AP CALCULUS
Stuff You **MUST** Know Cold

BASIC DERIVATIVES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$
$$\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$$
$$\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$$
$$\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)$$
$$\frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$$
$$\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$$
$$\frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$$
$$\frac{d(\ln(x))}{dx} = \frac{1}{x}$$
$$\frac{d}{dx}\left(e^x\right) = e^x$$

MORE DERIVATIVES

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\left(\cot^{-1}(x)\right) = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(\csc^{-1}(x)\right) = \frac{-1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(a^x\right) = a^x\ln(a)$$
$$\frac{d}{dx}\left(\log_a(x)\right) = \frac{1}{x\ln(a)}$$

+ = BC Topic Only

Differentiation Rules

Chain Rule

$$\frac{d}{dx}\left[f(u)\right] = f'(u)\frac{du}{dx}$$
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

Curve Sketching and Analysis

$y = f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or undefined.

local minimum:

$\frac{dy}{dx}$ (–,0,+) or (–,und,+) or $\frac{d^2y}{dx^2} > 0$.

local maximum:

$\frac{dy}{dx}$ (+,0,–) or (+,und,–) or $\frac{d^2y}{dx^2} < 0$.

pt of inflection: concavity changes.

$\frac{d^2y}{dx^2}$ (+,0,–), (–,0,+),(+,und,–), or (–,und,+)

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F'(x) = f(x)$.

Corollary to FTC

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt =$$
$$f(b(x))b'(x) - f(a(x))a'(x)$$

Intermediate Value Theorem

If the function f is continuous on [a, b], then for any number c between $f(a)$ and $f(b)$, there exists a number d in the open interval (a, b) such that $f(d) = c$.

Mean Value Theorem

If the function f is continuous on [a, b] and the first derivative exists on the interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Rolle's Theorem

If the function f is continuous on [a, b], the first derivative exists on the interval (a, b), and $f(a) = f(b)$; then there exists a number c in (a, b) such that

$$f'(c) = 0.$$

Cauchy's Mean Value Theorem

If the function f is continuous on [a, b], then there exists a number c in (a, b) such that

$$f(c) = \frac{\int_a^b f(x)dx}{(b-a)}.$$

This value $f(c)$ is the "average value" of the function on the interval $[a, b]$.

Trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)\right]$$

Solids of Revolution and friends

Disk Method

$$V = \pi\int_a^b \left[R(x)\right]^2 dx$$

Washer Method

$$V = \pi\int_a^b \left(\left[R(x)\right]^2 - \left[r(x)\right]^2\right)dx$$

Shell Method (no longer on AP)

$$V = 2\pi\int_a^b r(x)h(x)dx$$

ArcLength

$$L = \int_a^b \sqrt{1 + \left[f'(x)\right]^2}\,dx$$

Surface of revolution

$$S = 2\pi\int_a^b r(x)\sqrt{1 + \left[f'(x)\right]^2}\,dx$$

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Distance, velocity, and acceleration

Velocity = $\frac{d}{dt}$ (position).

Acceleration = $\frac{d}{dt}$ (velocity).

+ Velocity vector = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$.

+ Speed = $|v| = \sqrt{(x')^2 + (y')^2}$.

Distance = $\int_{\text{initial time}}^{\text{final time}} |v|\,dt$

+ $= \int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2}\,dt$

Average velocity $= \dfrac{\text{final position} - \text{initial position}}{\text{total time}}$

+ Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

+ Integral of Log

$$\int \ln(x)\,dx = x\ln(x) - x + C$$

+ l'Hopital's Rule

If $\dfrac{f(a)}{g(a)} = \dfrac{0}{0}$ or $= \dfrac{\infty}{\infty}$,

then $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$.

+ Taylor Polynomial

If the function f is "smooth" at $x = a$, then it can be approximated by the n^{th} degree polynomial

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

+ Taylor Polynomial Approximation

If $P_n(x)$ is the n_{th} degree Taylor polynomial of $f(x)$ about a then

$$f(x) = P_n(x) + \frac{f^{(n+1)}(t)}{(n+1)!}(x-a)^{(n+1)}$$

for some t between x and a.

So if $\left|f^{(n+1)}(t)\right| \le M$ for all t between x and a, then

$$\left|f(x) - P_n(x)\right| \le \frac{M}{(n+1)!}|x-a|^{n+1}$$

+ Maclaurin Series

A Taylor Series about $x = 0$ is called Maclaurin.

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

+ Alternating Series Error Bound

If $S_N = \sum_{k=1}^{N} (-1)^n a_n$ is the Nth partial sum of a convergent alternating series with "decreasing" terms, then

$$|S - S_N| \le |a_{N+1}|$$

+ Ratio Test

The series $\sum_{k=0}^{\infty} a_k$ converges if

$$\lim_{k \to \infty} \left|\frac{a_{k+1}}{a_k}\right| < 1.$$

If limit equals 1, you know nothing.

+ Euler's Method

If given that $\frac{dy}{dx} = f(x, y)$ and that the solution passes through (x_0, y_0),

$$y(x_0) = y_0$$

$$\vdots$$

$$y(x_n) = y(x_{n-1}) + f(x_{n-1}, y_{n-1}) \cdot (x_n - x_{n-1})$$

In other words:

$$x_{new} = x_{old} + \Delta x$$

$$y_{new} = y_{old} + \left.\frac{dy}{dx}\right|_{(x_{old}, y_{old})} \cdot \Delta x$$

+ Polar Curves

For a polar curve $r(\theta)$, the area inside a "leaf" is

$$\int_{\theta_1}^{\theta_2} \frac{1}{2}[r(\theta)]^2\,d\theta,$$

where θ_1 and θ_2 are the "first" two times that $r = 0$.

The slope of $r(\theta)$ at a given θ is

$$\frac{dy}{dx} = \frac{(dy/d\theta)}{(dx/d\theta)} = \frac{\frac{d}{d\theta}[r(\theta)\sin\theta]}{\frac{d}{d\theta}[r(\theta)\cos\theta]}$$

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