

Sect 3.5 - Order Relations and the Order of Operations

Objective a: Applying the order of operations to fractions.

Recall the order of operations:

Order of Operations

- 1) Parentheses - Do operations inside of Parentheses (), [], { }, | |
- 2) Exponents including square roots.
- 3) Multiplication or Division as they appear from left to right.
- 4) Addition or Subtraction as they appear from left to right.

Simplify the following:

Ex. 1 $7\frac{1}{3} \div 3\frac{1}{7} \cdot 21 - 16\frac{2}{3}$

Solution:

Since there are no parentheses or exponents, we start with step #3, multiply or divide as they appear from left to right:

$$7\frac{1}{3} \div 3\frac{1}{7} \cdot 21 - 16\frac{2}{3} \quad (\text{change } 7\frac{1}{3} \text{ \& } 3\frac{1}{7} \text{ to improper fractions})$$

$$= \frac{22}{3} \div \frac{22}{7} \cdot 21 - 16\frac{2}{3} \quad (\text{invert and change operation to } \bullet)$$

$$= \frac{22}{3} \cdot \frac{7}{22} \cdot 21 - 16\frac{2}{3} = \frac{1 \cdot 22}{3} \cdot \frac{7}{22 \cdot 1} \cdot 21 - 16\frac{2}{3}$$

$$= \frac{1}{3} \cdot \frac{7}{1} \cdot 21 - 16\frac{2}{3} \quad (\#3\text{-multiplication})$$

$$= \frac{7}{3} \cdot 21 - 16\frac{2}{3} \quad (\text{write 21 over 1})$$

$$= \frac{7}{3} \cdot \frac{21}{1} - 16\frac{2}{3} = \frac{7}{1 \cdot 3} \cdot \frac{3 \cdot 7}{1} - 16\frac{2}{3} \quad (\text{reduce})$$

$$= \frac{7}{1} \cdot \frac{7}{1} - 16\frac{2}{3} \quad (\#3\text{-multiplication})$$

$$= 49 - 16\frac{2}{3} \quad (\text{borrow one from 49})$$

$$= 48 \frac{1}{1} - 16\frac{2}{3} = 48\frac{1}{1} - 16\frac{2}{3} \quad (\text{L.C.D.} = 3)$$

$$= 48 \frac{1 \cdot 3}{1 \cdot 3} - 16\frac{2}{3} = 48\frac{3}{3} - 16\frac{2}{3} \quad (\#4\text{-subtraction})$$

$$= 32\frac{1}{3}.$$

$$\text{Ex. 2} \quad \left(1\frac{3}{4}\right)^2 \div 2\frac{4}{5} - \left(\frac{4}{\sqrt{9}} - 2\frac{3}{8} \cdot \frac{2^2}{19}\right)$$

Solution:

Let's first change all the mixed numbers into improper fractions:

$$\begin{aligned} & \left(1\frac{3}{4}\right)^2 \div 2\frac{4}{5} - \left(\frac{4}{\sqrt{9}} - 2\frac{3}{8} \cdot \frac{2^2}{19}\right) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{\sqrt{9}} - \frac{19}{8} \cdot \frac{2^2}{19}\right) \quad (\#1\text{-parentheses, } \#2\text{-exponents}) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{19}{8} \cdot \frac{4}{19}\right) \quad (\text{reduce inside the parentheses}) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{1 \cdot 19}{2 \cdot 4} \cdot \frac{1 \cdot 4}{1 \cdot 19}\right) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{1}{2} \cdot \frac{1}{1}\right) \quad (\#1\text{-parentheses, } \#3\text{-multiplication}) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4}{3} - \frac{1}{2}\right) \quad (\text{L.C.D.} = 6) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{4 \cdot 2}{3 \cdot 2} - \frac{1 \cdot 3}{2 \cdot 3}\right) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \left(\frac{8}{6} - \frac{3}{6}\right) \quad (\#1\text{-parentheses, } \#4\text{-subtraction}) \\ &= \left(\frac{7}{4}\right)^2 \div \frac{14}{5} - \frac{5}{6} \quad (\#2\text{-exponents}) \\ &= \frac{49}{16} \div \frac{14}{5} - \frac{5}{6} \quad (\text{invert and change operation to } \bullet) \\ &= \frac{49}{16} \cdot \frac{5}{14} - \frac{5}{6} \quad (\text{reduce}) \\ &= \frac{7 \cdot 7}{16} \cdot \frac{5}{7 \cdot 2} - \frac{5}{6} \\ &= \frac{7}{16} \cdot \frac{5}{2} - \frac{5}{6} \quad (\#3\text{-multiplication}) \\ &= \frac{35}{32} - \frac{5}{6} \quad (\text{L.C.D.} = 96) \\ &= \frac{35 \cdot 3}{32 \cdot 3} - \frac{5 \cdot 16}{6 \cdot 16} = \frac{105}{96} - \frac{80}{96} \quad (\#4\text{-subtraction}) \\ &= \frac{25}{96} \end{aligned}$$

Ex. 3 $\left(5\frac{1}{4} \cdot \frac{1}{6} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11}$

Solution:

Change $5\frac{1}{4}$ into an improper fraction:

$$\begin{aligned} & \left(5\frac{1}{4} \cdot \frac{1}{6} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} \\ &= \left(\frac{21}{4} \cdot \frac{1}{6} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} && \text{(reduce)} \\ &= \left(\frac{3 \cdot 7}{4} \cdot \frac{1}{3 \cdot 2} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} \\ &= \left(\frac{7}{4} \cdot \frac{1}{2} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} && \text{(#1-parentheses, #3-multiplication)} \\ &= \left(\frac{7}{8} - \frac{1}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} && \text{(#1-parentheses, #4-subtraction)} \\ &= \left(\frac{6}{8}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} = \left(\frac{2 \cdot 3}{2 \cdot 4}\right)^2 \div \frac{11}{24} - \frac{3}{11} && \text{(reduce)} \\ &= \left(\frac{3}{4}\right)^2 \div \frac{\sqrt{121}}{24} - \frac{3}{11} && \text{(#2-exponents)} \\ &= \frac{9}{16} \div \frac{11}{24} - \frac{3}{11} && \text{(invert and change operation to } \bullet \text{)} \\ &= \frac{9}{16} \bullet \frac{24}{11} - \frac{3}{11} = \frac{9}{2 \cdot 8} \bullet \frac{3 \cdot 8}{11} - \frac{3}{11} && \text{(reduce)} \\ &= \frac{9}{2} \bullet \frac{3}{11} - \frac{3}{11} && \text{(#3-multiplication)} \\ &= \frac{27}{22} - \frac{3}{11} && \text{(L.C.D. = 22)} \\ &= \frac{27}{22} - \frac{3 \cdot 2}{11 \cdot 2} = \frac{27}{22} - \frac{6}{22} && \text{(#4-subtraction)} \\ &= \frac{21}{22} \end{aligned}$$

Objective b: Applications.

Solve the following:

Ex. 4 If twenty-four pounds of candy is distributed in $\frac{3}{8}$ -pound bags, how many bags will be filled?

Solution:

Distributed means divide up so we need to take the total candy divide by the amount of candy in each bag;

$$24 \div \frac{3}{8} = \frac{24}{1} \cdot \frac{8}{3} = \frac{3 \cdot 8}{1} \cdot \frac{8}{3 \cdot 1} = \frac{8}{1} \cdot \frac{8}{1} = 64$$

Sixty-four bags can be filled.

- Ex. 5 A developer has 250 acres of land in the hill country that she is planning to develop into a new subdivision called Saddle Rock. If $30\frac{1}{12}$ acres of land will be reserved for roads and utilities, how many houses can she build in the subdivision if she wants the average lot size to be $2\frac{5}{12}$ acres?

Solution:

First, subtract $30\frac{1}{12}$ from 250 to find the number of acres that will be

$$\begin{array}{r} \text{used for houses: } 250 = 249 \ 1 = 249 \ \frac{1}{1} = 249 \frac{1 \cdot 12}{1 \cdot 12} = 249 \frac{12}{12} \\ - 30 \frac{1}{12} = - 30 \frac{1}{12} = - 30 \frac{1}{12} = - 30 \frac{1}{12} = - 30 \frac{1}{12} \\ \hline \end{array}$$

$$219 \frac{11}{12} \text{ Acres}$$

Now, take the total acres used for houses and divide it by $2\frac{5}{12}$:

$$\begin{aligned} 219 \frac{11}{12} \div 2 \frac{5}{12} &= \frac{2639}{12} \div \frac{29}{12} = \frac{2639}{12} \cdot \frac{12}{29} = \frac{29 \cdot 91}{1 \cdot 12} \cdot \frac{1 \cdot 12}{1 \cdot 29} \\ &= \frac{91}{1} \cdot \frac{1}{1} = 91 \text{ houses.} \end{aligned}$$

- Ex. 6 A countertop is made of $\frac{11}{16}$ -in particleboard and is covered with $\frac{1}{4}$ -in laminated plastic. If the length of the edge is 132 in, how many square inches of the metal edging are needed?

Solution:

To find the width of the metal edging, we need to add the thickness of the particleboard and the plastic:

$$\frac{11}{16} + \frac{1}{4} = \frac{11}{16} + \frac{4}{16} = \frac{15}{16} \text{ in}$$

To find the area, we multiply the length times the width:

$$\frac{15}{16} \cdot 132 = \frac{15}{16} \cdot \frac{132}{1} = \frac{15}{4 \cdot 4} \cdot \frac{4 \cdot 33}{1} = \frac{15}{4} \cdot \frac{33}{1} = \frac{495}{4} \text{ or } 123 \frac{3}{4}$$

The metal edging will be $123\frac{3}{4}$ in².