Ratios, Proportions, Geometric Mean & Similarity

A **RATIO** is a comparison of two quantities, usually written as a fraction like $\frac{2}{3}$ (can also written as $2:3$ or "$2\ to\ 3$")

A **PROPORTION** consists of two equivalent ratios.

The **GEOMETRIC MEAN** of a and b is x in the proportion $\frac{a}{x}=\frac{x}{b}$; that is, the geometric mean of a and b is $\sqrt{ab}$.

In **TWO SIMILAR FIGURES**, their angles are ≅ & their sides are proportional (i.e. you can make a proportion with their side lengths).

The ratio of the angles in a triangle is $1:2:3$. What is the measure of each angle?

This means that the measures of the angles must reduce to $1:2:3$. Since we don't know what they are, we can call them $1x, 2x$ and $3x$, which reduces to $1:2:3$. Since the sum of the measures of the angles in a triangle add up to $180°$,

$1x+2x+3x=180 \quad \rightarrow \quad 6x=180 \quad \rightarrow \quad x=30$

Thus, $1x=30$, $2x=60$, $3x=90$, so the measures of the angles of the triangle are $30°$, $60°$, and $90°$.

The ratio of the side lengths of a triangle is $3:4:5$. The smallest side is $18\ cm$ long. What is the measure of the other two sides?

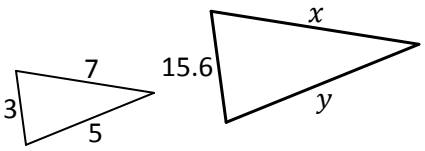
We can set up two proportions:

$\frac{3}{4}=\frac{18}{x}$ and $\frac{3}{5}=\frac{18}{y}$

$3x=72$ and $3y=90$

$x=24$ and $y=30$

The other two side lengths are $24\ cm$ and $30\ cm$.

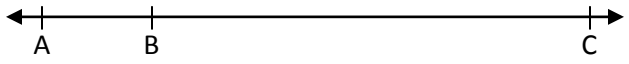


If the two triangles are similar, then find the missing side lengths x and y.

You can do this with a proportion: $\frac{3}{15.6}=\frac{7}{x}$ and $\frac{3}{15.6}=\frac{5}{y}$,

or by multiplying by the *scale factor* of $15.6\div 3$, which is 5.2.

Either way, $x=36.4$ and $y=26$.



$AB:BC=2:7$, and $AC=36$. Find AB and BC.

$AB:BC=2:7$, so we can say that $AB=2x$ and $BC=7x$.

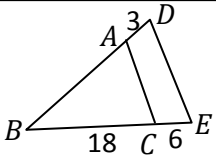
By the segment addition postulate, $2x+7x=36$.

This means that $9x=36$ or $x=4$.

$AB=2x=2(4)=8$

and

$BC=7x=7(4)=28$.

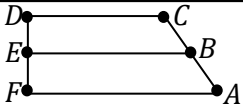


If $\Delta DBE\cong\Delta ABC$, find BA and BD?

You can do this with a proportion since

$\frac{BD}{AD}=\frac{BE}{CE} \quad \rightarrow \quad \frac{BD}{3}=\frac{18+6}{6} \quad \rightarrow \quad \frac{BD}{3}=\frac{24}{6} \quad \rightarrow \quad BD=12$

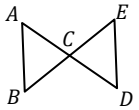
$BD=BA+AD \quad \rightarrow \quad 12=BA+3 \quad \rightarrow \quad BA=9$



$CB=6; DE=4; EF=7.\ Find\ BA.$

Solution: $\frac{BA}{CA}=\frac{EF}{DF} \quad \rightarrow \quad \frac{BA}{6+BA}=\frac{7}{11}$

$\rightarrow \quad 11BA=42+7BA \quad \rightarrow \quad 4BA=42 \quad \rightarrow \quad BA=\frac{21}{2}$



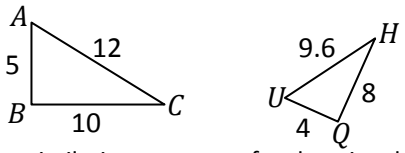
$CA=27;\ CB=21; CD=18; CE=14$

Are the two Δ's similar? If so, write a similarity statement.

Solution: $\frac{CA}{CD}=\frac{27}{18},\ and\ \frac{CB}{CE}=\frac{21}{14}$, both of which reduce to $\frac{3}{2}$.

Also, $<ACB\cong<DCE$ because vert. <'s are ≅. Thus, since two pairs of sides are PROPORTIONAL and the included angles are ≅,

$\Delta ACB\sim\Delta DCE$ by SAS Similarity Thm. Similarity Statement: $\frac{CA}{CD}=\frac{CB}{CE}=\frac{AB}{DE}$.



Write a similarity statement for the triangles. What is the scale factor?

$\frac{AB}{UQ}=\frac{5}{4}$, $\frac{BC}{QH}=\frac{10}{8}$, and $\frac{CA}{HU}=\frac{12}{9.6}$, and all of these fractions are equal to the decimal 1.25. Thus, the scale factor is 1.25.

Similarity statement: $\frac{AB}{UQ}=\frac{BC}{QH}=\frac{CA}{HU}$.

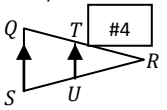
POSTULATES & THEOREMS

Ways that you can know that two triangles are similar:

1) *AA* Similarity Postulate: 2 pairs of <'s are ≅

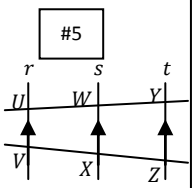
2) *SSS* Similarity Postulate: All pairs of sides are PROPORTIONAL

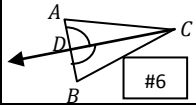
3) *SAS* Similarity Postulate: 2 pairs of sides are PROPORTIONAL & the INCLUDED <'s are ≅



Other important theorems:

4) $TU\ ||\ QS\ if and only if\ \frac{RT}{TQ}=\frac{RU}{US}$





5) If $r\ ||\ s\ ||\ t$, then $\frac{UW}{WY}=\frac{VX}{XZ}$

6) If $<ACD\ \cong<BCD$, then $\frac{AD}{DB}=\frac{CA}{CB}$

Notes/Other Things I Need to Remember: _____

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