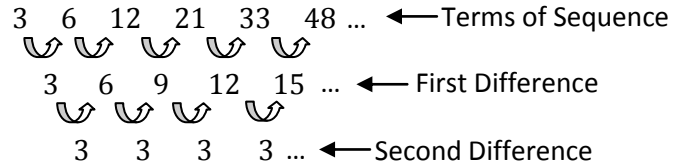


Problem: Find a formula or rule for the sequence
3, 6, 12, 21, 33, 48, ...



Constant difference was reached at 2nd difference \rightarrow Must be a Quadratic; let $f_n = an^2 + bn + c$
 (Note 1: If a constant was reached at the 1st difference, it's linear; 3rd difference, it's cubic; etc.)
 (Note 2: I'm using f_n instead of a_n since I'm using a in another way)

$$\begin{cases}
 a + b + c = 3 & \text{Since when } n = 1, f_n = 3 \\
 4a + 2b + c = 6 & \text{Since when } n = 2, f_n = 6 \\
 9a + 3b + c = 12 & \text{Since when } n = 3, f_n = 12
 \end{cases}$$

You can solve this system in many ways. I'm going to do this by the elimination method.

- 1) Multiply the 1st equation by -4 on both sides and add it to the 2nd equation.

$$\text{Result: } -2b - 3c = -6$$

Multiply the 1st equation by -9 on both sides and add it to the 3rd equation.

$$\text{Result: } -6b - 8c = -15$$

- 2) Take the two resulting equations from step (1) and solve for b and c .

Multiply $-2b - 3c = -6$ by -3 and add it to $-6b - 8c = -15$:

$$\text{Result: } c = 3$$

Plug $c = 3$ into $-2b - 3c = -6$ to find b .

$$\text{Result: } -2b - 9 = -6 \rightarrow b = -\frac{3}{2}$$

- 3) Use the results from step (2) to solve for a .

Use $4a + 2b + c = 6$:

$$4a + 2\left(-\frac{3}{2}\right) + (3) = 6 \rightarrow 4a - 3 + 3 = 6 \rightarrow a = \frac{3}{2}$$

This means that $a = \frac{3}{2}$, $b = -\frac{3}{2}$, and $c = 3$ in our formula:

$$f_n = \frac{3}{2}n^2 - \frac{3}{2}n + 3$$

Check it! Plug in $n = 1, 2, 3, 4, 5 \dots$ and see what terms you get!