Sect 8.2 – Triangles and the Pythagorean Theorem

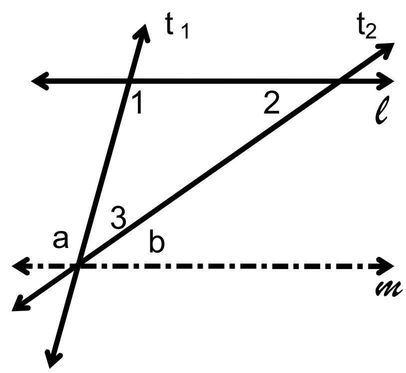
Objective a: Categorizing Triangles.

We first want to show an important property of the angles of a triangle. Consider the following example:

Ex. 1 In the triangle below, show that the sum of the measures of the angles is equal to 180˚.

Solution:

Draw m so that it is parallel to ℓ and passes through the vertex of the angle of the triangle not on ℓ. Notice that ∠ a, ∠ b, & ∠ 3 together form a line. This means that m∠ a + m∠ b + m∠ 3 = 180˚. Since $\ell \parallel m$ and t_1 is a transversal, then ∠ 1 and ∠ a are a pair of alternate interior angles. Hence, m∠ 1 = m∠ a and we can replace m∠ a by m∠ 1 in the formula above to get:



m∠ 1 + m∠ b + m∠ 3 = 180˚. But, t_2 is also a transversal, so ∠ 2 and ∠ b are a pair of alternate interior angles. Hence, m∠ 2 = m∠ b and we can replace m∠ b by m∠ 2 to get:

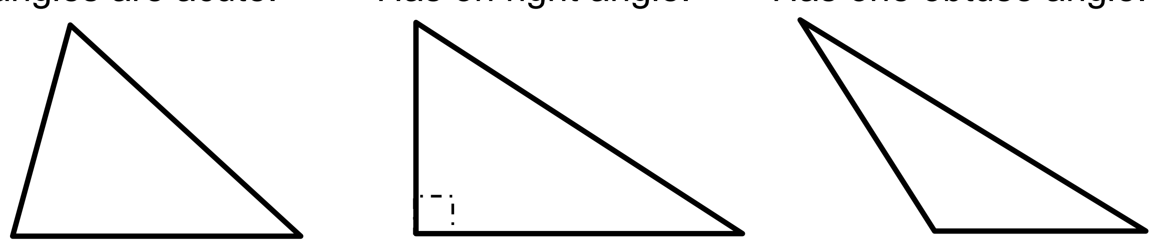
m∠ 1 + m∠ 2 + m∠ 3 = 180˚.

Thus, the sum of the measures of the angles is 180˚.

This property holds for any triangle.

We can classify triangles by their angles:

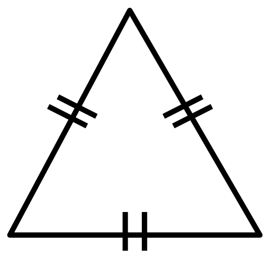
Acute Triangle	**Right Triangle**	**Obtuse Triangle**
All angles are acute.	Has on right angle.	Has one obtuse angle.



In a right triangle, the two sides that intersect to form the right angle are called the **legs** of the right triangle while the third side (the longest side) of a right triangle is called the **hypotenuse** of the right triangle.

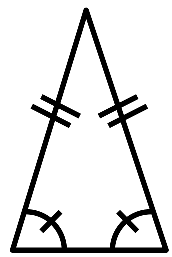
We can also classify triangles by their sides:

Equilateral Triangle
All sides are equal.



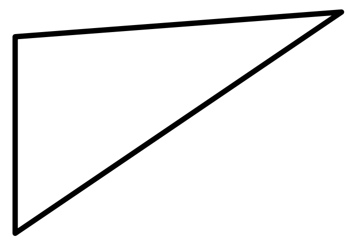
All angles measure 60˚.

Isosceles Triangle
Two sides are equal.



The two base angles are equal.

Scalene Triangle
No sides are equal.

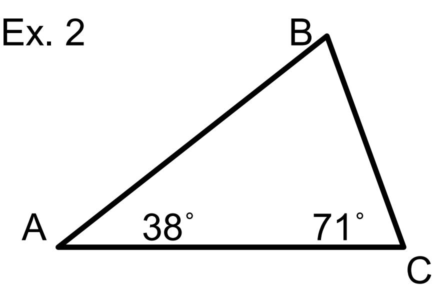


No angles are equal.

An equilateral triangle is also called an **equiangular triangle**. The third angle in an isosceles triangle formed by the two equal sides is called the **vertex angle**.

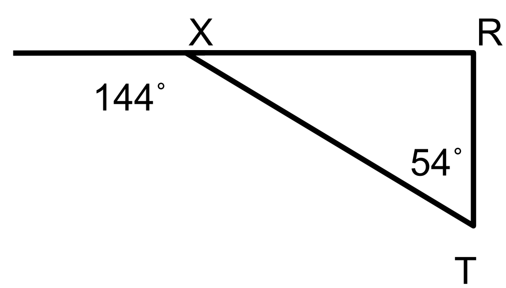
Determine what type of triangle is picture below:

Ex. 2



Solution:
Since the measures of the angles of a triangle total 180˚, then
$38^\circ + m\angle B + 71^\circ = 180^\circ$
$m\angle B + 109^\circ = 180^\circ$
$\underline{-109^\circ = -109^\circ}$
$m\angle B = 71^\circ$.
So, ΔABC is an Isosceles and an Acute Triangle.

Ex. 3



Solution:
Since ∠TXR and 144˚ are supplementary angles, then
$m\angle TXR = 180^\circ - 144^\circ = 36^\circ$.
But, $m\angle TXR + m\angle T = 54^\circ + 36^\circ = 90^\circ$. So, $\angle R = 180^\circ - 90^\circ = 90^\circ$.
Since the angles are different, then none of the sides are equal. Thus, ΔXRT is a scalene and a right triangle.

Objective b: Square Roots

Recall that the square root of a number a asks what number times itself is equal to a. For example, the square root of 25 is 5 since 5 times 5 is 25.

The **square root** of a number a, denoted $\sqrt{a}$, is a number whose square is a. So, $\sqrt{25}$ = 5. If the number under the square root sign is not a perfect square, we will need to use our calculator.

Simplify the following:

Ex. 4a $\sqrt{49}$ Ex. 4b $\sqrt{144}$

Ex. 4c $\sqrt{0}$ Ex. 4d $\sqrt{15}$

Solution:

a) $\sqrt{49} = 7$ b) $\sqrt{144} = 12$

c) $\sqrt{0} = 0$ d) $\sqrt{15} \approx 3.873$

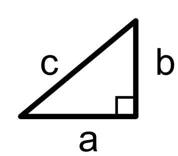
Objective c: The Pythagorean Theorem.

In a right triangle, there is a special relationship between the legs (a and b) and the hypotenuse (c). This is known as the Pythagorean Theorem:

Pythagorean Theorem

In a right triangle, the square of the hypotenuse (c^2) is equal to the sum of the squares of the legs ($a^2 + b^2$)

$c^2 = a^2 + b^2$



Keep in mind that the hypotenuse is the longest side of the right triangle.

Determine is the following triangle is a right triangle:

Ex. 5 A triangle with sides of 7 ft, 24 ft, and 25 ft.

Solution:

Plug the values in the Pythagorean Theorem and see if you get a true statement. Since c is the longest side, then c = 25:

$c^2 = a^2 + b^2$

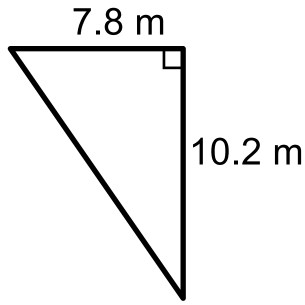
$(25)^2 = (7)^2 + (24)^2$

$625 = 49 + 576$

625 = 625, yes the triangle is a right triangle.

Find the length of the missing sides (to the nearest hundredth):

Ex. 6



Solution:

In this problem, we have the two legs of the triangle and we are looking for the hypotenuse:

$c^2 = a^2 + b^2$

$c^2 = (7.8)^2 + (10.2)^2$

$c^2 = 60.84 + 104.04$

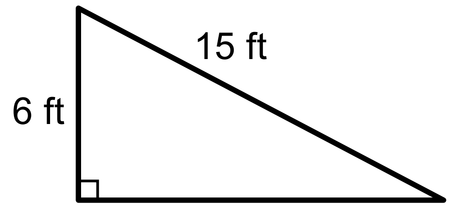
$c^2 = 164.88$

To find c, take the square root* of 164.88:

$c = \pm\sqrt{164.88} = \pm 12.8405...$

$c \approx 12.84$ m

Ex. 7



Solution:

In this problem, we have one leg and the hypotenuse and we are looking for the other leg:

$c^2 = a^2 + b^2$

$(15)^2 = (6)^2 + b^2$

$225 = 36 + b^2$ (solve for b^2)

$\underline{-36 = -36}$

$189 = b^2$

To find b, take the square root of 189:

$b = \pm\sqrt{189} = \pm 13.7477...$

$b \approx 13.75$ ft

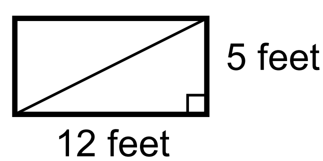
* - The equation $c^2 = 164.88$ actually has two solutions ≈ 12.84 and ≈ – 12.84, but the lengths of triangles are positive so we ignore the negative solution.

Find the length of the diagonal of the following rectangle:

Ex. 8 A rectangle that is 12 feet by 5 feet.

Solution:

First, draw a picture. Since the angles of a rectangle are right angles, then the two sides are the legs of a right triangle while the diagonal is the hypotenuse of the right triangle. Using the Pythagorean Theorem, we get:



$c^2 = a^2 + b^2$

$c^2 = (12)^2 + (5)^2$

$c^2 = 144 + 25 = 169$

$c = \pm\sqrt{169} = \pm 13$. So, the diagonal is 13 feet.