Some Stokes flow problems in coating: beyond lubrication theory

George M. Homsy* and Ali Mazouchi**
*University of California, Santa Barbara, California, USA
**Stanford University, Stanford, California, USA

Abstract

We study the contact line dynamics and free surface profile of a viscous gravity driven flow over both flat planes and topographies of wedges and steps. The boundary integral method is used to solve the full Stokes equations. For a moving contact line, we obtain fully developed steady solutions with an overhang and a prediction of the apparent contact angle as functions of Ca and contact angle ϕ. Flow along downward facing surfaces was found to lead to drips. Finally, Stokes flows over sharp steps were found to agree with lubrication theory for small Ca, and deviate only slightly from the topography at high Ca.

1. Introduction

Coating flows have a wide application in integrated circuit fabrication, manufacturing of mechanical and optical storage devices and photographic films. In fast or 'high speed' coating, a thin layer of viscous liquid spreads on a substrate due to body forces. In such flows, which typically have high capillary numbers, viscous stresses can become comparable to capillary stresses. Lubrication theory fails for high capillary number, and the full Stokes equations must be used. We develop a boundary integral approach and solve several time-dependent flow problems involving combinations of contact lines and topography.

2. Problem definition and numerical method

We study two dimensional motion of a viscous fluid over topography in the presence of an external body force as shown in Figure 1. The substrate is a flat plane, a wedge, or a step which makes an angle α with the body force. The liquid contacts the wall with a fixed contact angle ϕ and matches to a parallel flow far from the wedge. Length, velocity and pressure are scaled with the parallel flow thickness b, the mean inflow velocity $\bar{u} = \rho g b^2 \sin(\alpha)/\mu$ and σ / b, respectively.

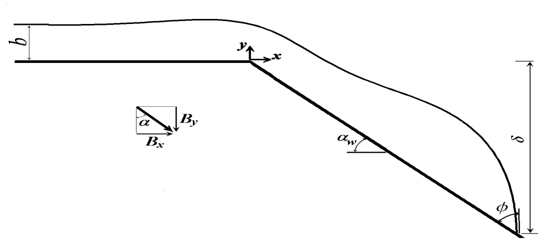

Figure 1: Schematic of flow over topography

We consider the problem to be governed by the full Stokes equations subject to the usual boundary conditions of no slip at solid boundaries, appropriate inflow and outflow conditions, and a slip condition at the contact line, if present. At the free surface, the normal and tangential stress balances apply, together with the kinematic condition. Our problems are governed by one dynamic dimensionless group, the capillary number, $Ca = \mu\bar{u}/\sigma$ and a number of geometric parameters. In the case of the dynamic contact line problems we use numerical slip on the order of the grid spacing.

Our approach to the numerical solution of these free and moving boundary problems relies on the use of boundary integral techniques [1-3]. As is well-known, because of the linearity of the Stokes equations, the field equations can be transformed to integral equations for the flow variables (stream function and vorticity in this case) on the boundary. Collocation of these equations at discrete points, (none of which are placed at either geometric singularities or on the contact line), results in a set of linear algebraic equations.

In the case of the moving contact line problems, these equations are solved at a given time step subject to all the dynamic boundary conditions of the problem. This results in knowledge of the velocities of all material points, which are then advected over a small time step using Euler's method. In the case of steady flow over topography, the flow equations are solved subject to the tangential stress condition and the kinematic condition. Iteration of the boundary location is accomplished by using the normal stress condition as a two point boundary value problem for the interfacial position. For further details, see Kelmanson [1], Pozrikidis [2], Goodwin and Homsy [3] and Mazouchi [4].

3. Results: dynamic contact lines

3.1 Flat substrates

Figure 2 shows a typical time evolution of a profile ($Ca = 0.1$, $\phi = 90^{\circ}$) in the fixed Eulerian frame. Initially the flat profile moves in the opposite direction of the body force, since the surface tension force pulls the contact line upward against the body force in order to develop a ridge by collecting the liquid from downstream. Then the film moves down due to gravity and the ridge evolves into its steady fully developed shape, which has an overhang.

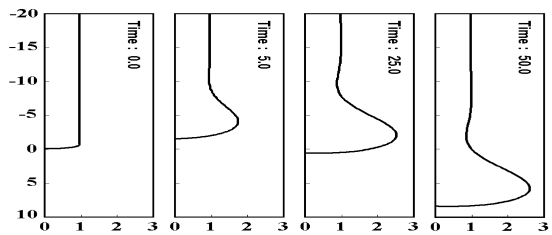

Figure 2: Time evolution of a moving contact line

Figure 3 illustrates the map of our simulations in the (Ca, ϕ) plane. The filled circles show the cases for which the profile has a fully developed steady state solution without an overhang in front, while the unfilled circles show the existence of stable solution with an overhang. By increasing Ca, the overhang limit moves back towards smaller contact angles due to viscous bending of the interface at finite Ca. The map also has regions where we could not capture any fully developed steady solutions.

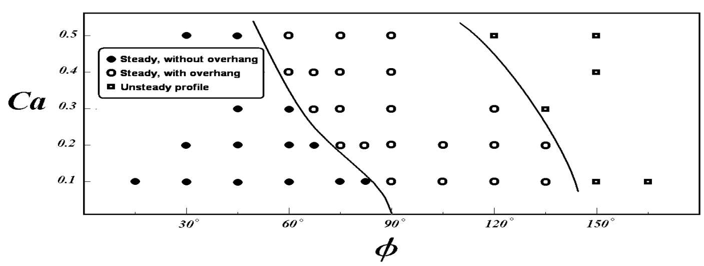

Figure 3: Map of solutions for flow over plane surfaces

Figure 4 shows a magnified region around the contact point. It can be seen that the slope of each profiles remains constant over a reasonable distance and then quickly takes on the prescribed value of the contact angle at the contact point. The apparent contact angle ϕ_d can be defined from the slope in this region. Figure 5 shows the behavior of ϕ_d versus Ca and ϕ.

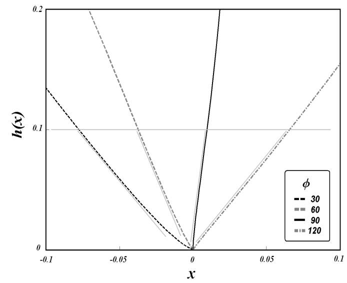

Figure 4: Apparent contact angle

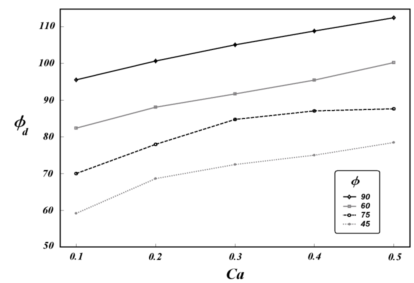

Figure 5: Apparent contact angle vs. Ca

3.2 Flow Over a Wedge

Figure 6 shows typical results for a flow over a right angle wedge. Due to gravity the liquid moves down and slowly turns the corner after which it is forced sideways by the ridge pressure (which is the only driving force in horizontal direction). Approximately at $t = 140$, when the contact point reaches the "run out" length $\delta_{\max} = 12.1$, the negative pressure formed at the contact point forces it to move back gradually and finally the liquid bulge grows and drips. Further results are given in Mazouchi and Homsy [5,6].

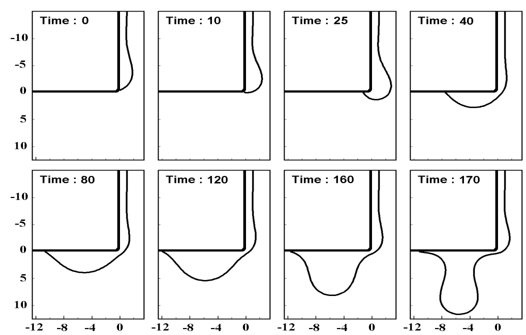

Figure 6: Time evolution of flow over a wedge for $\phi = 60^\circ$, $Ca = 0.1$.

4. Results: flow over downward facing steps

Our main motivation to study flow over topography is to scrutinize the free surface response to etch pits typified by a downward facing step. The profiles for a step with depth $D = 0.5$ at different Ca are presented in Figure 7. For larger Ca the profile closely follows the topography while for smaller Ca, a capillary ridge appears on the profile which moves upstream and becomes broader as Ca decreases. Lubrication theory predicts self-similarity of the profiles when $x / Ca^{1/3}$ is used as the horizontal length scale [7]. In Figures 8, we compare our scaled Stokes flow results with lubrication theory. Surprisingly, lubrication theory seems to be valid for small capillary numbers, $Ca < 10^{-2}$, in spite of the fact that the substrate is not a small sloped topography. As discussed in [8] the Stokes flow solutions show *a posteriori* that even for sharp topographies, the interface slope is small when Ca is small, a result that could not have been anticipated from lubrication theory alone.

Figure 9 illustrates how the ridge height increases and asymptotically reaches the one predicted by lubrication theory, valid as the capillary number goes to zero. Also of interest is the sharp decrease in $h_{\max} - 1$ with increasing Ca. Our results suggest that this quantity goes to zero exponentially with increasing Ca, corresponding to a solution in which the free surface conforms closely to the topography.

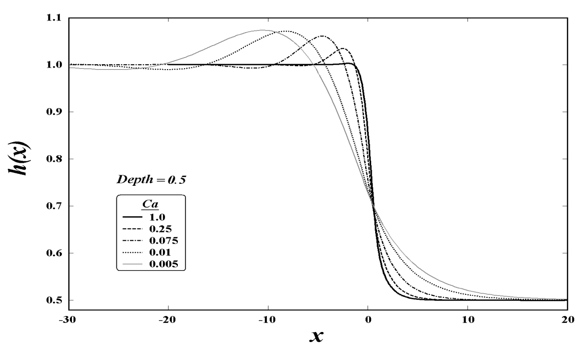

Figure 7: Steady flow over a step

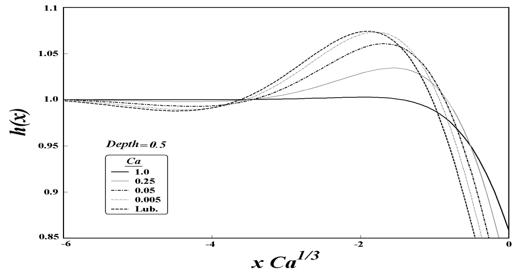

Figure 8: Scaled free surface profiles

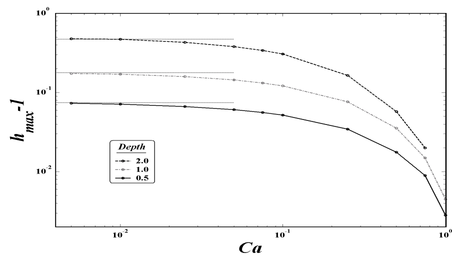

Figure 9: Size of the ridge vs. Ca

5. References

1. Kelmanson, A. Boundary integral equation solution of viscous flows with free surfaces, *J. Eng. Math.*, **17**, 329 (1983)
2. Goodwin, R. and Homsy, G. M. Viscous flow down a slope in the vicinity of a contact line, *Phys. Fluids A* , **3**, 515 (1991)
3. Pozrikidis C. *Boundary integral and singularity methods for linearized viscous flow*. Cambridge University Press (1992)
4. Mazouchi A. *Low Reynolds number interfacial flows*, Ph.D. Thesis, Stanford University (2001)
5. Mazouchi, A. and Homsy, G. M. Time-dependent free surface Stokes flows with a moving contact line, I. Flow over plane surfaces, in preparation.
6. Mazouchi, A. and Homsy, G. M. Time-dependent free surface Stokes flows with a moving contact line, II. Flow over a wedge, in preparation.
7. Kalliadasis, C.; Bielarz, C. and Homsy, G. M. Steady free surface thin film flows over topography, *Phys. Fluids*, **12**, 1898 (2000)
8. Mazouchi, A. and Homsy, G. M. Free surface Stokes flow over topography, to appear in *Phys. Fluids* (2001)