Problem: Find a formula or rule for the sequence 3, 6, 12, 21, 33, 48, …

3 6 12 21 33 48 … ← Terms of Sequence

3 6 9 12 15 … ← First Difference

3 3 3 3 … ← Second Difference

Constant difference was reached at 2nd difference → Must be a Quadratic; let $f_n = an^2 + bn + c$

(Note 1: If a constant was reached at the 1st difference, it's linear; 3rd difference, it's cubic; etc.)

(Note 2: I'm using f_n instead of a_n since I'm using a in another way)

$$\begin{cases} a + b + c = 3 & \text{Since when } n = 1,\ f_n = 3 \\ 4a + 2b + c = 6 & \text{Since when } n = 2,\ f_n = 6 \\ 9a + 3b + c = 12 & \text{Since when } n = 3,\ f_n = 12 \end{cases}$$

You can solve this system in many ways. I'm going to do this by the elimination method.

1) Multiply the 1st equation by -4 on both sides and add it to the 2nd equation.

Result: $-2b - 3c = -6$

Multiply the 1st equation by -9 on both sides and add it to the 3rd equation.

Result: $-6b - 8c = -15$

2) Take the two resulting equations from step (1) and solve for b and c.

Multiply $-2b - 3c = -6$ by -3 and add it to $-6b - 8c = -15$:

Result: $c = 3$

Plug $c = 3$ into $-2b - 3c = -6$ to find b.

Result: $-2b - 9 = -6 \quad \rightarrow \quad b = -\frac{3}{2}$

3) Use the results from step (2) to solve for a.

Use $4a + 2b + c = 6$:

$$4a + 2\left(-\frac{3}{2}\right) + (3) = 6 \quad \rightarrow \quad 4a - 3 + 3 = 6 \quad \rightarrow \quad a = \frac{3}{2}$$

This means that $a = \frac{3}{2}$, $b = -\frac{3}{2}$, and $c = 3$ in our formula:

$$f_n = \frac{3}{2}n^2 - \frac{3}{2}n + 3$$

Check it! Plug in $n = 1, 2, 3, 4, 5$ … and see what terms you get!