only in the case $V^r = 0$, $\mathbf{W}^r = 0$, i.e., in the absence of interactions. Therefore, in RQD, Lorentz transformations (40) - (43) cannot be applied to events with interacting particles. The violation of Lorentz transformations for interacting systems means, in particular, that propositions (G) and (H) are not valid. Let us first discuss proposition (H) (the retarded form of interactions). We will turn to proposition (G) (time dilation in a moving frame of reference) in the next subsection.

It is often declared that in QFT interactions between particles are retarded. The usual argument is that interactions are transmitted by (virtual) particles, so the action-at-a-distance is forbidden. This statement, however, is rather confusing. For example, it is known that virtual photons "transmitting" the Coulomb force between two charged particles in QED would have imaginary mass. It should be clear that such particles are just theoretical constructs which do not have any chance to be observed experimentally. In fact, the question about the speed of interaction cannot be answered by QFT because, as discussed in Section 2.9, it is not a dynamical theory, and particle trajectories can be calculated only in the asymptotic regimes, i.e., far from the region of interaction. On the other hand, RQD with interaction (34) gives an unambiguous answer: interaction propagates instantaneously. For example, the first term on the right hand side of (34) leads to the instantaneous Coulomb potential with proper relativistic and radiative corrections.[1] Contrary to common opinion, such action-at-a-distance potential does not contradict the principle of relativity. It is not difficult to build relativistic few-particle models in which interaction is instantaneous, while Poincaré commutation relations are valid.[18]

Moreover, since Lorentz transformations should be modified in the presence of interaction, as discussed above, such instantaneous interaction does not violate causality in any frame of reference. Consider two interacting quasiclassical electrons a and b whose wave function at time $t = 0$ is given by Eq. (46) (the electron spin is neglected). The dynamics of this system is described by the Hamiltonian H^r with interaction (34). In the classical limit, we can represent the movement of these two electrons by trajectories CAG and DBF, respectively (see Fig. 2). Suppose that due to some external influence (e.g., impact by a third neutral particle) at time $t = 0$ (point A) the electron a changes its course to AG'. This change will be felt by the electron b instantaneously due to the first (Coulomb) term in the interaction operator (34). Therefore, the electron b will also change its trajectory to BEF'. It is important to note that the electron-electron interaction has an "instantaneous" part $a^\dagger a^\dagger aa$ in *all* frames of reference. Thus, for observer O', the information about event at A will reach particle b instantaneously (of course, not earlier than event A has occurred), just as for observer O, and no contradictions with causality arise here.

In addition to the pure instantaneous interaction, there is a retarded interaction between electrons as well. The origin of this retarded interaction in QED can be explained as follows (see Fig. 2). The impact on the electron a at point A creates bremsstrahlung photons (dashed line). There is a chance that such a photon will reach the electron b at point E of its trajectory and force b to change its course again (EF''), e.g., due to the elastic photon-electron scattering described by the term $a^\dagger c^\dagger ac$ in the Hamiltonian (34). In this process, the force between two particles is not transmitted instantaneously; it is carried by photons (or by the transversal electromagnetic wave, in classical language) traveling with the speed of light. Therefore, in RQD, two kinds of interactions coexist: the instantaneous potential and the retarded interaction transmitted by real (not virtual!) particles.

6.2 Decay of unstable particles in the moving frame of reference

The non-decay probability ω is an expectation value of the projection operator Ω (see Eq. (35)). Therefore, we can find the decay law for a particle in the moving reference frame O' by using general formula (15)

$$\omega_\theta(t') = \langle \Psi | e^{-iK_x^r \theta} e^{-iH^r t'} \Omega e^{iH^r t'} e^{iK_x^r \theta} | \Psi \rangle. \tag{57}$$

If we now assume the validity of proposition (G) then, in particular, we obtain $1 = \omega_0(0) = \omega_\theta(0) = \langle \Psi | e^{iK_x^r \theta} \Omega e^{-iK_x^r \theta} | \Psi \rangle$, from which it follows that the subspace $\mathcal{H}^\alpha$ is invariant with respect to boosts, i.e., $[\Omega, K_x^r] = 0$. From (8) and (36), we then obtain, using Jacobi identity,

$$[\Omega, H^r] \;=\; i[\Omega, [K_x^r, (P_0)_x]] = i[K_x^r, [\Omega, (P_0)_x]] - i[(P_0)_x, [\Omega, K_x^r]] = 0$$

in contradiction to (37). Therefore, the probability ω is not invariant with respect to boosts. In other words, if observer O sees a pure particle α at time $t = 0$, then observer O' at $t' = 0$ will see a mixture of the particle α and its decay products β and γ. [13]

The conclusion from this analysis is that the decay law in a moving frame of reference is not given by the universal STR formula (44), but depends on interaction. Explicit calculations of the decay law (57) in different forms of relativistic dynamics confirm this conclusion. They will be reported elsewhere.

7 COMPARISON WITH EXPERIMENT

It is not difficult to show that predictions of RQD agree with Einstein's special relativity in all aspects concerning relationships between total mass, momentum, energy, and the center-of-mass velocity. These predictions include the relativistic law of addition of velocities (20) - (22), Lorentz transformations of the momentum-energy 4-vector (16) - (19), the Doppler effect, etc. The above effects cover a vast majority of experimental manifestations of STR; they were tested with very high precision in modern experiments.[19] However, there are experiments for which predictions of both theories (STR and RQD) are different. As discussed above, important examples of such experimental effects are i) superluminal propagation of interactions, and ii) decay of moving unstable particles. Our goal in this Section is to demonstrate that these RQD results do not contradict available experimental data.

7.1 Action-at-a-distance

The speed of propagation of interactions is still an intensely debated subject.[20, 21] There are certain indications that the speed of gravitational force may exceed the speed of light.[20] For example, the vector of gravitational acceleration of the Earth points to the instantaneous position of the Sun rather than to its visible position. However, the most direct information about the speed of propagation of interactions may come from laboratory experiments. There are numerous experiments indicating superluminal group and phase velocities of transverse electromagnetic waves (photons) in certain conditions.[22] However, to the best of our knowledge, there are only two experiments directly relevant to our discussion of the instantaneous Coulomb potential in Section 6.1.

Tzontchev, Chubykalo, and Rivera-Juárez found that the propagation speed of the electromagnetic pulse resulting from the discharge of the van de Graaff generator is equal to the speed of light.[23] An apparently contradicting result was obtained by Walker[24] who discovered that the longitudinal electromagnetic field propagates superluminally at distances less than one wavelength from the emitting antenna. Rather unsophisticated equipment has been used by these two experimental groups. This gives a hope that with more advanced techniques, the controversy surrounding the propagation speed of the Coulomb and gravitational force will be easily resolved.

7.2 Decay law of moving particles

Let us denote $\omega_0(t)$ the decay law for a particle at rest (see Eqs. (38) and (39)). Based on proposition (G), STR would predict that the decay law of the moving particle is *exactly* $\cosh\theta$ times slower than $\omega_0(t)$

$$\omega_\theta^{STR}(t) = \omega_0(\frac{t}{\cosh\theta}) = |\int dmc(m)\exp(-imt/\cosh\theta)|^2. \tag{58}$$

This "time dilation" was confirmed by experiments[25] and is usually considered to be the strongest argument in favor of STR. However, RQD can explain this effect as well. Before considering this explanation we need to mention that formula (57) (describing decay from the point of view of a moving observer, or, equivalently, decay of a particle prepared by a moving preparation device) does not apply here. In experiments, the preparation device (accelerator) and the measuring device (detector) are usually immobile. Therefore, we need to consider the decay of a *moving unstable particle*. If state vector $|\Psi\rangle$ describes particle α at rest, then the state vector $|\Psi'\rangle = e^{-i(K_x)_0\theta}|\Psi\rangle$ describes the same particle moving with velocity $\tanh\theta$. Its decay law measured by the stationary observer O is

$$\begin{aligned}\omega_\theta(t) &= \langle\Psi'|e^{-iH^r t}\Omega e^{iH^r t}|\Psi'\rangle \\ &= (\langle\Psi|e^{i(K_x)_0\theta})e^{-iH^r t}\Omega e^{iH^r t}(e^{-i(K_x)_0\theta}|\Psi\rangle).\end{aligned}$$

Assuming that interaction has a particular flavor of the instant form of dynamics we can obtain an exact formula[13]

$$\omega_\theta(t) = |\int dmc(m)\exp(-it((m^\alpha)^2\sinh^2\theta + m^2)^{1/2})|^2. \tag{59}$$

Although this is different from the STR result (58), the difference does not exceed ≈0.001% for all known unstable systems. Since accuracy of current experiments is not better than 0.1 %[25], the RQD result (59) is also consistent with experiment.

It has been shown[13] that the decay laws in other Dirac forms of dynamics may be significantly different from both (58) and instant form result (59). For example, the decay of a moving particle *accelerates* in the point form of dynamics, which is completely unacceptable from the experimental point of view. Therefore the observed slowing-down of the decay is not a result of the universal time dilation (as in Proposition (G)), but an indication that interactions responsible for particle decays have a specific (most likely, instant) form of relativistic dynamics.

8 DISCUSSION

It is often claimed that the Newton-Wigner position operator (13) does not correspond to physical position because under boosts this operator does not transform as a spatial part of a 4-vector. Various approaches were suggested to enforce the manifest covariance of position in quantum mechanics. They include introduction of the time operator. However, these attempts contradict quantum postulates. For example, in Lorentz transformations (53) - (55) obtained using the RQD approach, t_e is a value of a numerical parameter distinguishing two reference frames shifted in time with respect to each other. On the other hand, in quantum theory, the observable of position $\mathbf{R}$ must be represented by a Hermitian operator, because any measurement of position is associated with quantum-mechanical uncertainty. Therefore, in (53) - (55), $\mathbf{r}_e$ is the expectation value of quantum observable $\mathbf{R}$ in a given state of the system. The quantities t_e and $\mathbf{r}_e$ have completely different status in quantum theory and it is not correct to consider them as components of the same 4-vector.

We should also note that two important conclusions of this work: the superluminal propagation of interactions and the existence of quantum corrections to the decay law of moving particles do not depend on the choice of the position operator. Therefore, these conclusions will remain valid even if the assumed properties of position (10) - (12) will be found not accurate.

There are also attempts to interpret the 4-vector argument of the quantum field $\phi_i(t, \mathbf{x})$ as manifestly covariant time and coordinate.[26] It is rather difficult to assign any direct physical interpretation to quantum fields $\phi_i(t, \mathbf{x})$ and to parameters $\mathbf{x}$, but, fortunately, there is no need to do that according to the interpretation of the QFT formalism suggested by Weinberg.[4] In this interpretation (fully consistent with the views expressed in the present paper), quantum fields are just auxiliary constructs which help us to build interaction operators $V(t, \mathbf{x})$ and $\mathbf{W}(t, \mathbf{x})$ and to ensure that generators $(H, \mathbf{P}, \mathbf{J}, \mathbf{K})$ satisfy Poincaré commutation relations. Then parameters $\mathbf{x}$ should be considered just as integration variables used in (32) and (33) to write down expressions for the potential energy $V(t)$ and potential boost $\mathbf{W}(t)$. QFT does not require any connection between these parameters and particle coordinates measured in experiments. Thus, the 4-dimensional "space-time" M in QFT has only formal resemblance to real space and time.

The manifest covariance should not be considered as indispensable ingredient of a relativistic physical theory. We saw that, in the presence of interaction, the time-position transformations have a complex form depending on the interaction and on the state of the system. Similarly, the time dilation is a dynamical effect not described by simple formula (44). On the other hand, the Poincaré group (which is, incidentally, also the symmetry group of the "space-time" M) expresses exact relations between different inertial observers and remains valid independent on the form and strength of interactions. These relations between inertial observers are what is really important to ensure the relativistic invariance of the theory. The manner in which various observables of the theory transform from one reference frame to another should be derived from the Poincaré group properties and from quantum postulates. In some cases (e.g., the total energy-momentum vector) these transformation properties are manifestly covariant, in other cases (e.g., for time and position of events) they are not. The fact that manifestly covariant quantum fields are not essential components in interacting relativistic quantum theories is clearly demonstrated in the clothed particle approach in which quantum fields are not present explicitly and the manifest covariance is lost.

9 CONCLUSIONS

Relativistic quantum dynamics successfully combines quantum mechanics with the special principle of relativity and accurately describes all available experimental data. In this theory, transformations for time and position of events between different reference frames do not have the form of Lorentz transformations. This is not a flaw of RQD, as is often thought, but a real physical fact. The common mistake made in theoretical derivations of special relativity is an unjustified generalization of Lorentz transformations (derived in STR for photonic events and valid for quasiclassical states and for non-interacting systems only) to quantum states and to interacting systems. This generalization, formalized in Proposition (F), does not have any theoretical or experimental support. Therefore, we conclude that the Minkowski space-time is an approximate concept valid only in the limit of vanishing interactions between particles.

Some results of RQD (e.g., the decay law of a moving unstable system, faster-than-light spreading of wave packets, and instantaneous action-at-a-distance) contradict orthodox special relativity, but they are consistent with currently available experiments and with the principle of causality. The most promising way to observe clear deviations from predictions of special relativity is to confirm experimentally that the propagation speed of the Coulomb interaction is greater than the speed of light.

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Figure captions

Fig.1. Trajectories of two non-interacting quasiclassical particles a and b having wave function (46), and the event defined as intersection of particle trajectories in the reference frame O (full lines) and in the moving reference frame O' (dotted lines).

Fig.2. Trajectories of two interacting quasiclassical electrons a and b. An external impact and the trajectory change of the electron a at point A causes instantaneous change of the trajectory of the electron b at point B due to the Coulomb potential. A bremsstrahlung photon (dashed line) emitted at point A causes a retarded influence on the electron b at point E. Dotted lines are trajectories in the absence of the external impact. Dashed-dotted lines are trajectories without taking into account the photon-transmitted interaction. Full lines are exact trajectories.