Change of Base Formula for Logarithms:

Suppose that a and b are real numbers with $a > 0$, $b > 0$, $a \neq 1$, and $b \neq 1$. Also, suppose that x is a real number with $x > 0$. Then

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}.$$

Proof of the Validity of the Change of Base Formula:

Let $q = \log_a(x)$, $s = \log_b(x)$, and $t = \log_b(a)$. Then

$$a^q = x$$
$$b^s = x$$
$$b^t = a.$$

This implies that

$$b^{t \cdot q} = (b^t)^q = a^q = x = b^s.$$

Since $b^{t \cdot q} = b^s$, we conclude that $t \cdot q = s$. Furthermore, since $t \neq 0$ (because $a \neq 1$), we can conclude that

$$q = \frac{s}{t}.$$

By definition of q, s, and t, this shows that

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}.$$