Quantum Field Theory without Infinities. II. A Simple Model.

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A general method for elimination of infinities from quantum field theory (E. V. Stefanovich, 2001, *Annals of Physics* **292**, 139) is illustrated here by studying a simple model of interacting electrons and photons. All calculations are performed up to the 4th order of the perturbation theory. The renormalization program (including both mass and charge renormalizations) is carried through and radiative corrections to the electron-electron scattering are explicitly calculated. The "clothed particle" finite Hamiltonian of the theory is obtained. The electron-electron interaction in this Hamiltonian has the form of the Coulomb potential with relativistic and radiative corrections.

Key Words: quantum field theory; renormalization; Hamiltonian; time evolution; S-operator; clothed particles, ultraviolet divergences.

1. INTRODUCTION

Quantum field theory (QFT) achieved an unprecedented success in describing certain types of experiments in subatomic physics (scattering processes, energies of bound states, etc.). However, this theory is far from being satisfactory, especially due to the problem of ultraviolet divergences. Although the S-matrix calculated in renormalized QFT, e.g., in quantum electrodynamics (QED), is extremely accurate, the Hamiltonian H^c of the theory contains infinite counterterms. This means that an important class of measurements, namely those concerned with time evolution of states and observables [1] cannot be described. This seemingly paradoxical situation was analyzed in [2] where it was suggested to redefine the Hamiltonian of QFT to get rid of divergences, while keeping the S-matrix intact. This redefinition used the "clothed particle" approach developed in [3, 4]. The resulting theory was formulated in a general form for a broad class of QFT Hamiltonians in all orders of the perturbation theory. Understandably, in this general setting, many computational details were omitted. Thus, it is desirable to illustrate the general approach of Ref. [2] on a simple example. In this paper, we consider a model quantum field theory which has mass and charge renormalization problems similar to those in more complex theories, such as QED. We show, step by step, how all ultraviolet infinities can be eliminated from the theory and how a finite clothed particle Hamiltonian H^r can be derived. All calculations are performed up to the fourth order in the coupling constant. This is the lowest order in which non-trivial cancellations should occur to make the renormalization theory work.

The paper is organized as follows. In section 2, we discuss the motivation of the clothed particle approach to the elimination of divergences in QFT. In Section 3, we introduce the bare particle Hamiltonian of our model theory and formulas for scattering operators. In Section 4, some important properties of operators in the Fock space are discussed. In section 5, we introduce a diagram technique which greatly simplifies calculations in our model. In Section 6, we present the traditional method of the S-matrix renormalization in the diagram notation and calculate the infinite counterterms in the Hamiltonian H^c and the finite renormalized scattering operator S^c. In Section 7, the clothed particle approach is employed in order to obtain the finite Hamiltonian H^r which is scattering-equivalent to H^c. Section 8 is devoted to discussion of our results.

2. HAMILTONIAN AND S-OPERATOR

The essence of the problem of infinities in QFT is better understood by considering two ways in which quantum dynamics is described: the time evolution operator and the S-operator. In a generic quantum theory, the

Hamiltonian H and the time evolution operator e^{-iHt} provide a detailed dynamical description: If $|\Psi(0)\rangle$ is a state vector at time $t = 0$, then the state vector at any other time t is given by $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$. However, in many cases, this description is too detailed. For example, in current experiments with energetic particles, it is not possible to observe the time evolution during interaction: particle reactions occur almost instantaneously and we can only observe reactants and products which move freely before and after the collision. Then the theory is not required to describe the actual evolution of particles during the short interval of collision. It is sufficient to provide a mapping of free states before interaction onto the free states after the interaction. This mapping is given by the S-operator.

Since dynamical, i.e., time-dependent, information is present in the S-operator only in the averaged form (e.g., time delay of scattering [5]), it is not surprising that the same S-operator can be obtained from different Hamiltonians. These scattering-equivalent Hamiltonians are connected to each other by unitary transformations U [6, 7]. The set of transformations U conserving the S-operator is very large. As Ekstein remarked [6], "The conditions on the operator U are so unspectacular that one may be inclined to think that "any reasonable" unitary transformation will lead to the same S operator." In QFT, there are numerous examples of such unitary transformations of the Hamiltonian aimed to simplify calculations or achieve other goals without affecting the S-operator. For example, in gauge theories, Hamiltonians obtained in different gauges are different, but the S-matrix is gauge-independent [8]. Another example is the unitary- and scattering-equivalence of different Dirac forms of dynamics [9]. Due to this apparent insensitivity of theoretical predictions to the choice of the Hamiltonian, the Hamiltonian in QFT is often considered just as an auxiliary tool for obtaining the S-operator which, allegedly, contains all relevant physical information.

However, it is important to realize that scattering-equivalent theories with unitarily equivalent Hamiltonians are not *physically* equivalent. They yield different descriptions of dynamics in the interaction region. Current experiments in high energy physics cannot distinguish these different dynamics due to a limited resolution of instruments. However this fact is not a sufficient ground to believe that all measurable quantities are contained exclusively in the S-operator. For example, time dependence of observables is easily measurable at the macroscopic level. Moreover, rapidly developing time-resolved spectroscopy techniques [1] may soon be able to observe dynamics of wave packets involving atomic levels affected by radiative corrections. Therefore, there is a need for finding a *unique* correct Hamiltonian H^r which is capable to describe both the time evolution and the scattering.

The Hamiltonian H^c of QFT is normally obtained through canonical quantization of a classical field theory. Since classical theory (e.g., Maxwell electrodynamics) is supposed to be a low-energy approximation of the quantum theory (QED), such derivation is by no means rigorous. Moreover, infinite counterterms must be present in H^c in order to obtain a finite and accurate operator S^c. The presence of infinities in H^c implies that this Hamiltonian is not the correct generator of time evolution. However, the above discussion suggests that we may try to find the correct finite Hamiltonian H^r by applying a certain unitary scattering-preserving transformation $U = e^{i\Phi}$ to H^c

$$H^r = e^{i\Phi} H^c e^{-i\Phi}.$$

We can properly choose infinite terms in the Hermitian operator Φ, so that these infinities cancel infinite counterterms in H^c and the resulting H^r is finite. It was shown in [2] that, indeed, such an operator Φ can be found for a rather general class of quantum field theories in each order of the perturbation theory and that such properties of QFT as cluster separability and relativistic invariance are preserved during this transformation as well as the exact form of the S-operator. In what follows, we will consider an application of this general theory to a simple QFT model.

3. THE MODEL THEORY

We will study a model quantum field theory describing spinless fermions with mass $m > 0$ and spinless massless bosons. Although such particles do not exist in Nature, they can be considered as approximate (spin is neglected) representatives of electrons and photons, respectively. The Fock space $\mathcal{H} = |0> \oplus \mathcal{H}_e \oplus \mathcal{H}_p \oplus \mathcal{H}_{ee} \oplus \mathcal{H}_{ep} \oplus \mathcal{H}_{pp} \ldots$ is a direct sum of subspaces with 0, 1, 2,... particles. Creation and destruction operators for our "electrons" (a) and "photons" (c) satisfy anticommutation and commutation relations

$$\{a_{\mathbf{p}}, a^\dagger_{\mathbf{p}'}\} = [c_{\mathbf{p}}, c^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p} - \mathbf{p}'), \tag{1}$$

$$\{a_{\mathbf{p}}, a_{\mathbf{p}'}\} = \{a^\dagger_{\mathbf{p}}, a^\dagger_{\mathbf{p}'}\} = 0, \tag{2}$$

$$[c_{\mathbf{p}}, c_{\mathbf{p}'}] = [c^\dagger_{\mathbf{p}}, c^\dagger_{\mathbf{p}'}] = 0,$$

$$[a^\dagger_{\mathbf{p}}, c^\dagger_{\mathbf{p}'}] = [a^\dagger_{\mathbf{p}}, c_{\mathbf{p}'}] = [a_{\mathbf{p}}, c^\dagger_{\mathbf{p}'}] = [a_{\mathbf{p}}, c_{\mathbf{p}'}] = 0.$$

The Hamiltonian has the form $H = H_0 + \hat{V}_1$, where the free Hamiltonian is

$$H_0 = \int d\mathbf{p}\omega_{\mathbf{p}} a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + \int_{\mathbf{k}\neq 0} d\mathbf{k} k c^\dagger_{\mathbf{k}} c_{\mathbf{k}},$$

($\omega_{\mathbf{p}} = \sqrt{m^2+p^2}$) and the interaction

$$\hat{V}_1 = e(2\pi)^{-3/2} \int_{\mathbf{k}\neq 0} \frac{d\mathbf{p}d\mathbf{p}'d\mathbf{k}}{\sqrt{2k}} [a^\dagger_{\mathbf{p}} c^\dagger_{\mathbf{k}} a_{\mathbf{p}'} \delta(\mathbf{p}+\mathbf{k}-\mathbf{p}') + a^\dagger_{\mathbf{p}} a_{\mathbf{p}'} c_{\mathbf{k}} \delta(\mathbf{p}-\mathbf{k}-\mathbf{p}')].$$

is first order in the coupling constant e (electron charge). Here and in what follows the order of an operator is shown by the subscript. We also write all operators in the normal form, i.e., all creation operators to the left from all destruction operators.

The above theory satisfies the total momentum and charge conservation laws

$$[H,Q] = [H,\mathbf{P}] = 0, \tag{3}$$

where

$$\mathbf{P} = \int d\mathbf{p}\mathbf{p} a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + \int_{\mathbf{k}\neq 0} d\mathbf{k}\mathbf{k} c^\dagger_{\mathbf{k}} c_{\mathbf{k}}$$

is operator of total momentum and $Q = -e\int d\mathbf{p} a^\dagger_{\mathbf{p}} a_{\mathbf{p}}$ is operator of total charge.

To describe scattering experiments we are primarily interested in the S-operator [10]

$$S = \lim_{t_1\to-\infty, t_2\to+\infty} e^{iH_0 t_2} e^{-i(H_0+\hat{V}_1)(t_2-t_1)} e^{iH_0 t_1}.$$

Using convenient symbols for time integrals $\underline{Y(t)} = \int_{-\infty}^{t} Y(t')dt'$ and $\underbrace{Y} = \int_{-\infty}^{+\infty} Y(t')dt'$, the "old-fashioned" perturbation expansion of the S-operator is given by $S = 1 + \underbrace{\Sigma}$, where

$$\Sigma = -iV_1 - V_1\underline{V_1} + iV_1\underline{\underline{V_1\underline{V_1}}} + V_1\underline{\underline{\underline{V_1\underline{V_1\underline{V_1}}}}} + \ldots, \tag{4}$$

$$V_1(t) = e^{iH_0 t}\hat{V}_1 e^{-iH_0 t} e^{\epsilon t},\ \epsilon\to 0, \tag{5}$$

and factor $e^{\epsilon t}$ describes the adiabatic switching of the interaction.

We will also use another equivalent representation of the S-operator which explicitly shows its unitarity $S = e^{2iG}$ [11], where the *phase shift* operator is

$$G = -\frac{1}{2}\underbrace{F},$$

and Hermitian operator F can be represented as a series of multiple commutators

$$\begin{aligned} F \;=\;& V_1 + \frac{i}{2}[\underline{V_1}, V_1] - \frac{1}{6}[\underline{\underline{\underline{V_1}}, [V_1, V_1]}] - \frac{1}{6}[\underline{[\underline{V_1}, V_1]}, V_1] - \frac{i}{12}[\underline{\underline{\underline{V_1}}, [[V_1, V_1], V_1]}] \\ &- \frac{i}{12}[\underline{\underline{[\underline{\underline{V_1}}, [V_1, V_1]}]}, V_1] - \frac{i}{12}[\underline{\underline{[\underline{V_1}, V_1]}, [V_1, V_1]}] + \ldots \end{aligned} \tag{6}$$

It follows from equation $1 + \underline{\Sigma} = \exp(-i\underline{F})$ that operators Σ and F are related to each other

$$\Sigma \;=\; -iF - \frac{1}{2!}\frac{d}{dt}(\underline{F})^2 + \frac{i}{3!}\frac{d}{dt}(\underline{F})^3 + \frac{1}{4!}\frac{d}{dt}(\underline{F})^4 + \ldots, \tag{7}$$

so finding F or Σ are equivalent tasks.

4. PROPERTIES OF OPERATORS IN THE FOCK SPACE

In calculations of operators (4), (6), and (7) we meet normal products, commutators, time integrals and derivatives of $V_1(t)$. These operations result in linear combinations of operators Y each of which contains a product of N creation and M destruction operators and satisfies conditions (3). Let us discuss the general structure of such operators Y using $V_1(t)$ as an example.

$$\begin{aligned} V_1(t) \;=\;& e(2\pi)^{-3/2}\int_{\mathbf{k}\neq 0}\frac{d\mathbf{p}d\mathbf{p}'d\mathbf{k}}{\sqrt{2k}}[a^\dagger_{\mathbf{p}}c^\dagger_{\mathbf{k}}a_{\mathbf{p}'}e^{i(\omega_{\mathbf{p}}-\omega_{\mathbf{p}'}+k-i\epsilon)t}\delta(\mathbf{p}+\mathbf{k}-\mathbf{p}') \\ &+a^\dagger_{\mathbf{p}}a_{\mathbf{p}'}c_{\mathbf{k}}e^{i(\omega_{\mathbf{p}}-\omega_{\mathbf{p}'}-k-i\epsilon)t}\delta(\mathbf{p}-\mathbf{k}-\mathbf{p}')]. \end{aligned} \tag{8}$$

The operator part of expression (8) is a normal product of creation and destruction operators of particles. The numerical factor in the integrand consists of three parts. First, there is an exponential factor $e^{i(E_Y - i\epsilon)t}$ which describes the time dependence of the expression. Due to (5) the *energy function*

$$E_Y(\mathbf{q}'_1, \dots, \mathbf{q}'_N, \mathbf{q}_1, \dots, \mathbf{q}_M) \equiv \sum_{i=1}^{N} \sqrt{m_i^2 + \mathbf{q}'^2_i} - \sum_{j=1}^{M} \sqrt{m_j^2 + \mathbf{q}_j^2} \qquad (9)$$

is the difference of energies of particles created and destroyed by the operator part. We will call *regular* those operators Y whose time dependence is given by $e^{i(E_Y - i\epsilon)t}$ as in (8). It is easy to show that product and commutator of two regular operators is regular. Time derivative of a regular operator is also regular. However, this is not always true for time integrals (see below). The hypersurface of solutions

$$E_Y(\mathbf{q}'_1, \dots, \mathbf{q}'_N, \mathbf{q}_1, \dots, \mathbf{q}_M) = 0 \qquad (10)$$

(if exists) is called the *energy shell* of the operator Y. The numerical factor in (8) also contains a *coefficient function* D_Y. For each of the two operators in (8) the coefficient function is

$$D_Y(\mathbf{p}, \mathbf{p}', \mathbf{k}) = \frac{e}{(2\pi)^{3/2}\sqrt{2k}}.$$

Finally, there is a momentum delta function in each term in (8), which is needed to satisfy the momentum conservation law (3). The product of all mentioned factors is integrated by momenta which are present as arguments of the particle operators. Finally, since the resulting expression contains a momentum delta function in the integrand, the integration by one arbitrarily chosen momentum can be performed explicitly. After integration by momentum $\mathbf{p}'$, eq. (8) becomes

$$\begin{aligned} V_1 &= e(2\pi)^{-3/2} \int\limits_{k \neq 0} \frac{d\mathbf{p}d\mathbf{k}}{\sqrt{2k}} [a^\dagger_{\mathbf{p}} c^\dagger_{\mathbf{k}} a_{\mathbf{p}+\mathbf{k}} e^{i(\omega_{\mathbf{p}} - \omega_{\mathbf{p}+\mathbf{k}} + k - i\epsilon)t} \\ &+ a^\dagger_{\mathbf{p}} a_{\mathbf{p}-\mathbf{k}} c_{\mathbf{k}} e^{i(\omega_{\mathbf{p}} - \omega_{\mathbf{p}-\mathbf{k}} - k - i\epsilon)t}]. \end{aligned} \qquad (11)$$

Let us divide all operators into three categories: physical (ph), unphysical (unp), and renorm (ren), so that any linear combination of operators can be uniquely decomposed into three parts $Y = Y^{ph} + Y^{unp} + Y^{ren}$. Note that *decay* type operators [2] are not present in this model theory.

An operator will be called *physical* if it has at least two creation operators and at least two destruction operators ($N \geq 2$ and $M \geq 2$). It is easy to see that the energy function (9) of a regular physical operator may be zero at some values of momenta of created and destroyed particles, so that particles can be destroyed and created without changing the total energy. Therefore the solutions of equation (10) exist, and the energy shell of a physical operator is not empty. For a regular physical operator Y^{ph}

$$\underline{Y^{ph}} = Y^{ph} \circ \frac{-i}{E_{Y^{ph}} - i\epsilon}, \tag{12}$$

$$\underbrace{Y^{ph}} = Y^{ph} \circ 2\pi\delta(E_{Y^{ph}}). \tag{13}$$

The notation (small circle) introduced in (12) means that the coefficient function of $\underline{Y^{ph}}$ differs from the coefficient function of Y^{ph} by a factor $-i(E_{Y^{ph}} - i\epsilon)^{-1}$. Hence for a regular physical Y^{ph}, integral $\underline{Y^{ph}}$ is regular and physical, and integral $\underbrace{Y^{ph}}$ is non-zero only on the energy shell.

By definition, a *renorm* operator either has one creation operator and one destruction operator of the same kind ($a^\dagger a$ or $c^\dagger c$) or is equal to a numerical constant. H_0, $\mathbf{P}$, and Q are examples of renorm operators. The energy function of a regular renorm operator is identically zero. Such operators Y^{ren} are time-independent, therefore the corresponding integrals $\underline{Y^{ren}}$ and $\underbrace{Y^{ren}}$ are ill-defined and non-regular. The theory must be formulated in such a way that these renorm integrals are not present (see section 6.1).

All operators that are not renorm or physical will be called *unphysical*. Examples of unphysical operators are two terms in the interaction operator (11). In our model theory, the energy function $E_{Y^{unp}}$ of an unphysical operator Y^{unp} is nonzero everywhere (for example, the energy function of the first term in (10) is $\omega_{\mathbf{p}} - \omega_{\mathbf{p}+\mathbf{k}} + |\mathbf{k}|$ which turns to zero only at $\mathbf{k} = 0$, but this point is excluded from the integral). So, the energy shell of an unphysical operator is empty, and unphysical operators describe processes in which the energy is not conserved. For a regular unphysical operator Y^{unp}, the equations (12) and (13) simplify (the term $-i\epsilon$ can be omitted because the denominator $E_{Y^{unp}}$ never vanishes).

$$\underline{Y^{unp}} = Y^{unp} \circ \frac{-i}{E_{Y^{unp}}},$$

$$\underbrace{\underline{Y^{unp}}} = 0. \tag{14}$$

Operator $\underline{Y^{unp}}$ is regular and unphysical.

5. RULES OF THE DIAGRAM TECHNIQUE

Calculations of commutators and products of operators in (4) and (6) are greatly facilitated by the diagram technique presented below.

Drawing a diagram. In Fig. 1 we draw two diagrams corresponding to two terms in the interaction operator (11). Particle operators are represented by oriented lines (full arcs for electrons and broken arrows for photons) either coming into the diagram (destruction operators) or going out of the diagram (creation operators). Each line is marked with the momentum carried by the corresponding particle operator. The order of these external lines (from top to bottom of the diagram) corresponds to the order of particle operators in the algebraical expression (from left to right). Relative positions of electron and photon external lines are not important, since corresponding operators commute. For clarity, electron arcs always start and end on the vertical "time line", and free ends of photon external lines are on the right hand side of the diagram. All three particle lines meet at the interaction vertex which corresponds to the factor $e(2\pi)^{-3/2}$ in the expression (11). An additional numerical factor is indicated in the upper left corner of the diagram.

Time integration. It follows from (12) that the time integral $\underline{Y}$ differs from Y only by the factor $-i(E_Y - i\epsilon)^{-1}$ which is represented in the diagram by drawing a box that crosses all external lines. A line entering (leaving) the box contributes its energy with the negative (positive) sign to the denominator. For example, the diagram representation of the integral $\underline{V_1}$ is shown in Fig. 2.

Normal product of two diagrams. The product of two diagrams AB is obtained by simply placing diagram B below diagram A on the same time line. For example, the product of terms in Fig. 1 (b) and 2 (a) is shown in Fig. 3 (a). This product should be further converted to the normal form, i.e., all creation operators should be moved to the left from all destruction operators (outgoing lines should be moved upwards until all of them appear above the incoming lines). Due to the anticommutation relations (1) and (2) each exchange of positions of electron particle operators (full external lines on the diagram) changes the total sign of the expression. Each exchange of positions of destruction and creation operators (incoming and outgoing lines) of similar particles creates an additional expression and a new diagram in which the switched lines are joined together.

Following these rules, in Fig. 3 (b) we change the order of electron lines $a^\dagger_{\mathbf{p}} a_{\mathbf{p}-\mathbf{k}} a^\dagger_{\mathbf{p}'} a_{\mathbf{p}'+\mathbf{k}'} \to a^\dagger_{\mathbf{p}'} a^\dagger_{\mathbf{p}} a_{\mathbf{p}-\mathbf{k}} a_{\mathbf{p}'+\mathbf{k}'}$. In Fig. 3 (c) we merge incoming electron line carrying momentum $\mathbf{p}-\mathbf{k}$ with outgoing electron line carrying momentum $\mathbf{p}'$. This results in delta function $\delta(\mathbf{p}'-\mathbf{p}+\mathbf{k})$ that is integrated by $\mathbf{p}'$. In the diagram notation this produces an internal electron line carrying momentum $\mathbf{p}-\mathbf{k}$ between vertices. In Fig. 3(b) and (c) the electron lines are in the normal order, however, the photon lines are not. Normal ordering of photon operators in 3 (b) yields diagrams 3 (d) and (e) according to equation $c_{\mathbf{k}} c^\dagger_{\mathbf{k}'} = c^\dagger_{\mathbf{k}'} c_{\mathbf{k}} + \delta(\mathbf{k}-\mathbf{k}')$. Diagrams 3 (f) and (g) are obtained from 3 (c) in a similar way.

Using the diagram notation we can perform all operations required in (4) and (6), such as time integrals, normal products, etc. During these operations we do not need to keep momentum labels of lines in the intermediate diagrams. The momentum labels can be assigned in the final result based on the topology of the diagram, and the algebraic expression of the result can be easily restored, as described below.

Reading the diagram. 1) Assign a distinct momentum label to each external line. 2) Assign momentum labels to internal lines so that momentum conservation law is satisfied at each vertex. If there are loops, one needs to introduce new independent "loop momenta" [see diagram 3 (g) in which $\mathbf{h}$ is the loop momentum]. 3) Read external lines from top to bottom of the diagram and write corresponding particle operators from left to right. 4) Write a factor $(2k)^{-1/2}$ for each external photon line carrying momentum $\mathbf{k}$ and a factor $(2k)^{-1}$ for each internal photon line carrying momentum $\mathbf{k}$. 5) For each box write a factor $-i(E_f - E_i - i\epsilon)^{-1}$, where E_f is a sum of energies of particles going out of the box and E_i is a sum of energies of particles coming into the box. 6) Write a factor $e^{iE_Y t}$, where E_Y is given by (9). 7) Write a delta function whose argument is a sum of momenta going out of the diagram minus sum of momenta coming into the diagram. 8) Multiply the obtained expression by the factor $(2\pi)^{-3j/2}e^j$, where j is the total number of vertices, and integrate by all external and loop momenta. 9) One external momentum integral can be performed explicitly to cancel the delta function as described in section 3. According to these rules, the diagram 3 (g) is

$$(V_1\underline{V_1})^{ren} = -\frac{ie^2}{16\pi^3}\int d\mathbf{p}d\mathbf{h}\frac{a^\dagger_{\mathbf{p}}a_{\mathbf{p}}}{(\omega_{\mathbf{p}-\mathbf{h}} - \omega_{\mathbf{p}} + h)h}. \tag{15}$$

6. RENORMALIZATION OF THE S-OPERATOR

If we naively used the above technique and calculated scattering operators in Eqs. (4) and (6), we would immediately encounter familiar ultraviolet divergences. Consider for example diagram 3 (g) which provides a renorm contribution (15) to the second-order term $-V_1\underline{V_1}$ in (4). The integral by $\mathbf{h}$ in (15) is divergent because the integrand behaves as h^{-2} at large values of $\mathbf{h}$. The divergence is due to the slow asymptotic decay of the integrand which is determined by two factors in the denominator. The factor $(\omega_{\mathbf{p}-\mathbf{h}} - \omega_{\mathbf{p}} + h)$ results from the perturbation theory, while the origin of the factor h can be traced back to the factor $k^{-1/2}$ in the interaction operator (11).

This observation suggests a simple way how one may try to formulate a finite theory in which all integrals are convergent. This can be done by modifying the coefficient functions in (11) to make their decay at large values of momenta more rapid (e.g., multiply them by e^{-k^2}). However, such a modification would not be helpful, even if we ignore the comparison with experiment. Indeed, renorm time-independent terms like (15) cannot be tolerated in Σ and F even if they were finite, because they would yield an infinite result when time integration $\underbrace{F}$ or $\underbrace{\Sigma}$ is performed to obtain scattering operators. The correct way to eliminate infinities in the scattering operators is by the traditional renormalization procedure [10] in which the Hamiltonian is modified by adding infinite Hermitian counterterms $R_2, U_3, R_4, \ldots$

$$H \to H^c = H_0 + V^c = H_0 + V_1 + R_2 + U_3 + R_4 + \ldots,$$

so that finite scattering operators Σ^c, S^c, F^c, and G^c are obtained from H^c and, in particular,

$$(\Sigma^c)^{ren} = (F^c)^{ren} = 0.$$

In the rest of this section we will consider this standard renormalization procedure in greater detail.

6.1. Mass renormalization

We obtain formulas for F_i^c $(i = 1, 2, 3, \ldots)$ by inserting interaction $V^c = V_1 + R_2 + U_3 + R_4 + \ldots$ in (6) instead of V_1 and collecting terms of equal order.

$$
\begin{aligned}
F_1^c = & \quad V_1, && (16)\\
F_2^c = & \quad \frac{i}{2}[\underline{V_1}, V_1] + R_2, &&\\
F_3^c = & - \frac{1}{6}[\underline{\underline{V_1}, [V_1, V_1}]] - \frac{1}{6}[[\underline{\underline{V_1}, V_1]}, V_1] + \frac{i}{2}[\underline{V_1}, R_2] + \frac{i}{2}[\underline{R_2}, V_1] + U_3, && (17)\\
F_4^c = & - \frac{i}{12}[\underline{\underline{\underline{V_1}, [[V_1, V_1]}, V_1}]] - \frac{i}{12}[[\underline{\underline{\underline{V_1}, [V_1, V_1]}], V_1}] - \frac{i}{12}[[\underline{\underline{\underline{V_1}, V_1]}, [V_1, V_1}]] &&\\
& - \frac{1}{6}[\underline{\underline{R_2}, [V_1, V_1}]] - \frac{1}{6}[\underline{\underline{V_1}, [R_2, V_1}]] - \frac{1}{6}[\underline{\underline{V_1}, [V_1, R_2}]] - \frac{1}{6}[[\underline{\underline{R_2}, V_1]}, V_1] &&\\
& - \frac{1}{6}[[\underline{\underline{V_1}, R_2]}, V_1] - \frac{1}{6}[[\underline{\underline{V_1}, V_1]}, R_2] + \frac{i}{2}[\underline{R_2}, R_2] + \frac{i}{2}[\underline{U_3}, V_1] &&\\
& + \frac{i}{2}[\underline{V_1}, U_3] + R_4. &&
\end{aligned}
$$

Our first goal is to choose counterterms R_2, U_3, and R_4, so that all renorm operators in F_i^c cancel. First-order term (16) is unphysical and does not contribute to $G^c = (-1/2)\underbrace{F^c}$ due to property (14). To ensure that F_2^c does not have a renorm part we choose the counterterm

$$R_2 = -\frac{i}{2}[\underline{V_1}, V_1]^{ren} = i(V_1\underline{V_1})^{ren}, \tag{18}$$

where the last equality follows from the time independence of renorm operators: $0 = \frac{d}{dt}(\underline{V_1}\ \underline{V_1})^{ren} = (\underline{V_1}V_1)^{ren} + (V_1\underline{V_1})^{ren}$. Infinite operator $(V_1\underline{V_1})^{ren}$ is shown in the diagram 3 (g). First four terms on the right hand side of (17) have odd number of particle operators, hence they are free of renorm parts (which have either zero or two particle operators). Therefore, to ensure that F_3^c does not contain renorm parts we must choose

$$U_3^{ren} = 0. \tag{19}$$

Finally, we can always choose a renorm counterterm R_4 which simply cancels all renorm terms which may be present in F_4^c. With the above choices, operators F and G do not contain renorm terms up to the 4th order.

Let us show that the same is true for Σ^c and S^c. Using V^c in place of V_1 in (4) and taking into account (18) we calculate Σ^c

$$\begin{aligned}
\Sigma_1^c &= - \ iV_1, \\
\Sigma_2^c &= - \ (V_1\underline{V_1})^{ph} - (V_1\underline{V_1})^{unp}, \\
\Sigma_3^c &= + \ iV_1(\underline{V_1\underline{V_1}})^{ph} + iV_1(\underline{V_1\underline{V_1}})^{unp} - R_2\underline{V_1} - iU_3, \\
\Sigma_4^c &= \quad \sigma_4^c - U_3\underline{V_1} - V_1\underline{U_3} - iR_4,
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
\sigma_4^c = \quad & V_1\underline{V_1(\underline{V_1\underline{V_1}})^{ph}} + V_1\underline{V_1(\underline{V_1\underline{V_1}})^{unp}} + iV_1\underline{R_2\underline{V_1}} + iR_2\underline{(V_1\underline{V_1})^{ph}} \\
+ \ & iR_2\underline{(V_1\underline{V_1})^{unp}}.
\end{aligned} \tag{21}$$

Σ_3^c does not have renorm parts because the third term has odd number of particle operators and $U_3^{ren} = 0$ (eq. (19)). To show that Σ_4^c also has this property we rewrite it using (3)

$$\begin{aligned}
\Sigma_4^c \ &= \ -iF_4^c - \frac{1}{2!}\frac{d}{dt}(\underline{F_2^c}\ \underline{F_2^c} + \underline{F_1^c}\ \underline{F_3^c} + \underline{F_3^c}\ \underline{F_1^c}) \\
&+ \ \frac{i}{3!}\frac{d}{dt}(\underline{F_1^c}\ \underline{F_1^c}\ \underline{F_2^c} + \underline{F_1^c}\ \underline{F_2^c}\ \underline{F_1^c} + \underline{F_2^c}\ \underline{F_1^c}\ \underline{F_1^c}) + \frac{1}{4!}\frac{d}{dt}(\underline{F_1^c})^4.
\end{aligned} \tag{22}$$

The first term on the right hand side is not renorm as shown above. The other terms are time derivatives of expressions which are regular and may, in principle, contain renorm terms. However, since latter terms are time-independent, the time derivatives on the right hand side of (22) cannot be renorm.

So, with our choice of R_2, R_4, and eq. (19), renorm operators are eliminated from the phase operator G^c and scattering operator S^c. This is equivalent to the removal of the electron self-energy diagrams (mass renormalization) in the traditional renormalization approach [10, 12].

Unphysical terms vanish in $\underbrace{F^c}$ and $\underbrace{\Sigma^c}$ due to (14), therefore operators G^c and $S^c - 1$ are purely physical

$$
\begin{aligned}
G^c &= -\frac{1}{2}(\underbrace{F_2^c} + \underbrace{F_3^c} + \underbrace{F_4^c} + \ldots)^{ph}, \\
S^c - 1 &= (\underbrace{\Sigma_2^c} + \underbrace{\Sigma_3^c} + \underbrace{\Sigma_4^c} + \ldots)^{ph}.
\end{aligned}
\tag{23}
$$

Simple calculations show that the scattering operator S^c contains terms corresponding to all possible scattering processes. For example, S_2^c has terms describing the elastic electron-electron scattering $a^\dagger a^\dagger aa$ (see Fig. 3 (e)) and the Compton scattering $a^\dagger ac^\dagger c$ (Fig. 3(c) and 3(f)). Diagram 3(d) describes two independent scattering processes each of which does not conserve energy: we are not interested in discussing such terms here. The third order scattering operator S_3^c has terms with 5 and 7 particle operators, e.g., those responsible for bremsstrahlung $a^\dagger a^\dagger c^\dagger aa$ and $a^\dagger a^\dagger aac$. The fourth-order operator S_4^c contains various terms with 4, 6, 8, and 10 particle operators. The terms with 4 particle operators $a^\dagger a^\dagger aa$ and $a^\dagger c^\dagger ac$ provide fourth-order *radiative corrections* to the second-order electron-electron and Compton scattering, respectively.

6.2. Electron-electron scattering and charge renormalization

Consider for example the elastic electron-electron scattering which is described by the terms in S^c having general form

$$
\begin{aligned}
S^c & \; (e^-e^- \to e^-e^-) = \\
(2\pi)^3 & \int d\mathbf{p} d\mathbf{q} d\mathbf{s} K(\mathbf{p},\mathbf{q},\mathbf{s}) \delta(\omega_{\mathbf{p}-\mathbf{s}} + \omega_{\mathbf{q}+\mathbf{s}} - \omega_{\mathbf{q}} - \omega_{\mathbf{p}}) a^\dagger_{\mathbf{p}-\mathbf{s}} a^\dagger_{\mathbf{q}+\mathbf{s}} a_{\mathbf{q}} a_{\mathbf{p}}.
\end{aligned}
\tag{24}
$$

The hypersurface of solutions of $\omega_{\mathbf{p}-\mathbf{s}} + \omega_{\mathbf{q}+\mathbf{s}} = \omega_{\mathbf{q}} + \omega_{\mathbf{p}}$ defines the energy shell for the electron-electron scattering, and only values of the coefficient function K in the vicinity of the energy shell are important for scattering. The second-order contribution to $S^c(e^-e^- \to e^-e^-)$ is given by the term in Fig. 3 (e) which yields

$$
K_2 = \frac{ie^2}{2(2\pi)^3(\omega_{\mathbf{p}-\mathbf{s}} - \omega_{\mathbf{p}} + s)s}. \tag{25}
$$

In the non-relativistic region ($q, s \ll m$) this reduces to $K_2 \approx ie^2/[2(2\pi)^3 s^2]$ which is characteristic for scattering of two indistinguishable electrons interacting via Coulomb potential

$$\frac{e^2}{4\pi|\mathbf{r}_1 - \mathbf{r}_2|}.$$

Let us now calculate the radiative corrections to the elastic electron-electron scattering due to the 4th order terms. We can neglect all terms that are zero on the energy shell, since they do not contribute to the scattering. When doing diagram calculations on the energy shell one can also use the following trick: If there is a box surrounding a subset of vertices in the diagram, then one can erase this box and draw another box surrounding the complement of this subset and simultaneously change the total sign of the diagram. This follows from the fact that the energy function of a diagram (which is equal zero on the energy shell) is a sum of vertex energies at all vertices.

Calculation of σ_4^c in (21) results in 9 diagrams shown in Fig. 4. The contribution of diagrams 4 (a) - (d) to the function K on the energy shell is

$$K_4^{(a-d)} = -\frac{ie^4}{4(2\pi)^6(\omega_{\mathbf{p}-\mathbf{s}} - \omega_{\mathbf{p}} + s)s}\int \frac{d\mathbf{h}}{h}(A^2 + BC + D^2 + EF),$$

where $A^{-1} = \omega_{\mathbf{q}+\mathbf{s}-\mathbf{h}} - \omega_{\mathbf{q}+\mathbf{s}} + h$, $B^{-1} = \omega_{\mathbf{p}-\mathbf{h}} - \omega_{\mathbf{p}} + h$, $C^{-1} = \omega_{\mathbf{p}-\mathbf{s}} - \omega_{\mathbf{p}-\mathbf{h}-\mathbf{s}} - h$, $D^{-1} = \omega_{\mathbf{p}-\mathbf{s}-\mathbf{h}} - \omega_{\mathbf{p}-\mathbf{s}} + h$, $E^{-1} = \omega_{\mathbf{q}-\mathbf{h}} - \omega_{\mathbf{q}} + h$, and $F^{-1} = \omega_{\mathbf{q}+\mathbf{s}} - \omega_{\mathbf{q}+\mathbf{s}-\mathbf{h}} - h$. This contribution is infinite (at large values of $\mathbf{h}$ the integrand behaves as h^{-3}) and has a singularity s^{-2} at $s \to 0$. Thus we conclude that the mass renormalization procedure described in Section 6.1 has not removed all divergences. The remaining infinities are removed by the *charge renormalization* procedure explained below.

The presence of the $s \to 0$ singularity would not be acceptable in K_4 even if this function was finite. Indeed, such a singularity is responsible for electron-electron scattering at large distances. From classical physics we know that these processes depend on e^2, i.e., they are accurately described by the 2nd order term (25). Therefore, the 4th order contribution to the scattering should be short-range in the position space and thus non-singular at $s \to 0$. The cancellation of this singularity can be achieved by a proper selection of the counterterm U_3. Note that so far we have fixed only the renorm part of U_3 in eq. (19), and we are free to choose its physical and unphysical parts. We may choose the (infinite) unphysical counterterm U_3 as shown in Fig. 5. Then unwanted diagrams 4 (a) - (d) cancel with expression $-U_3\underline{V_1} - V_1\underline{U_3}$ on the energy shell in eq. (20). Therefore, the role of the charge renormalization counterterm U_3 is to effectively modify the electron charge and to ensure that electron-electron interaction agrees with the Coulomb law at low energies and large distances.

Remaining diagrams 4 (e)-(i) are the fourth-order radiative corrections to the electron-electron scattering. On the energy shell they yield

$$K_4^{(e-i)} = \frac{ie^4}{4(2\pi)^6}\int \frac{d\mathbf{h}}{h|\mathbf{h}+\mathbf{s}|}A(BC+DC+EF+DG+EG),$$

where $A^{-1} = \omega_{\mathbf{q}-\mathbf{h}} - \omega_{\mathbf{q}} + h$, $B^{-1} = \omega_{\mathbf{p}-\mathbf{s}-\mathbf{h}} - \omega_{\mathbf{p}} + |\mathbf{h}+\mathbf{s}|$, $C^{-1} = \omega_{\mathbf{q}-\mathbf{h}} + \omega_{\mathbf{p}-\mathbf{s}} + |\mathbf{h}+\mathbf{s}| - \omega_{\mathbf{q}} - \omega_{\mathbf{p}} - i\epsilon$, $D^{-1} = \omega_{\mathbf{p}+\mathbf{h}} + \omega_{\mathbf{q}-\mathbf{s}} - \omega_{\mathbf{q}} - \omega_{\mathbf{p}} - i\epsilon$, $E^{-1} = \omega_{\mathbf{q}+\mathbf{s}} - \omega_{\mathbf{q}} + |\mathbf{h}+\mathbf{s}| + h$, $F^{-1} = \omega_{\mathbf{q}+\mathbf{s}} + \omega_{\mathbf{p}-\mathbf{h}-\mathbf{s}} + h - \omega_{\mathbf{q}} - \omega_{\mathbf{p}} - i\epsilon$, and $G^{-1} = \omega_{\mathbf{q}+\mathbf{s}} + \omega_{\mathbf{p}+\mathbf{h}} + |\mathbf{h}+\mathbf{s}| - \omega_{\mathbf{q}} - \omega_{\mathbf{p}} - i\epsilon$. $K_4^{(e-i)}$ is not singular at $\mathbf{s} = 0$ as expected. The integrand is proportional to h^{-5} and the integral is convergent at large values of the loop momentum $\mathbf{h}$. Therefore $\Sigma_4^c(a^\dagger a^\dagger aa)$ is finite on the energy shell. The same is true for $F_4^c(a^\dagger a^\dagger aa)$. Incidentally, the same counterterms U_3 (Fig. 5) are sufficient to make radiative corrections to the Compton scattering $\Sigma_4^c(a^\dagger c^\dagger ac)$ and $F_4^c(a^\dagger c^\dagger ac)$ to be finite as well.

The process of elimination of infinities in S^c by introducing infinite mass renormalization R_i and charge renormalization U_i counterterms can be repeated in all higher orders of the perturbation theory. As any renormalizable theory, our model has a finite number of types of counterterms: all odd-order charge renormalization counterterms U_i have the same types ($a^\dagger c^\dagger a$ and $a^\dagger ac$) as V_1; all even-order mass renormalization counterterms are renorm $R_i \propto a^\dagger a$.

In realistic renormalizable quantum field theories, such as QED, the S-operator S^c obtained via above renormalization procedure agrees with experiment for all kinds of processes and in the whole range of particle energies. We therefore postulate that S^c and G^c (Eq. (23)) are exact scattering operators for our model theory of electrons and photons.

7. CONSTRUCTION OF THE CLOTHED PARTICLE HAMILTONIAN

Thus far we discussed the usual S-matrix renormalization approach and found that the Hamiltonian H^c of the theory is infinite due to counterterms, although the scattering operator S^c is finite and very accurate. This apparent paradox can be resolved if one can find a finite Hamiltonian $H^r = H_0 + V^r$ that is scattering equivalent to H^c. In addition to being finite we demand that V^r destroys vacuum and one-particle states: $V^r(|0> \oplus \mathcal{H}_e \oplus \mathcal{H}_p) = 0$. This reflects a physical requirement that there must be no (self-) interactions when less than two particles are present [3]. This also means that V^r is physical in our terminology. The operator H^r will be referred to as the "clothed particle" Hamiltonian [3].

More specifically, we will try to find a finite physical interaction operator

$$V^r = V_2^r + V_3^r + V_4^r + \ldots$$

which being substituted in (6) in place of V_1 results in the same phase shift operator as G^c.

$$G^c = G^r = -\frac{1}{2}(\underbrace{V_2^r}_{} + \underbrace{V_3^r}_{} + \underbrace{V_4^r}_{} + \frac{i}{2}\underbrace{[\underline{V_2^r}, V_2^r]}_{} + \ldots). \qquad (26)$$

To determine V_2^r, V_3^r, and V_4^r we compare (23) and (26) order by order

$$\begin{aligned} \underbrace{V_2^r}_{} &= \underbrace{(F_2^c)^{ph}}_{}, \\ \underbrace{V_3^r}_{} &= \underbrace{(F_3^c)^{ph}}_{}, \\ \underbrace{V_4^r}_{} &= \underbrace{(F_4^c)^{ph}}_{} - \frac{i}{2}\underbrace{[\underline{V_2^r}, V_2^r]}_{}. \end{aligned}$$

These equations define operators V_i^r only on the energy shell where they are physical and Hermitian, because F_i^c are Hermitian. The most general expressions satisfying these properties are

$$\begin{aligned} V_2^r &= (F_2^c)^{ph} \circ \xi_2 + \Sigma_2, \\ V_3^r &= (F_3^c)^{ph} \circ \xi_3 + \Sigma_3, \\ V_4^r &= \{(F_4^c)^{ph} - \frac{i}{2}[\underline{V_2^r}, V_2^r]\} \circ \xi_4 + \Sigma_4, \end{aligned} \qquad (27)$$

where ξ_2, ξ_3, and ξ_4 are arbitrary real functions which depend on the same arguments as coefficient functions of F_2^c, F_3^c, and F_4^c, respectively, and are equal unity ($\xi_i = 1$) on the energy shell. Σ_i are any physical Hermitian operators which are equal zero on the energy shell and connected (i.e., represented by a connected diagram). The latter property is important to ensure that the S-matrix has the property of cluster separability [10].

We are free to choose any behavior of Σ_i and ξ_i outside the energy shell. In order to ensure that coefficient functions of V_i^r are finite in the entire range of arguments, we will set $\Sigma_i = 0$ and choose ξ_i to tend to zero rapidly as their arguments move away from the energy shell. As an example we can choose $\xi_i = e^{-\alpha E_i^2}$ with $\alpha > 0$ for all orders i. Let us prove that if these conditions are satisfied for orders $i = 2, 3, 4$, then the interaction operator V^r is finite up to the 4th order. We saw that $(F_2^c)^{ph}$ and $(F_3^c)^{ph}$ are finite on the energy shell. Therefore, with our choice of ξ_2 and ξ_3, operators V_2^r and V_3^r are finite everywhere. $(F_4^c)^{ph}$ is also finite on the energy shell, and the same is true for the commutator $[\underline{V_2^r}, V_2^r]$, because coefficient functions of all terms in V_2^r fall off rapidly at large values of momenta, so that involved loop integrals are finite [2]. Therefore the term in braces on the right hand side of (27) is finite on the energy shell. With our choice of ξ_4, the coefficient functions of the 4th order interaction operator V_4^r are finite for all values of arguments.

Thus, we have shown that H^r can be made finite up to the 4th order by a proper choice of ξ_i and Σ_i. Since quantities Σ_i and ξ_i are rather arbitrary, there exists a class of physical Hamiltonians $\{H^r\}$ all of which are finite and yield correct scattering G^c. The above construction of the clothed particle Hamiltonian completely solves the problem of ultraviolet infinities in QFT (we do not discuss here the problem of infrared infinities, assuming that they can be treated by well-known techniques [10, 13]): Any Hamiltonian from the class $\{H^r\}$ can be used to calculate the S-matrix and related quantities without encountering divergent integrals. This result, however, does not provide an answer to the question: which member of the class $\{H^r\}$ is the unique *correct* Hamiltonian H^r capable of describing the time evolution of observables? Note that scattering experiments would not help us in establishing H^r: the freedom of choice of ξ_i, and Σ_i is not reflected in the S-matrix. All these choices are scattering-equivalent. Therefore, to find the correct H^r one needs the dynamical information, i.e., highly non-trivial measurements of the time evolution in the region of interaction.

However, this does not mean that we cannot say anything about the exact Hamiltonian H^r of the theory. There are certain features of H^r which do not depend on the freedom of choosing Σ_i and ξ_i. For example, the choice of ξ_2 and Σ_2 does not affect the behavior of the 2nd order electron-electron interaction near the energy shell, because in this region $\xi_2 \approx 1$ and $\Sigma_2 \approx 0$ and the electron-electron interaction part in V_2^r has the form

$$V_2^r\ (e^-e^- \to e^-e^-) = -\int d\mathbf{p}d\mathbf{q}d\mathbf{s}\frac{e^2}{4(2\pi)^3 s}(\frac{1}{\omega_{\mathbf{p}-\mathbf{s}} - \omega_{\mathbf{p}} + s} - \frac{1}{\omega_{\mathbf{q}+\mathbf{s}} - \omega_{\mathbf{q}} - s})$$
$$e^{i(\omega_{\mathbf{p}-\mathbf{s}}+\omega_{\mathbf{q}+\mathbf{s}}-\omega_{\mathbf{q}}-\omega_{\mathbf{p}})t}a^\dagger_{\mathbf{p}-\mathbf{s}}a^\dagger_{\mathbf{q}+\mathbf{s}}a_{\mathbf{q}}a_{\mathbf{p}}. \tag{28}$$

In the non-relativistic limit, the coefficient function in (28) reduces to $-e^2/[2(2\pi)^3 s^2]$ which is a momentum space representation of the Coulomb potential between identical electrons. There are no $a^\dagger a^\dagger aa$ terms in V_3^r and it is easy to show that such terms in V_4^r are not singular at $\mathbf{s} \to 0$. Thus there is no fourth-order contribution to the long-range behavior of the potential, as expected. Therefore, higher order corrections cannot change the Coulomb form of the potential acting between electrons at low energies and long distances.

The short-range behavior of the potential is, of course, different from $e^2/(4\pi r)$. First, there are relativistic corrections which in the 2nd order are described by Eq. (28). Second, there are radiative corrections due to higher order (4th, 6th, ...) terms in $V^r(e^-e^- \to e^-e^-)$. In the exact QED theory, such corrections (in the electron-proton potential) are responsible, for instance, for the Lamb shifts of atomic energies. These higher order corrections remain undetermined in the clothed particle approach due to the uncertainty of Σ_i and ξ_i. Apparently, the shifts of atomic energies resulting from these radiative corrections are not sensitive to the choice of Σ_i and ξ_i (because these energies are reflected in the S-matrix as positions of poles on the complex energy plane). However, their contributions to the wave functions remain unspecified from purely theoretical considerations.

8. DISCUSSION

Results of our studies in this paper and in Ref. [2] are summarized in Table I where three approaches to QFT ("naive" QFT without renormalization, QFT with a renormalized S-operator, and presented here approach with a clothed particle Hamiltonian) are compared by the way they describe the Hamiltonian and scattering operator. In each approach, the S-operator is derived from the corresponding Hamiltonian as shown by horizontal arrows. In calculations of the naive S-operator (first row in Table I) one obtains integrals divergent at large values of integration momenta as described in section 6.

The elimination of infinities in the S-operator is achieved by the usual renormalization procedure (second row of Table I): the naive Hamiltonian H is changed to H^c (arrow I) by adding infinite counterterms in the 2nd and higher orders of the perturbation theory. It appears that H^c yields correct scattering S^c (arrow II) due to cancellation of infinities. However H^c is infinite and the corresponding time evolution operator $e^{-iH^c t}$ is ill-defined. So, this solution is not satisfactory.

The clothed particle approach (third row in Table I) establishes existence of a finite Hamiltonian H^r that is scattering equivalent to H^c and yields correct time evolution. Finding H^r is not an easy task. Our approach was to fit H^r to the operator S^c (arrow III) requiring in addition its finiteness and "physical" form of the interaction V^r. Such fitting does not define H^r unambiguously. More experimental information is needed for full determination of functions ξ_i and operators Σ_i. Such experiments are bound to be rather challenging because they must go beyond usual information contained in the S-matrix and should be capable of measuring relativistic and radiative corrections to the form of wave functions and to the time evolution of observables.

However, even without specifying the exact form of ξ_i and Σ_i, it is clear that ultraviolet infinities are not present in the theory with a clothed particle Hamiltonian H^r: the correct S-operator $S^r = S^c$ can be calculated from the finite H^r by a straightforward application of quantum mechanical rules without renormalization corrections, and the time dependence of observables can be evaluated by a direct application of the time evolution operator $\exp(-iH^r t)$. In this formulation, QFT becomes conceptually not very different from ordinary quantum mechanics. The only significant difference is that QFT interactions may not conserve the number of particles. For example, wave functions and energies of bound states can be found by diagonalization of the low-energy approximation (the terms which do not conserve the number of particles of each type are neglected) to the Hamiltonian H^r [2].

As pointed above, the Hamiltonians H and H^c are not correct time evolution generators. However they still play critical roles in the formulation of realistic quantum field theories of the Standard Model. The importance of H is that it can be obtained by using the canonical quantization procedure and the gauge principle. Then the clothed particle Hamiltonian can be found by following steps I $\to$ II $\to$ III indicated in Table 1. In any realistic QFT, it seems impossible to arrive to the rather complex form of H^r without these intermediate steps.

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TABLE 1.

Comparison between the clothed particle approach and standard QFT. Infinite operators are shown in roman typeface. Correct finite operators are shown in bold typeface.

Hamiltonian		S-operator	Theory
$H = H_0 + V_1$	$\longrightarrow$	$\mathrm{S} = 1 - i \underbrace{V_1} + \ldots$	naive QFT
$\downarrow$ I	II		
$\mathrm{H}^{\mathrm{c}} = H_0 + V^c$	$\longrightarrow$	$\mathbf{S}^{\mathbf{c}} = 1 - i \underbrace{V^c} + \ldots$	renormalized QFT
	III $\swarrow$	$\Vert$	
$\mathbf{H}^{\mathbf{r}} = H_0 + V^r$	$\longrightarrow$	$\mathbf{S}^{\mathbf{r}} = 1 - i \underbrace{V^r} + \ldots$	clothed particle approach

FIGURE CAPTIONS

Fig.1. Interaction operator V_1.

Fig.2. Integral $\underline{V_1}$.

Fig.3. Normal product of operators in Fig. 1(b) and 2(a).

Fig.4. Operator σ_4^c on the energy shell.

Fig.5. Charge renormalization counterterms U_3.