Tests For Convergence - Calculus Chapter 8 Gover 10/04

nth Term (p564) (Page nos. refer to Larson 5th ed.)	$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$. An easy first test to see if further tests for convergence are required. If $\lim_{n\to\infty} a_n = 0$, no conclusion about convergence or divergence.
Geom. Series (P562)	$\sum_{n=0}^{\infty} ar^n$ converges to $S = \frac{a}{1-r}$ if $0<\lvert r\rvert<1$ and diverges if $\lvert r\rvert \geq 1$.
Telescoping (p561)	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$ converges to $S = b_1 - L$ if $\lim_{n\to\infty} b_n = L$, otherwise diverges.
p series (p572)	$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p>1$ and diverges if $0<p\leq 1$. If $p=1$, called a harmonic series.
Alternating Series (p582)	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges if $0 \leq a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$, otherwise divergent. If convergent, remainder (error) is $\lvert R_n\rvert \leq a_{n+1}$. Use when series has positive and negative alternating terms .
Absolute Convergence (p585)	If $\sum_{n=1}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
Conditional Convergence (p586)	$\sum_{n=1}^{\infty} a_n$ is conditionally convergent if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} \lvert a_n\rvert$ diverges. Must use alternating series tests to see if $\sum_{n=1}^{\infty} a_n$ converges.
Integral (p570)	If $f>0$, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both converge or both diverge. Remainder is $0 < R_n < \int_N^{\infty} f(x)dx$. Use for functions that are easily integrated.
Root (p592)	$\sum_{n=1}^{\infty} a_n$ converges absolutely if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$, diverges if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$ and is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$. Use for variables raised to the nth power.
Ratio (p589)	$\sum_{n=1}^{\infty} a_n$ converges absolutely if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$ and diverges if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$. Inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$. Use when nth term involves a factorial.
Direct Comparison (p575)	If $0 \leq a_n \leq b_n$ for all n, then (1) if $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges and (2) if $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges. Compare with a p series or geom. series known to converge.
Limit Comparison (p577)	If a_n and b_n >0 and $\lim_{x\to\infty} \left(\frac{a_n}{b_n} \right) = L$ where L is finite and positive, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge. Compare using a series known to converge or diverge. If you use a p series, compare with an nth term of the same magnitude as the nth term of the given series. In selecting the comparison p series, disregard all but the highest power terms in the numerator and denominator of the given series.