Sect 2.1 - Addition, Subtraction, Multiplication, and Division Properties of Equality

Concept #1 Definition of a Linear Equation in One Variable

An **equation** is a statement that two quantities are equal. An equation can be as simple as 4 quarters = $1, or it could be more complex like $3x = 2.7$ and $2x^2 – 5x + 4 = 23$. A **solution** to an equation is the value of x that makes the equation true. For example, $x = 0.9$ is a solution to the equation $3x = 2.7$ since if we replace x by 0.9 and do the multiplication, we get: $3(0.9) = 2.7$. Some examples of equations are:

Ex. 1a $3x + 11 = 8$ Ex. 1b $9y – 6z + 2 = 0$

Ex. 1c $\frac{4}{13}w = 52$ Ex. 1d $8x^3 – 27 = 37$

Determine whether the given number is a solution of the equation:

Ex. 2 $7x – 5 = 4x + 10$; 5

Solution:
Replace x by 5 and work out the left and right sides:
$7x – 5 = 4x + 10$
$7(5) – 5 = 4(5) + 10$
$35 – 5 = 20 + 10$
$30 = 30$ True
So, 5 is a solution to $7x – 5 = 4x + 10$.

Ex. 3 $0.3y – 1.1 = 1.5y – 2.5$; – 2

Solution:
Replace y by – 2 and work out the left and right sides:
$0.3y – 1.1 = 1.5y – 2.5$
$0.3(– 2) – 1.1 = 1.5(– 2) – 2.5$
$– 0.6 – 1.1 = – 3 – 2.5$
$– 1.7 = – 5.5$ False
So, – 2 is not a solution to $0.3y – 1.1 = 1.5y – 2.5$.

Ex. 4 $– \frac{1}{6}t + \frac{2}{3} = \frac{7}{6}$; – 3

Solution:
Replace t by – 3 and work out the left and right sides:
$– \frac{1}{6}t + \frac{2}{3} = \frac{7}{6}$
$– \frac{1}{6}(– 3) + \frac{2}{3} = \frac{7}{6}$

$\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

$\frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

$\frac{7}{6} = \frac{7}{6}$ True; So, – 3 is a solution to $-\frac{1}{6}t + \frac{2}{3} = \frac{7}{6}$.

In this chapter, we will focus on a particular type of equation called a linear equation in one variable:

Definition of a Linear Equation in One Variable
Let a and b be real numbers such that a ≠ 0. A **Linear Equation in One Variable** is an equation that can written in the form:

$$ax + b = 0$$

These types of equations have only one variable and the power of the variable is one. We can also call these equations "first-degree equations."

Are the following linear equations in one variable?

Ex. 5a $3x + 11 = 8$

Ex. 5b $9y - 6z + 2 = 0$

Ex. 5c $\frac{4}{13}w = 52$

Ex. 5d $8x^3 - 27 = 37$

Solution:

a) There is only one variable x and the power of x is one, so this is a linear equation in one variable. **Yes**

b) Although the power of the variables are one, there are two variables y and z, so this is not a linear equation in one variable. **No** This equation is actually a linear equation in *two variables*.

c) There is only one variable w and the power of w is one, so this is a linear equation in one variable. **Yes**

d) Although there is only one variable x, the power of x is three and not one, so this is not a linear equation in one variable. **No**

Concept #2 The Addition and Subtraction Properties of Equality.

The goal in this section is to use some properties of equations to isolate the variable on one side of the equation. Let us start with a definition.

Equivalent Equations are equations that have exactly the same solutions. So, for example, 7x – 5 = 4x + 10 and x = 5 are equivalent equations since they have the same solution (x = 5). We need to develop some properties that will allow us to take with a more complicated equation and then write it as a series of simpler equivalent equations:

Solve the following:

Ex. 6 $x + 8 = 11$

Solution:
Since $3 + 8 = 11$, then $x = 3$
Or, $11 - 8 = 3$.

Ex. 7 $x - 3.2 = 4.8$

Solution:
Since $8 - 3.2 = 4.8$, then $x = 8$
Or, $4.8 + 3.2 = 8.0 = 8$.

Ex. 8 $x + \frac{8}{9} = \frac{7}{12}$

This problem we cannot find the answer just by looking at it. We will need to develop some properties to help us solve it. In example #6, the second way we solved the problem was by subtracting 8 from 11. On the left side, if we subtract 8 from x + 8, we get x. So, by subtracting 8 from both sides of x + 8 = 11, we get the equation x = 3. In example #7, the second way we solved the problem was by adding 3.2 to 4.8. On the left side. if we add 3.2 to x – 3.2, we get x. So, by adding 3.2 to both sides of x – 3.2 = 4.8, we get x = 8. This means we can add or subtract the same quantity from both sides of an equation without changing the answer. These are known as the Addition and Subtraction Properties of Equality.

Addition and Subtraction Properties of Equality:

If a, b, and c are algebraic expressions and if a = b, then

1. $a + c = b + c$ is equivalent to $a = b$. **Addition Property**

2. $a - c = b - c$ is equivalent to $a = b$. **Subtraction Property**

So, to solve $x + \frac{8}{9} = \frac{7}{12}$, we will subtract $\frac{8}{9}$ from both sides:

$$\begin{array}{r} x + \frac{8}{9} = \frac{7}{12} \\ -\frac{8}{9} = -\frac{8}{9} \\ \hline x = -\frac{11}{36} \end{array}$$

But $\frac{7}{12} - \frac{8}{9} = \frac{21}{36} - \frac{32}{36}$
$= -\frac{11}{36}$

Solve the following:

Ex. 9 $-9.2 + r = -3.1$

Solution:
Add 9.2 to both sides to get r by itself:

$$\begin{array}{r} -9.2 + r = -3.1 \\ +9.2 \quad = +9.2 \\ \hline r = 6.1 \end{array}$$

Ex. 10 $x + 9 = -25$

Solution:
Subtract 9 from both sides to get x by itself:

$$\begin{array}{r} x + 9 = -25 \\ -9 = -9 \\ \hline x \quad = -34 \end{array}$$

Check:

$-9.2 + r = -3.1$

$-9.2 + (6.1) = -3.1$

$-3.1 = -3.1$ True

So, r = 6.1

Check:

$x + 9 = -25$

$(-34) + 9 = -25$

$-25 = -25$ True

So, x = – 34

Ex. 11 $\frac{2}{3} = p - \frac{3}{4}$

Solution:

Add $\frac{3}{4}$ to both sides to get p by itself:

$$\frac{2}{3} = p - \frac{3}{4}$$

$$+\frac{3}{4} = \ +\frac{3}{4}$$

$$\frac{17}{12} = p$$

So, $p = \frac{17}{12}$

But, $\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$

Check: $\frac{2}{3} = \left(\frac{17}{12}\right) - \frac{3}{4}$

$$\frac{2}{3} = \left(\frac{17}{12}\right) - \frac{9}{12} = \frac{8}{12}$$

$$\frac{2}{3} = \frac{2}{3} \text{ True}$$

Ex. 12 $6x + 8 = 7x$

Solution:

We want to have the x terms on one side of the equation. Subtract 6x from both sides to get 8 by itself:

$6x + 8 = 7x$

$-6x \quad = -6x$

$8 = x$

Check: $6(8) + 8 = 7(8)$

$48 + 8 = 56$

$56 = 56$ True

So, x = 8.

Ex. 13 $-3x - 9.4 = -4x + 5.1$

Solution:

We want to get the x terms on one side of the equation and the constant terms on other. Add 4x to both sides & then add 9.4 to both sides:

$-3x - 9.4 = -4x + 5.1$

$+4x \quad = +4x$

$x - 9.4 = 5.1$

$+9.4 = +9.4$

$x = 14.5$

Check:

$-3(14.5) - 9.4 = -4(14.5) + 5.1$

$-43.5 - 9.4 = -58 + 5.1$

$-52.9 = -52.9$ True

So, x = 14.5.

Concept #3 Multiplication and Division Properties of Equality.

Multiplication and division properties of equality work in a similar fashion to the addition and subtraction properties of equality. Consider the following:

Solve the following:

Ex. 14 $3x = 27$

Solution:

Since $3 \cdot 9 = 27$, then $x = 9$.

Or, $27 \div 3 = 9$.

Ex. 15 $\frac{w}{5} = 1.1$

Solution:

Since $5.5 \div 5 = 1.1$, then $x = 5.5$. Or, $1.1 \cdot 5 = 5.5$.

In example #14, the second way we solved the problem was by dividing 27 by 3. On the left side, if we divide 3x by 3, we get x. So, by dividing both sides of $3x = 27$, we get the equation $x = 9$. In example #15, the second way we solved the problem was by multiplying 1.1 by 5. On the left side. if we multiply $\frac{w}{5}$ by 5, we get w. So, by multiplying both sides of $\frac{w}{5} = 1.1$ by 5, we get $x = 5.5$. This means we can multiply or divide both sides of an equation by any non-zero quantity without changing the answer. These are known as the Multiplication and Division Properties of Equality.

Multiplication and Division Properties of Equality:

If a, b, and c are algebraic expressions with $c \neq 0$, and if $a = b$, then

1. $ac = bc$ is equivalent to $a = b$. **Multiplication Property**

2. $\frac{a}{c} = \frac{b}{c}$ is equivalent to $a = b$. **Division Property**

There are several comments that need to be made about these properties. First, we cannot multiply both sides of an equation by zero. If we do, we will get an equation that has a different solution than the original equation and hence it will not be an equivalent equation. We also cannot divide both sides by zero since division by zero is undefined. Finally, since dividing by a number is the same as multiplying by its reciprocal, we may find it easier to multiply both sides by the reciprocal of a number.

Solve the following:

Ex. 16 $-1.875x = -0.465$

Solution:

Divide both sides by – 1.875 to solve for x:

$$\frac{-1.875x}{-1.875} = \frac{-0.465}{-1.875}$$

$x = 0.248$

Check: $-1.875(0.248) = -0.465$

$-0.465 = -0.465$ True

So, $x = 0.248$.

Ex. 17 $-17x = 45$

Solution:

Divide both sides by – 17 to solve for x:

$$\frac{-17x}{-17} = \frac{45}{-17}$$

$x = -\frac{45}{17}$

Check: $-17\left(-\frac{45}{17}\right) = 45$

$45 = 45$ True

So, $x = -\frac{45}{17}$.

Ex. 18 $\frac{x}{-11} = -4.532$

Solution:

Multiply both sides by – 11 to solve for x:

$-11\left(\frac{x}{-11}\right) = -11(-4.532)$

$x = 49.852$

Check: $\frac{49.852}{-11} = -4.532$

-4.532

So, $x = 49.852$.

Ex. 19 $\frac{4}{7}t = -\frac{14}{9}$

Solution:

Multiply both sides by the reciprocal of $\frac{4}{7}$ to solve for t:

$\frac{7}{4}\left(\frac{4}{7}t\right) = \frac{7}{4}\left(-\frac{14}{9}\right)$

$t = -\frac{49}{18}$

Check: $\frac{4}{7}\left(-\frac{49}{18}\right) = -\frac{14}{9}$

$\frac{4}{1}\left(-\frac{7}{18}\right) = -\frac{14}{9}$

$\frac{2}{1}\left(-\frac{7}{9}\right) = -\frac{14}{9}$

$-\frac{14}{9} = -\frac{14}{9}$ True

So, $t = -\frac{49}{18}$.

Ex. 20 $-x = -6.5$

Solution:

Divide by – 1:

$\frac{-x}{-1} = \frac{-6.5}{-1}$

$x = 6.5$

Ex. 21 $-3.2p = 0$

Solution:

Divide by – 3.2:

$\frac{-3.2p}{-3.2} = \frac{0}{-3.2}$

$p = 0$

Check: $-(6.5) = -6.5$
$-6.5 = -6.5$ True
So, $x = 6.5$.

Check: $-3.2(0) = 0$
$0 = 0$ True
So, $p = 0$.

<u>Solve, but do not check the answer:</u>

Ex. 22 $-7x = 0.462$

Solution:
Divide both sides by – 7:

$$\frac{-7x}{-7} = \frac{0.462}{-7}$$

$$x = -0.066$$

Ex. 23 $-7 + x = 0.462$

Solution:
Add 7 to both sides:

$$-7 + x = 0.462$$
$$+7 \quad = +7$$
$$x = 7.462$$

Ex. 24 $-\frac{x}{7} = 0.462$

Solution:
Multiply both sides by – 7:

$$-7\left(-\frac{x}{7}\right) = -7(0.462)$$

$$x = -3.234$$

Ex. 25 $-7 - x = 0.462$

Solution:
Add 7 to both sides:

$$-7 - x = 0.462$$
$$+7 \quad = +7$$
$$-x = 7.462$$

To solve for x, divide both sides by – 1

$$\frac{-x}{-1} = \frac{7.462}{-1}$$

$$x = -7.462$$

Ex. 26 $4y = 0$

Solution:
Divide both sides by 4:

$$\frac{4y}{4} = \frac{0}{4}$$

$$y = 0$$

Ex. 27 $y + 6.4 = 6.4$

Solution:
Subtract 6.4 from both sides:

$$y + 6.4 = 6.4$$
$$-6.4 = -6.4$$
$$y = 0$$

Concept #4 Translation

<u>Write the equation and solve:</u>

Ex. 28a The quotient of a number and five is seven.
Ex. 28b Negative eleven equals the total of eight and a number.
Ex. 28c The product of negative four and a number is 4.2.
Ex. 28d $\frac{2}{3}$ subtracted from a number is $-\frac{3}{5}$.

Solution:

a) The quotient of a number and five is seven:

Let n = the number

$\frac{n}{5} = 7$ (multiply both sides by 5)

$5\left(\frac{n}{5}\right) = 5(7)$

$n = 35$

b) Negative eleven equals the total of eight and a number:

Let n = the number

$-11 = 8 + n$

$-11 = 8 + n$ (subtract 8 from both sides)

$\underline{-8 \quad = -8}$

$-19 = n$

c) The product of negative four and a number is 4.2:

Let n = the number

$-4n = 4.2$

$\frac{-4n}{-4} = \frac{4.2}{-4}$ (divide both sides by – 4)

$n = -1.05$

d) $\frac{2}{3}$ subtracted from a number is $-\frac{3}{5}$:

Let n = the number

$n - \frac{2}{3} = -\frac{3}{5}$

$n - \frac{2}{3} = -\frac{3}{5}$ (add $\frac{2}{3}$ to both sides)

$\underline{\quad + \frac{2}{3} = + \frac{2}{3}}$

$n = -\frac{3}{5} + \frac{2}{3} = -\frac{9}{15} + \frac{10}{15} = \frac{1}{15}$

$n = \frac{1}{15}$