

# Appendix: Formulae

## ⊕ Formulae For Reference

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

## ⊕ Functions of $2x$ , $\frac{1}{2}x$ and $3x$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

⊕ **Values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  for selected values of  $\theta$**

Degrees	0	30	45	60	90	135	180
$\theta$ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		-1	0

⊕ **Formulae For Differentiation**

( $u, v$  denote differentiable functions of  $x$  and  $k$  denotes constant.)

$$\frac{dk}{dx} = 0$$

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(ku) = k \frac{du}{dx}$$

$$\frac{d}{dx}(uv) = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{\log_a e}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(u^v) = vu^{v-1} \frac{du}{dx} + u^v \ln u \frac{dv}{dx}$$

**⊕ Formulae For Integration**

( $u, v$  denote differentiable functions of  $x$  and  $k, C$  denote constant.)

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx \quad \int \cot x dx = \ln|\sin x| + C$$

$$\int kf(x) dx = k \int f(x) dx$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C & \text{if } n \neq -1, \\ \ln|x| + C & \text{if } n = -1. \end{cases}$$

$$\int \csc x dx = \ln|\csc x - \cot x| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln|x + \sqrt{x^2 + a^2}| + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$$

$$\int e^x dx = e^x + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln|x + \sqrt{x^2 + a^2}| + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$$