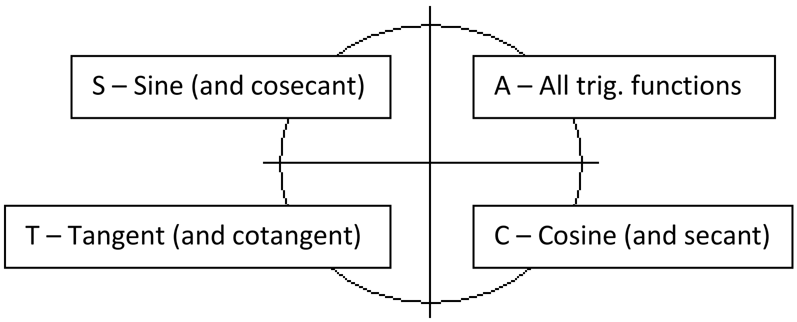
Some Pre-Calculus Ideas to Remember

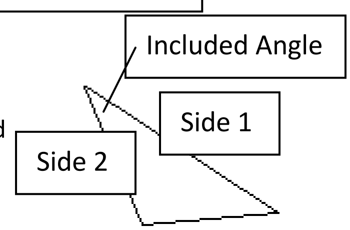
- TRIGONOMETRY: Use that "trig cheat sheet" located on my website. In addition to that cheat sheet, remember:
  - The half-angle formulas can also be written:
    - $\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$; $\sin(\theta) = \pm\sqrt{\frac{1-\cos(2\theta)}{2}}$; $\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$; $\cos\theta = \pm\sqrt{\frac{1+\cos(2\theta)}{2}}$
  - To remember when the trigonometric functions are **positive** in the four quadrants, remember CAST:



  - How to Derive Trigonometric Identities
    - Notes: Almost always start with the side that is more complicated because there are more options as to what you can try on the complicated side.
    - Always use the Pythagorean Identities if you can, because they can make terms cancel nicely.
    - Don't forget the "conjugate" trick: $(1-\sin\theta)(1+\sin\theta) = 1-\sin^2\theta$

> Show that $\tan^2\theta + \sin^2\theta = (\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$.
>
> $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$
>
> $= \sec^2\theta - \sec\theta\cos\theta + \sec\theta\cos\theta + \cos^2\theta$ *Distrubuted (FOIL)*
>
> $= \sec^2\theta - \cos^2\theta$ *Cancelled out middle terms*
>
> $= (\tan^2\theta + 1) - (1 - \sin^2\theta)$ *Substituted two Pythagorean Identities (use your sheet)*
>
> $= \tan^2\theta + \sin^2\theta$ *Combined like terms*

  - Law of Sines & Law of Cosines



    - Use the Law of Cosines:
      - If you KNOW two sides lengths of a triangle AND their included angle measure (see picture at right)…
        …and you WANT TO KNOW the third side length
    - Use the Law of Sines:
      - If you KNOW two side lengths of a triangle AND the angle measure opposite ONE of those sides…
        …and you WANT TO KNOW the other opposite angle measure
      - If you KNOW two angle measures of a triangle AND the side length opposite ONE of those angles…
        …and you WANT TO KNOW the other opposite side length
  - Inverse Trigonometry
    - How to Use It
      - If you want to solve the equation $\tan x = 1$, you need to find the angle x that will make the equation true. You can guess by using the unit circle that the tangent (remember that the tangent is $\frac{y}{x}$) of 45° (or $\frac{\pi}{2}$) is equal to 1. To solve using inverse trigonometry:

$$\tan x = 1$$
$$\tan^{-1}(\tan x) = \tan^{-1} 1$$
$$x = \tan^{-1} 1$$

        If you plug this into your calculator, you will get 45° if you're in degree mode and $\frac{\pi}{2}$ (about 1.571) if you're in radian mode.
    - Note: remember that in NORMAL trig. functions (i.e. sine, cosine), the input is an ANGLE MEASURE. When working with INVERSE trig. functions, you're NOT inputting an angle measure.

- OTHER PRE-CALCULUS CONCEPTS
  - Exponentials and Logarithms
    - Remember that e to the 1st power is just a number, and it is approximately 2.71828 (it doesn't end)
    - Other properties & concepts to remember:

"ln 2" means "$\log_e 2$" or "the **natural log** of 2;" likewise, "log 3" means "$\log_{10} 3$" or "the **common log** of 3."

Product Property Example: $\log_7 20 = \log_7(4 \cdot 5) = \log_7 4 + \log_7 5$ *The property is "$\log_b ac = \log_b a + \log_b c$."*

Quotient Property Example: $\log_6 1.5 = \log_6(\frac{3}{2}) = \log_6 3 - \log_6 2$ *The property is "$\log_b \frac{a}{c} = \log_b a - \log_b c$."*

Exponent Property Example: $\log_5 16 = \log_5 4^2 = 2 \cdot \log_5 4$ *The property is "$\log_b a^c = c \cdot \log_b a$."*

Change of Base Formula Example: $\log_9 5 = \frac{\log 5}{\log 9}$ *The formula is "$\log_b a = \frac{\log a}{\log b}$."*

    - When most people see the problem $2^x = 8$, they can guess that $x = 3$ is the only solution. But what about the problem $2^x = 7$? The solution isn't 2 or 3; it's somewhere in between. This is why we need logarithms. Here are some example problems:

Solve $2^x = 8$ using logarithms.

$2^x = 8$

$\log_2(2^x) = \log_2(8)$ *Found log-base-2 of both sides*

$\cancel{\log_2(2}^x) = \log_2(8)$ *Unnecessary step; shows what cancels*

$x = \log_2(8)$ *Only x remains on the left-hand side*

$x = \log_2(2^3)$ *Substituted $8 = 2^3$*

$x = \cancel{\log_2(2}^3)$ *Unnecessary step; shows what cancels*

$x = 3$ *Only the 3 remains on the right-hand side*

Solve $2^x = 7$ using logarithms.

$2^x = 7$

$\log_2(2^x) = \log_2(7)$ *Found log-base-2 of both sides*

$\cancel{\log_2(2}^x) = \log_2(7)$ *Unnecessary step; shows what cancels*

$x = \log_2(7)$ *Only x remains on the left-hand side*

$x = \frac{\log 7}{\log 2}$ *Used change-of-base formula*

$x \approx 2.807$ *Used calculator to find the answer as a decimal*

Solve $\log_4 x = 5$ by performing "exponentiation" both sides.

$\log_4 x = 5$

$4^{(\log_4 x)} = 4^5$ *Exponentiation (base 4)*

$\cancel{4^{(\log_4}} {}^{x)} = 4^5$ *Unnecessary step; shows what cancels*

$x = 4^5$ *Only x remains on the left-hand side*

$x = 1024$ *Since $4^5 = 1024$*

Condense $\log_4 x - 2\log_4 y + 3\log_4 x + 2$ as much as possible.

$\log_4 x - 2\log_4 y + 3\log_4 x + 2$

$\log_4 x - \log_4 y^2 + \log_4 x^3 + 2$ *Exponent Property*

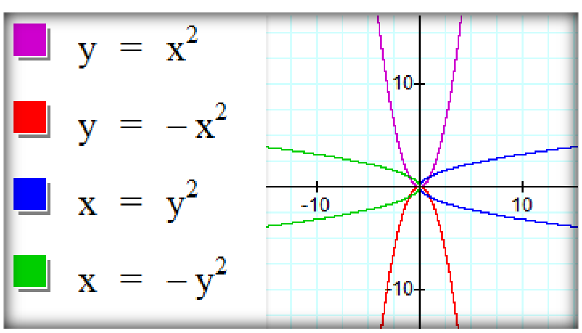
$\log_4 \frac{x}{y^2} + \log_4 x^3 + 2$ *Quotient Property*

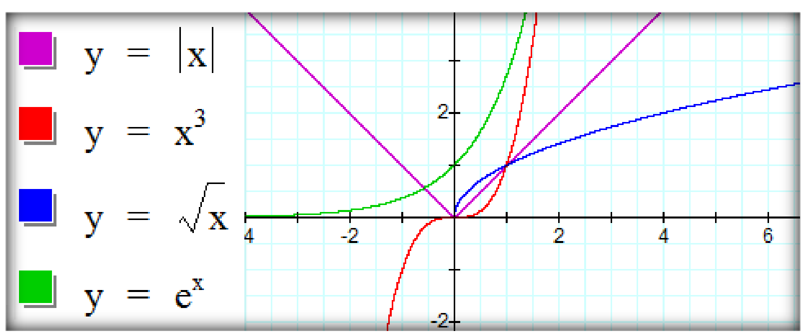
$\log_4(\frac{x}{y^2} \cdot x^3) + 2$ *Product Property*

$\log_4(\frac{x^4}{y^2}) + \log_4 16$ *Simplifying & Rewriting 2 as "$\log_4 4^2$"*

$\log_4(\frac{16x^4}{y^2})$ *Product Property & Simplifying*

  - Graphs of Common Functions





  - Complex Numbers
    - $i = \sqrt{-1}$; $i^2 = -1$; $i^3 = -\sqrt{-1}$; $i^4 = 1$
    - The standard form of a complex number is "$a + bi$," where a is the "real part" & b is the "imaginary part"
    - Since radicals (like square roots) cannot remain in denominators, you cannot keep "i" in the denominator (see example at right)

Simplify $\frac{3-i}{3+i}$ (rationalize).

$= \frac{3-i}{3+i} \cdot \frac{(3-i)}{(3-i)}$ *Multiply by the complex conjugate*

$= \frac{(3-i)(3-i)}{(3+i)(3-i)}$ *Prepare to FOIL*

$= \frac{9-3i-3i+i^2}{9+3i-3i-i^2}$ *FOIL*

$= \frac{9-6i-1}{9-(-1)}$ *Simplify and write "i" as "-1"*

$= \frac{8-6i}{10}$ *Combine like terms*

$= \frac{8}{10} - \frac{6i}{10}$ *Write in standard form*

$= \frac{4}{5} - \frac{3i}{5}$ *Reduce fractions*