

Sect 9.4 - Mean, Median, and Mode

Objective a - c: Understanding Mean, Median, and Mode.

Let's begin by defining the mean and median.

The **mean** is just another name for the average of a set of numbers. To find the mean, we simply add the numbers together and then divide by the number of numbers.

The **median** is the "middle number" of a set of numbers. To find the median of a set of numbers, first list the numbers from smallest to largest. If there is an odd number of numbers in the list, the median is the middle number. If there is an even number of numbers in the list, the median is the average of the two middle numbers.

Ex. 1 Find the mean and the median of 3.4, 9.7, 4.2, 1.6, and 1.8.

Solution:

First, list the numbers from smallest to largest: 1.6, 1.8, 3.4, 4.2, 9.7

To find the mean, find the sum of the numbers and then divide by 5:

Mean = $(1.6 + 1.8 + 3.4 + 4.2 + 9.7) \div 5 = 20.7 \div 5 = 4.14$, so the

Mean = 4.14.

Since there are five numbers and five is an odd number, then the median is the middle number which is 3.4, so the **Median = 3.4.**

Ex. 2 Find the mean and the median of 9.3, 8.2, 7.4, 9.3, 9.3, and 6.1.

Solution:

First, list the numbers from smallest to largest: 6.1, 7.4, 8.2, 9.3, 9.3, 9.3

To find the mean, find the sum of the numbers and then divide by 6:

Mean = $(6.1 + 7.4 + 8.2 + 9.3 + 9.3 + 9.3) \div 6$

$= 49.6 \div 6 = 8.266666\dots$, so the **Mean = 8.26 $\bar{6}$.**

Since there are six numbers and six is an even number, then the median is the average of the two middle numbers:

Median = $(8.2 + 9.3) \div 2 = 17.5 \div 2 = 8.75$, so the **Median = 8.75.**

Now, let's define the mode.

Mode* - the mode is the number that occurs most often in a set of numbers. If no number appears more often than any other, then there is no mode.

Ex. 3 Find the mode of 9.3, 8.2, 7.4, 9.3, 9.3, and 6.1.

Solution:

Since 9.3 appears three times whereas all the other numbers appear once, then the mode is 9.3. **Mode = 9.3.**

Ex. 4 Find the mode of $\frac{1}{3}$, $\frac{7}{8}$, $\frac{5}{24}$, $\frac{1}{6}$, and $\frac{3}{4}$.

Solution:

Since every number occurs only one, there is no mode. **Mode = None.**

*In certain applications (biology for example), a set of data can have more than one mode. If there are two modes, the data is said to be "bimodal." Here's an example:

Ex. 5 Find the mode of 9, 11, 4, 3, 4, 11, 8, 1, 11, 5, and 4.

Solution:

Since both 4 and 11 occur three times and all the other numbers occur once, the modes are 4 and 11. **Mode = 4 and 11.**

Ex. 6 Find the mean, median and mode of 6.3, 1.75, 4.26, 9, 0.72, and 1.1.

Solution:

First write in order from smallest to largest: 0.72, 1.1, 1.75, 4.26, 6.3, 9.

Mean = $(0.72 + 1.1 + 1.75 + 4.26 + 6.3 + 9) \div 6 = 23.13 \div 6 = 3.855$.

Median = $(1.75 + 4.26) \div 2 = 6.01 \div 2 = 3.005$.

Mode = None.

Thus, the Mean = 3.855, the Median = 3.005, and the Mode = None.

Ex. 7 Find the mean, median, and mode of $\frac{3}{7}$, $\frac{2}{21}$, $\frac{5}{6}$, $\frac{5}{21}$, $\frac{3}{7}$, $\frac{3}{14}$, and $\frac{13}{14}$.

Solution:

The LCD = 42, so $\frac{3}{7} = \frac{18}{42}$, $\frac{2}{21} = \frac{4}{42}$, $\frac{5}{6} = \frac{35}{42}$, $\frac{5}{21} = \frac{10}{42}$, $\frac{3}{7} = \frac{18}{42}$,

$\frac{3}{14} = \frac{9}{42}$, and $\frac{13}{14} = \frac{39}{42}$. Listing them from smallest to largest, we get:

$\frac{4}{42}$, $\frac{9}{42}$, $\frac{10}{42}$, $\frac{18}{42}$, $\frac{18}{42}$, $\frac{35}{42}$, $\frac{39}{42}$

$$\begin{aligned}\text{Mean} &= \left(\frac{4}{42} + \frac{9}{42} + \frac{10}{42} + \frac{18}{42} + \frac{18}{42} + \frac{35}{42} + \frac{39}{42} \right) \div 7 = \frac{133}{42} \div \frac{7}{1} \\ &= \frac{7 \cancel{1}9}{7 \cancel{1}6} \cdot \frac{1}{7} = \frac{19}{6} \cdot \frac{1}{7} = \frac{19}{42}.\end{aligned}$$

$$\text{Median} = \frac{18}{42} = \frac{3}{7} \text{ since it is the middle number.}$$

$$\text{Mode} = \frac{18}{42} = \frac{3}{7} \text{ since it occurs most often.}$$

$$\text{Thus, the Mean} = \frac{19}{42}, \text{ the Median} = \frac{3}{7}, \text{ and the Mode} = \frac{3}{7}.$$

Ex. 8 Find the mean, median, and mode of 0.25, $\frac{3}{8}$, $\frac{1}{4}$ and $\frac{5}{16}$.

Solution:

Since $\frac{1}{4} = 0.25$, $\frac{3}{8} = 0.375$, and $\frac{5}{16} = 0.3125$ are nice, terminating

decimals, we can convert the fractions into decimal.

From smallest to largest: 0.25, 0.25, 0.3125, 0.375.

$$\text{Mean} = (0.25 + 0.25 + 0.3125 + 0.375) \div 4 = 1.1875 \div 4 = 0.296875.$$

$$\text{Median} = (0.25 + 0.3125) \div 2 = 0.5625 \div 2 = 0.28125.$$

$$\text{Mode} = 0.25.$$

Thus, the Mean = 0.296875, the Median = 0.28125, and the Mode = 0.25.

Ex. 9 Find the mean, median, and mode of $\frac{6}{13}$, 0.4, 1.8, and $\frac{3}{26}$.

Solution:

Since $\frac{6}{13} = 0.4615384 \dots$ and $\frac{3}{26} = 0.1153846 \dots$ are non-terminating

decimals, we will need to convert the decimals into fractions:

$0.4 = \frac{4}{10} = \frac{2}{5}$ and $1.8 = 1\frac{8}{10} = 1\frac{4}{5} = \frac{9}{5}$. Thus, the numbers are

$\frac{6}{13}$, $\frac{2}{5}$, $\frac{9}{5}$, and $\frac{3}{26}$.

The LCD is 130. Thus, $\frac{6}{13} = \frac{60}{130}$, $\frac{2}{5} = \frac{52}{130}$, $\frac{9}{5} = \frac{234}{130}$, and $\frac{3}{26} = \frac{15}{130}$.

From smallest to largest: $\frac{15}{130}$, $\frac{52}{130}$, $\frac{60}{130}$, $\frac{234}{130}$

$$\text{Mean} = \left(\frac{15}{130} + \frac{52}{130} + \frac{60}{130} + \frac{234}{130} \right) \div 4 = \frac{361}{130} \div \frac{4}{1} = \frac{361}{130} \cdot \frac{1}{4} = \frac{361}{520}.$$

$$\text{Median} = \left(\frac{52}{130} + \frac{60}{130} \right) \div 2 = \left(\frac{112}{130} \right) \div \frac{2}{1} = \frac{56}{65} \cdot \frac{1}{2} = \frac{28}{65}.$$

Mode = None.

Objective d: Weighted Mean.

If some of the data has a stronger bearing on the average than the other data, then the weight of that data must be used. Typically, one takes the data and multiply it by its weight. Add the results and then divide by the total of the weights.

Ex. 10 For the fall semester, Pearl received the following grades:

Number of Credit Hours	Grade
4	A
2	B
3	C
3	B
1	A

Calculate Pearl's GPA.

Solution:

At most universities, an A is worth 4 points, a B is worth 3 points, a C is worth 2 points, a D is worth 1 point, and an F is worth 0 points. We begin by multiplying the number of credits hours of each course by the grade points received for that course and add the results. Then, we divide the answer by the total number of credits:

Number of Credit Hours	Grade	Total Points
4	A = 4	$4 \cdot 4 = 16$
2	B = 3	$2 \cdot 3 = 6$
3	C = 2	$3 \cdot 2 = 6$
3	B = 3	$3 \cdot 3 = 9$
1	A = 4	$1 \cdot 4 = 4$
Total: 13 credits		Total: 41 points

$$\text{GPA} = 41 \text{ points} \div 13 \text{ credits} = 3.153846 \dots \approx 3.15.$$