Properties of Circles

Definitions

- Circle: all points in a plane that are equidistant from a given point, called the circle's center
- Radius: the distance between the center and all points on a circle
- Chord: a segment whose endpoints are on a circle
- Diameter: a chord that passes through the center of a circle
- Secant: a line that intersects a circle in at two points
- Tangent: a line that intersects a circle at only one point called a point of tangency; there are also tangent rays and tangent segments
- Tangent circles: circles that intersect at exactly one point
- Concentric circle: circles that have a common center
- Central angle: an angle whose vertex is the center of a circle and whose sides are two radii
- Minor arc: a curve of the circle whose central angle has a measure of less than 180°
- Major arc: a curve of the circle whose central angle has a measure of more than 180°
- Semicircle: an arc whose central angle is the diameter and is measured to be 180°
- Inscribed angle: an angle whose vertex is a point on the circle and whose sides are chords
- Intercepted arc: the arc than a central angle or inscribed angle intersects in a given circle

Other Postulates & Concepts

- Arc Addition Postulate: the measure of an arc formed by two smaller, adjacent arcs is equal to the sum of the measures of the two adjacent arcs
- Congruence of Circles: two circles are congruent if they have congruent radii
- Congruence of Arcs: two arcs are congruent if they are located in congruent circles and have the same measure
- Equation of a Circle:
  $(x-h)^2+(y-k)^2=r^2$
  - (h,k) is the center of the circle
  - r is the radius of the circle
  - Points (x,y) make up the circle

Theorems (picture numbers, in red, correspond to the theorem numbers below)

- Theorem 1: m is tangent to Q if and only if m is perpendicular to $\overline{QP}$.
- Theorem 2: If $\overline{SR}$ and $\overline{ST}$ are tangent segments and have a common point outside the circle, then $\overline{SR} \cong \overline{ST}$.
- Theorem 3: $\widehat{AB} \cong \widehat{CD}$ if and only if $\overline{AB} \cong \overline{CD}$ (this is also true if $\overline{AB}$ and $\overline{CD}$ are in congruent circles).
- Theorem 4: If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter.
- Theorem 5: If $\overline{EG}$ is a diameter and EG is perpendicular to DF, then $\overline{HD} \cong \overline{HF}$ and $\widehat{GD} \cong \widehat{GF}$.
- Theorem 6: $\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.
- Theorem 7: $m < ADB = \frac{1}{2} m\widehat{AB}$
- Theorem 8: $< ADB \cong < ACB$ since the two angles share the intercepted arc $\widehat{AB}$.
- Theorem 9: $m < ABC = 90°$ if and only if $\overline{AC}$ (the hypotenuse) is a diameter.
- Theorem 10: The vertices of the quadrilateral D, E, F, and G lie on the circle if and only if opposite angles are supplementary (meaning $m < EFG + m < GDE = 180°$ and $m < DEF + m < FGD = 180°$).
- Theorem 11: $m < 1 = \frac{1}{2} m\widehat{AB};\ m < 2 = \frac{1}{2} m\widehat{BCA}$.
- Theorem 12: $m < 1 = \frac{1}{2}(m\widehat{CD} + m\widehat{AB});\ m < 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$
- Theorem 13: $m < 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC});\ m < 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR});\ m < 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$
- Theorem 14: $EA \cdot EB = EC \cdot ED$
- Theorem 15: $EA \cdot EB = EC \cdot ED$
- Theorem 16: $EA^2 = EC \cdot ED$