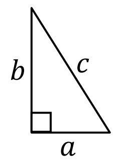
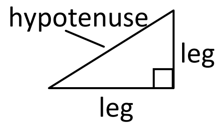
Right Triangles & Trigonometry

If you have a right triangle and…

- You know 2 sides lengths and want the third side length – use the Pythagorean Theorem
- You know 1 side length and 1 angle that is either 30°, 45°, or 60° and want to know a side – use the Special Right Triangle proportions
- You know 1 side length and 1 angle that is NOT 30°, 45°, nor 60° and want to know a side – use SOH-CAH-TOA (trigonometry)
- You know 1 angle and want to know the other acute angle – do 90 minus the measure of the angle you know

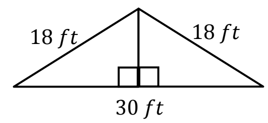


Pythagorean Theorem
$$\boldsymbol{a^2 + b^2 = c^2}$$



The hypotenuse is always the side opposite the right angle, and the length of the hypotenuse must always be substituted in for c. The other sides are the legs and are substituted in for a and b.
"Pythagorean Triples" are groups of 3 whole numbers that make $a^2 + b^2 = c^2$ true. The three most common "triples" are 3,4,5; 5,12,13; and 7,24,25. Any multiple of a triple is also a triple; for example, if you multiply "3,4,5" by 2, you get "6,8,10."

Example: Find the area of the figure below.



$$Triangle\ Area = \frac{1}{2}bh$$

The base of each triangle is 15 ft, but we need the Pythagorean Theorem to find the height:

$a^2 + b^2 = c^2$, where $a = 15$ and $c = 18$.

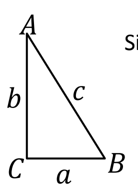
$15^2 + b^2 = 18^2 \rightarrow 225 + b^2 = 324 \rightarrow b^2 = 99$

So the height is $\sqrt{99} = \sqrt{9}\sqrt{11} = 3\sqrt{11}$.

Then the area of the triangle is $\frac{1}{2}bh = \frac{1}{2}(30)(3\sqrt{11})$
$\approx 149.248ft^2$

If $c^2 = a^2 + b^2$, then $m < C\ equals\ 90°$
If $c^2 < a^2 + b^2$, then $m < C\ is\ less\ than\ 90°$
If $c^2 > a^2 + b^2$, then $m < C\ is\ greater\ than\ 90°$

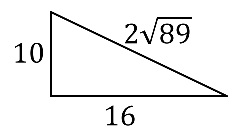
Example:
If the side lengths are 4, 5 and 7, classify the triangle.



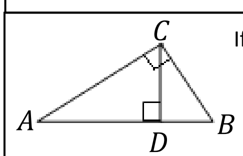
Since 7 is the largest number, $a = 4, b = 5$, and $c = 7$.
$a^2 + b^2 = 4^2 + 5^2 = 16 + 25 = 41$
$c^2 = 7^2 = 49$
Then we have $49 > 41$, which means $c^2 > a^2 + b^2$.
Thus, angle C is an obtuse angle.
ΔABC is an obtuse triangle.

Is the triangle a right triangle? If it is, find the length of one **altitude**.

$a = 10, b = 16$, and $c = 2\sqrt{89}$.

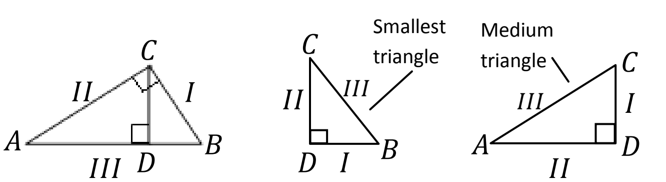


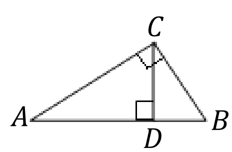
$a^2 + b^2 = 10^2 + 16^2 = 100 + 256 = 356$
$c^2 = (2\sqrt{89})^2 = 4 \cdot 89 = 356$
$356 = 356$, which means $c^2 = a^2 + b^2$.
This means that the angle between the sides with lengths 10 and 16 is a right angle.
Thus, ΔABC is a right triangle.
The easiest altitude of which we can find the length is actually a side of the triangle; the side with length 10 is perpendicular to the base of the triangle.
This means that the length of one of the altitudes of the triangle is 10



If $< ACB$ is a right angle and CD is an altitude of ΔACB that intersects AB, then all 3 right triangles in the picture at the left are similar.

Since the Δ's are similar, the sides are proportional (see below).
Some important proportions are: $\frac{BD}{CD} = \frac{CD}{AD}, \frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.
The 1[st] is the "altitude" proportion and the 2[nd] two are "leg" proportions.



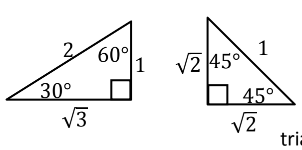


If $AD = 8, BD = x + 4$ and $CD = 6$, find the value of x.

$\frac{BD}{CD} = \frac{CD}{AD} \rightarrow \frac{x+4}{6} = \frac{6}{8}$

$\rightarrow 8(x + 4) = 6 \cdot 6 \rightarrow 8x + 32 = 36 \rightarrow 8x = 4 \rightarrow x = \frac{1}{2}$

SPECIAL RIGHT TRIANGLES



When you have a 30-60-90 or a 45-45-90 triangle, the sides are proportional to the sides of the similar triangles shown at the left.
For example: If you have a 30-60-90 triangle and the side across from the 30°-angle is 15, find the length of the hypotenuse: $\frac{15}{x} = \frac{1}{2} \rightarrow x = 30$.
Thus, the hypotenuse is 30 units long.

TRIGONOMETRY – SOH-CAH-TOA

$\sin A = \frac{opposite}{hypotenuse} = \frac{BC}{AB}$

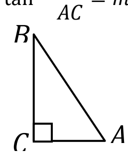
$\cos A = \frac{adjacent}{hypotenuse} = \frac{AC}{AB}$

$\tan A = \frac{opposite}{adjacent} = \frac{BC}{AC}$

$\sin^{-1}\frac{BC}{AB} = m < A$

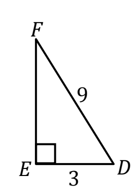
$\cos^{-1}\frac{AC}{AB} = m < A$

$\tan^{-1}\frac{BC}{AC} = m < A$



Example: Find $m < D$ using inverse trigonometry.

$m < D$
$= \cos^{-1}(\frac{DE}{DF})$
$= \cos^{-1}(\frac{3}{9})$
$\approx 70.529°$



Example: Find a and b.

$\cos 48° = \frac{a}{10} \rightarrow a = 10\cos 48° \approx 6.669$

$\sin 48° = \frac{b}{10} \rightarrow b = 10\sin 48° \approx 7.431$