Conic Sections

Midpoint Formula: $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ **Distance Formula:** $\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

Completing the Square: How do you get an equation in standard form to look like a specific conic section?

$9x^2-16y^2+108x+128y=76$ Need to put all x's and y's together

$9x^2+108x-16y^2+128y=76$ Need to get a leading coefficient (the a in ax^2) of 1

$9(x^2+12x+?)-16(y^2-8y+?)=76$ Need to complete the square (add to BOTH sides!)

$9(x^2+12x+36)-16(y^2-8y+16)=76+(9*36)+(-16*16)$ Need to simplify

$9(x+6)^2-16(y-4)^2=144$ The "—" tells us this is a hyperbola, so we need to get 1 on the right-hand side

$\frac{(x+6)^2}{16}-\frac{(y-4)^2}{9}=1$ Since the x term is positive, this is a horizontal hyperbola, where:

$$a=4, b=3, c=5;\ center: (-6,4);\ slopes\ of\ asymptotes: \pm\frac{3}{4};\ foci: (-11,4), (-1,4)$$

Equation of a Circle: $(x-h)^2+(y-k)^2=r^2;\ Center=(h,k);\ Radius=r$

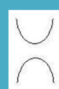
Equations of an Ellipse:

$Center=(h,k); c^2=a^2-b^2; a>b; 2a=length\ of\ major\ axis; 2b=length\ of\ minor\ axis$

- Horizontal Major Axis: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ 
  - Distinguishing Characteristic:
    - a^2 is the greatest denominator & it is under the x term
  - To Graph:
    1. Graph the center
    2. Create points that are a units to the left and right of the center
    3. Create points that are b units up and down from the center
    4. Connect the points in steps 2 & 3
    5. To graph the **FOCI**, create points c units to the left and right of the center
       Coordinates of the FOCI are $(h\pm c, k)$

- Vertical Major Axis: $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ 
  - Distinguishing Characteristic:
    - a^2 is the greatest denominator and it is under the y term
  - To Graph:
    1. Graph the center
    2. Create points that are a units up and down from the center
    3. Create points that are b units to the left and right of the center
    4. Connect the points in steps 2 & 3
    5. To graph the **FOCI**, create points c units up and down from the center
       Coordinates of the FOCI are $(h, k\pm c)$

Equations of a Hyperbola: $Center = (h,k); c^2 = a^2 + b^2; a\ isn't\ always\ greater\ than\ b$

- Horizontal: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ 
  - Distinguishing Characteristic:
    - The x term is the positive term
  - To Graph:
    1. Graph the center
    2. Create points (**VERTICES**) that are a units to the left and right of the center
    3. Create points that are b units up and down from the center
    4. Create a dotted rectangle where the points from steps 2 & 3 are the midpoints of each side
    5. Draw in the (dotted) diagonals of the rectangle from step 4
    6. Draw 2 parabolas whose vertices are the points from step 2 and that approach the dotted diagonals from step 5
    7. To graph the **FOCI**, create points c units to the left and right of the center
       Coordinates of the FOCI are $(h \pm c, k)$
- Vertical: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ 
  - Distinguishing Characteristic:
    - The y term is the positive term
  - To Graph:
    1. Graph the center
    2. Create points (**VERTICES**) that are a units up and down from the center
    3. Create points that are b units to the left and right of the center
    4. Create a dotted rectangle where the points from steps 2 & 3 are the midpoints of each side
    5. Draw in the (dotted) diagonals of the rectangle from step 4
    6. Draw 2 parabolas whose vertices are the points from step 2 and that approach the dotted diagonals from step 5
    7. To graph the **FOCI**, create points c units up and down from the center
       Coordinates of the FOCI are $(h \pm c, k)$

Equations of a Parabola: $Vertex = (h,k)$

- Opening Up or Down: $4p(y-k) = (x-h)^2$; (distinguishing characteristic: there is no y^2 term)
  - If $p > 0$: 
    - Focus: p units up from vertex; Directrix: p units down from vertex
  - If $p < 0$: 
    - Focus: p units down from vertex; Directrix: p units up from vertex
- Opening Up or Down: $4p(x-h) = (y-k)^2$; (distinguishing characteristic: there is no x^2 term)
  - If $p > 0$: 
    - Focus: p units to the right of the vertex; Directrix: p units to the left of the vertex
  - If $p < 0$: 
    - Focus: p units to the left of the vertex; Directrix: p units to the right of the vertex

PARABOLAS

Horizontal Axis of Symmetry

$(y-k)^2 = 4p(x-h)$

$vertex: (h, k)$

$focus: (h+p, k)$

$directrix: x = h - p$

Vertical Axis of Symmetry

$(x-h)^2 = 4p(y-k)$

$vertex: (h, k)$

$focus: (h, k+p)$

$directrix: y = k - p$

CIRCLES

$(x-h)^2 + (y-k)^2 = r^2;\ \ center: (h, k);\ \ radius: r$

ELLIPSES

Horizontal Major Axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a^2 > b^2; a^2\ is\ under\ the\ x - term$

$center: (h, k)$

$vertices: (h \pm a, k)$

$covertices: (h, k \pm b)$

$c^2 = a^2 - b^2$

$Foci: (h \pm c, k)$

Vertical Major Axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a^2 > b^2; a^2\ is\ under\ the\ y - term$

$center: (h, k)$

$vertices: (h, k \pm a)$

$covertices: (h \pm b, k)$

$c^2 = a^2 - b^2$

$Foci: (h, k \pm c)$

HYPERBOLAS

Horizontal Transverse Axis

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$The\ x - term\ is\ positive; a^2\ is\ under\ the\ x - term$

$center: (h, k)$

$vertices: (h \pm a, k)$

$c^2 = a^2 + b^2$

$Foci: (h \pm c, k)$

$Conjugate\ axis\ is\ vertical$

Vertical Transverse Axis

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$The\ y - term\ is\ positive; a^2\ is\ under\ the\ y - term$

$center: (h, k)$

$vertices: (h, k \pm a)$

$c^2 = a^2 + b^2$

$Foci: (h, k \pm c)$

$Conjugate\ axis\ is\ horizontal$