CHAPTER FOUR

Methodology

Introduction

The purpose of this chapter is to detail the methodological components of my study. First, an overview is given of the university course from which the subjects will be chosen, along with some general characteristics of the students who take the course. Next, case studies are discussed as my choice for overall research design. Then specific procedures for gathering and analyzing the data to address the research questions of my study are described.

Subjects

The subjects will be chosen from a section of Portland State University's Math 212 course, which is entitled *Foundations of Elementary Mathematics II.* The course is intended for prospective teachers, and is a required math course for those wanting to enroll in the Graduate Teacher Education Program (GTEP). Completion of GTEP leads to issuance of an Initial Teaching License. To give a sense of the environment common to the subjects, the content and pedagogy of Math 212 are next described. Subsequently, the range of student backgrounds is discussed, as well as some commonalities in general math experience and attitude.

Content and Pedagogy of Math 212

The content for Math 212 includes geometry, probability, and statistics. Instructors usually divide the ten-week course into two parts of roughly equal length, with one part dealing with geometry and the other part dealing with stochastics. The probability component includes single-stage and multistage experiments, and students investigate both theoretical and experimental probabilities. Theoretical probabilities are obtained by deriving the sample space and considering the possible outcomes of a specific event, while experimental probabilities are obtained by repeatedly conducting trials and dividing the number of favourable outcomes by the total number of trials. Students also consider disjoint events as well as independent and dependent events. The statistics component includes descriptive measures of central tendency

(mean, median, and mode) and spread (range, interquartile range, and standard deviation). Students also analyze data using a variety of graphs, such as boxplots, line plots, bar charts, histograms, pie graphs, and scatterplots. Themes of sampling, such as random sampling, stratified sampling, and making predictions based on sampling, are also a part of the statistics component.

The pedagogy for Math 212 varies with the instructor, but there are common themes which I'll address based on my experience in teaching the course and on discussions with other instructors. One theme is that students participate in activities, both as a class and in smaller groups. The Math 212 classroom is arranged so that students sit in groups, which can be as large as six in number. Once a problem has been posed, or an activity given, students typically will work singly, then together in their groups, and finally share ideas with each other in a class-wide discussion. The sharing of ideas points to a second common theme, which is communication. Students are expected to communicate what they are thinking about and to ask questions of one another so as to understand each other's reasoning. A third common theme is that the teacher acts more as an inquiring guide than as a lecturer. Ideally, the teacher facilitates discussion by asking questions, encouraging other students to ask questions, and generally guiding the class to consensus where possible.

Student Characteristics

The urban setting of Portland State University fosters a wide range of student backgrounds, and it is difficult to describe typical demographic characteristics of the students who take Math 212. Past students in my classes have ranged in age from the early twenties to the middle fifties. Some are undergraduates and some are graduate students. Some want to become teachers and some are just filling a university math requirement.

The majority of Math 212 students have taken Math 211 at PSU, a course taught in a similar style to Math 212. The content of Math 211 includes whole-number arithmetic, number theory, fractions, decimals, and ratios. Other than Math 211, most students have taken few if

any post-secondary math classes. It may have been as many as twenty or more years since they have had *any* mathematics class. At the outset of Math 211 I have always asked students to write a "mathography," which entails a description of their past math classes and their feelings about past math experiences. Most students describe themselves as not having been very good at math in the past, and most Math 212 students share negative memories of precollege mathematics. Some students say they feared or hated their math classes, and others say they were bored. The attitude of most beginning Math 211 students is that mathematics is a rule-oriented discipline. The role of the teacher is to reveal the rules, and the role of the students is to memorize and apply the rules.

The Math 211 experience helps to give most students a new and different vision of what "doing mathematics" entails. Because the pedagogy in Math 211 is similar to that in Math 212, most students come to expect a learning environment in Math 212 where they will be active participants and where their thinking strategies are validated. They have been enculturated to the process of problem-solving and communicating their reasoning. Although not all Math 212 students are comfortable with the learning environment, those who have come through Math 211 at least know what to expect.

Research Design: Case Studies

As mentioned in Chapter Three, qualitative methods will be used in my study. In either quantitative or qualitative research paradigms, multiple methods exist. Thus, some additional justification is needed in defending the specific choices within a given paradigm. The design for my research uses case studies. This section describes what a case study is, the method of choosing the cases, and why case studies are useful for my research.

Definition of Case Studies

Case studies are often associated with ethnographies, grounded theories, or exploratory research. Some authors refer to case studies as a type of qualitative methodology (Merriam,

1998), and others assert that the "case study is not a methodological choice, but a choice of object to be studied" (Stake, 1994, p. 236). Creswell (1998) offers this definition:

> "a *case study* is an exploration of a 'bounded system' or case (or multiple cases) over time through detailed, in-depth data collection involving multiple sources of information rich in context" (p. 61).

Stake (1994) agrees that a case study should represent "a specific, unique, bounded system" (p. 237). The boundaries can be defined by time and place. The Math 212 course, conducted in the same location over ten weeks with the same students and instructor, represents the kind of bounded system needed to conduct a case study. Moreover, there will be in-depth data collection, described later in this chapter, which involves several different sources.

Choice of Cases

I intend to use multiple case studies, choosing eight Math 212 students for my eight cases. Having a total of eight cases is largely a pragmatic decision, determined by the resources available in conducting my research. Qualitative inquiry generally involves relatively small samples, and the key in purposeful sampling is to select "information-rich cases whose study will illuminate the questions under study" (Patton, 2002, p. 230). Creswell (1998) discusses the value in purposeful sampling, saying that cases can be chosen to provide different perspectives on the phenomena under study. For my research, I want my cases (students) to interact in two groups of four, and this grouping is desired because it corresponds with the physical arrangement of the Math 212 classroom. Since the students sit in groups, it is natural to select a whole group of cases who are going to be working together anyhow.

Having cases which work together as a group highlights the social interactions which are emphasized in my theoretical framework. That is, my assumptions are that learning takes place both individually and through social mediation. Thus, there should be a methodological consideration of the social aspects of the Math 212 experience, as well as a way to capture individual conceptions. Choosing cases who sit together as a group also facilitates analysis of the data across cases. Creswell (1998) writes that

> "When multiple cases are chosen, a typical format is to first provide a detailed description of each case and themes within the case, called a *within-case analysis*, followed by a thematic analysis across the cases, called a *cross-case analysis*" (p. 63).

In addition to describing themes of conceptions which emerge from the within each case, I also want to capture the discourse between cases which are grouped together. Therefore, my cases are the individual students, but specifically chosen so that I have two table groups of four cases each . An informed consent form is listed in Appendix A.

Usefulness of Case Studies

There are three reasons why case studies are useful for my research: case studies are descriptive, they allow theory to be generated, and they use different techniques for gathering data. These three reasons are next elaborated.

First of all, recall that my research questions are

1) What conceptions of variation are held by EPSTs in the three contexts of data sets, sampling, and chance situations?
2) How can the conceptions of variation held EPSTs in these three contexts be characterized?
   - A) What variation do EPSTs *expect*, prior to seeing the data or carrying out an experiment?
   - B) What variation do EPSTs *notice*, given different types of displays of data?
   - C) How do EPSTs *display* data so as to emphasize the variation which is present?
   - D) How do EPSTs *describe* the variation which they do notice?
   - E) How do EPSTs *interpret* variation in terms of its causes and effects?

These questions require descriptions of the conceptions of variation held by EPSTs, and "case studies can provide a detailed description and analysis of processes or themes voiced by participants in a particular situation" (McMilan & Schumacher, 1997, p. 395). Case studies are useful in "areas of education where little research has been conducted" (Merriam, 1998, p. 38). Since not much previous work has been done finding out EPSTs' conceptions of variation, case studies are ideal for exploring, describing, and characterizing this knowledge. McMillan and

Schumacher (1997) wrote that “case studies are appropriate for exploratory and discovery-oriented research” (p. 395).

Second, in addition to providing a rich description of the subjects, case studies also allow the use of descriptive data “to develop conceptual categories or to illustrate, support, or challenge theoretical assumptions held prior to the data gathering” (Merriam, 1998, p. 38). For my research, there are five aspects of understanding variation which are hypothesized as a useful construct with which to view EPSTs conceptions. The data will determine which aspects do emerge, and these aspects could be different from the five that were previously conjectured. Case studies are consistent with my study’s goal of developing a theory, grounded in the data, which is useful for describing EPSTs conceptions of variation.

Third, case studies are well-suited to multiple sources of data, especially observations, document review, and interviews (Stake, 1995). These are the types of sources of data for my research, and are explained in more detail in the next section. The use of different data sources is referred to as *triangulation*, which serves to “clarify meaning by identifying different ways the phenomenon is being seen” (Stake, 1994, p. 241). Triangulation enhances internal validity, reliability, and generalizability, which many consider to be the criteria of research soundness (Romberg, 1992). Internal validity refers to how well the findings of the research match the reality of what went on during the research process. Reliability refers to the extent in which the research findings can be replicated, but “seems to be something of a misfit when applied to qualitative research” (Merriam, 1998, p. 206). Merriam advocates the term ‘consistency’ in place of ‘reliability’, saying that “the question then is not whether findings will be found again but *whether the results are consistent with the data collected*” (p. 206, italics in original). Regarding generalizability, it should be noted that whereas a case study represents a specific, unique, and bounded circumstance, the “transferability or generalizability to other settings may be problematic” (Marshall & Rossman, 1995, p. 144). Many writers contend that the notion of generalizability can only be applied to qualitative methodology in a theoretical sense. Some

qualitative methodologists prefer to talk instead of working hypotheses, while others prefer to talk in terms of naturalistic generalizations, which "include the kind of learning that readers take from their encounters with specific case studies" (Patton, 2001, p. 583). Still, "triangulating multiple sources of data…can enhance a study's generalizability," claim Marshall and Rossman (1995, p. 144). Moreover, "especially in terms of using multiple methods of data collection and analysis, triangulation strengthens reliability as well as internal validity" (Merriam, 1998, p. 207). To gain triangulation, the three methods of data gathering in this study are observations, document reviews, and interviews. These three methods, as well as the class activities concomitant to the data collection, are described in the next section.

Procedures for Data Gathering

The three data gathering methods of observation, document review, and interviews all require three activities that will be done in class. The three activities have specific themes which correspond to the three contexts for looking at conceptions of variation: variation in data sets, variation in sampling, and variation in probability situations.

The overall design for the data gathering can be summarized as follows. Prior to doing any class activities, at the beginning of the academic quarter, baseline information will be collected from all Math 212 students. The information will be collected via a survey of prior mathematical experience and a pretest about variation. During the class activities, the cases will have their small-group discussions audiotaped, and the class-wide discussions will be videotaped. After the class activities, homework based on the themes of the activities is then completed by all Math 212 students. The homework doesn't make as much sense without the context provided by the activities. After all three class activities have been completed, and all of my cases' written homeowork responses have been at least initially analyzed, individual interviews with my cases will be conducted. The interviews will also be videotaped. Questions which are asked during the class activities, in the homework, and during the interviews collectively probe the five aspects of understanding variation hypothesized as a part of my

conceptual framework. The types of data collected, when they will be gathered, and why they are useful are summarized in Table 1 below.

Data Collection Summary Table					
When	What	How	Who	Why	
PreActivities	Informational Survey	Written Response	All Students	Provides • demographical data • list of prior math experiences • attitudes towards data and chance	
	Pretest	Written Response	All Students	Establishes baseline information on students responses to questions about variation	
Activity:*Looking at Data Sets*	Classroom Observation	Videotape	All Students & Instructor	Captures the overall learning environment and the class-wide discussion of the activity	Emphasizes • context of data sets • aspects of displaying, noticing, describing
	Small-Group Discussion	Audiotape	All Cases	Captures discourse between cases as they engage in the activity in small groups	
PostActivity: *Looking at Data Sets*	Homework	Written Response	All Students	Extends the activity, allows individual cases opportunity to put their thinking down on paper	
Activity: *The M&M Investigation*	(Same as in *Looking at Data Sets)*				Emphasizes • context of sampling • aspects of expecting, noticing, describing, interpreting
PostActivity: *The M&M Investigation*					
Activity: *The River Crossing Game*	(Same as in *Looking at Data Sets)*				Emphasizes • context of chance situations • aspects of expecting, noticing, describing, interpreting
PostActivity: *The River Crossing Game*					
PostActivities	Individual Interviews	Videotape	All Cases	Probes further thoughts about activities and homework	

Table 1

The remainder of this section provides further elucidation on the data gathering, and begins with a discussion of the class activities used to situate the study of conceptions of variation within the three contexts of my conceptual framework.

Class Activities

The three class activities used in my research are referred to in Appendix B as *Looking at Data Sets, The M&M Investigation,* and *The River Crossing Game*. The reason these three activities have been chosen is because they provide many opportunities for looking at conceptions of variation in the three contexts of data sets, sampling, and chance situations, respectively. In order for the homework to make sense, the activities have to be done first. Similarly, in order for the interviews to be fruitful, the homework has to be completed. Thus, the activities act as starting point to get students started in thinking about variation. Moreover, as I have used the activities before in the Math 212 class, I have found them to be a good fit for the pedagogical and content goals of the class. Not only do they allow learning of stochastical skills and concepts such as graphicacy and measures of center and spread, but they also help facilitate discussion on students' conjectures and reasoning.

My theoretical framework holds that knowledge is constructed both personally and socially, and so it is important to offer an environment where my cases can interact with other students in contexts where questions about variation arise. Instructor notes for conducting the activities are in Appendix B. The following paragraphs give an overview of each activity and point out how the aspects of understanding variation arise.

Looking at Data Sets The activity *Looking at Data Sets* is partly derived by Lesson 17, "Data Centers, Spreads, and Plots," from the *Visual Math Course II* curriculum (Foreman & Bennett, 1995). As with the other activities, this one is also partly derived from my own experience in teaching Math 212 and generating ideas with other Math 212 instructors. The activity involves students in Math 212 gathering data from one another, then graphing the data,

and finally discussing the data and the graphs. Each student answers the following four questions:

1. What is your armspan ?
2. What is your height ?
3. What is your head circumference ?
4. How many times in one minute does your heart beat?

Before all the data is made known, students make conjectures about the center and spread of the entire class's data. Then, the set of class data for each of the four questions is graphed on large poster paper so that graphs can be seen by everyone in class. Two different graph types are used: histograms and boxplots. Also, the interval width in the histograms varies. For example, in a pilot for my study, the same set of data for question four resulted in the following two different histograms (see Fig. 13).

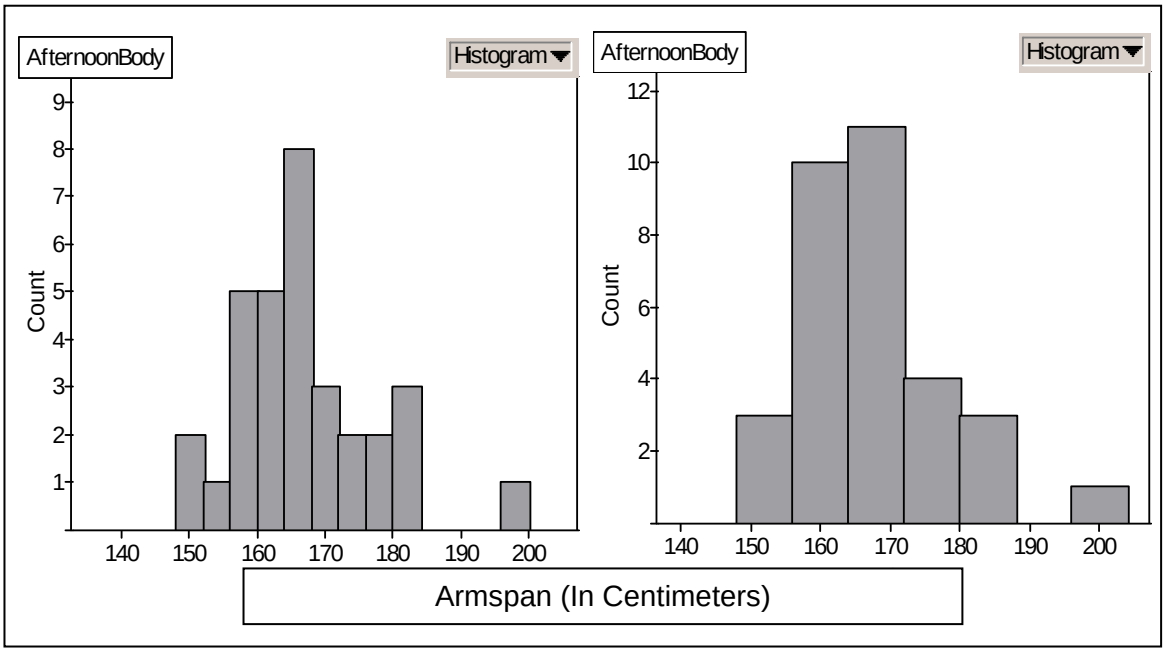

Fig. 13

After the graphs are available, a small-group discussion next ensues, followed by class-wide discussion. The themes of the discussions are about what the graphs reveal about representative measures as well as the variation in the data. For instance, a boxplot is used to

show the armspan data used in Figure 13, and we talk about whether one graph or the other did a better job of showing what armspan was typical for a member of the class. Also we discuss how alike or unalike our armspans were, and which graph does a better job of showing differences in the data. Table 2 shows a set of questions for *Looking at Data* Sets which are used in small-group and class-wide discussion, as well the aspects of understanding variation that are addressed by the questions.

Looking at Data Sets Discussion Guide	
Aspect	Sample Questions
1.Expecting	a. What do you think will be the range for the class armspan data? b. What do you think will be a reasonable interval of heights to capture the middle 50% of the class data?
2.Noticing	a. Look at the graphs for the heartbeat data. What features of the graphs do you notice? b. Find the median height for the class. Now look at the graphs for the height data. Would you say the median height is typical for a member of the class?
3.Displaying	a. Compare the boxplot and the histogram for the head circumference data. Which graph better shows how the data are spread out from the median? b. With the height data, compare two histograms of different interval widths. Which one does a better job of disguising the differences among the data?
4.Describing	a. How alike or unalike would you say our head circumferences are? b. Does it look like there is more variation in one set of data versus another set? Explain.
5.Interpreting	a. Why aren't the armspan data closer to one another? b. Let's suppose another person joins the class. How confident are you that the person's head circumference is within 2cm of the class mean height?

Table 2

The M&M Investigation This activity is motivated in part by an extension in "Investigation 4: Solving Real-World Problems" in the eighth grade stochastics unit for the *Connected Mathematics* curriculum (Lappan, Fey, Fitzgerald, Friel, & Philips, 1998). Each table group receives two regular-size (1.69 ounce) bags of M&Ms, so the class is using a total of sixteen bags. Before opening the bags, students make conjectures about what is inside their bag. For example, they guess about how many pieces of candy are inside, which color is predominant, and what percentage of the bag is represented by each color. Then the bags are opened. The number of pieces of candies in each bag are determined, and bar charts are constructed which

show the relative frequency of each color type (red, orange, yellow, blue, green, and brown) for each bag. Based on these small samples (the number of candies in a regular bag is about 57), the class makes conjectures about the true color distributions as reported by the candy company.

Sixteen regular bags tend to show a large amount of variation in their respective color distributions. The percentage of candies which are red, for example, ranged from about twelve to thirty in the sixteen bags examined during the pilot for my study (see left side of Fig. 14).

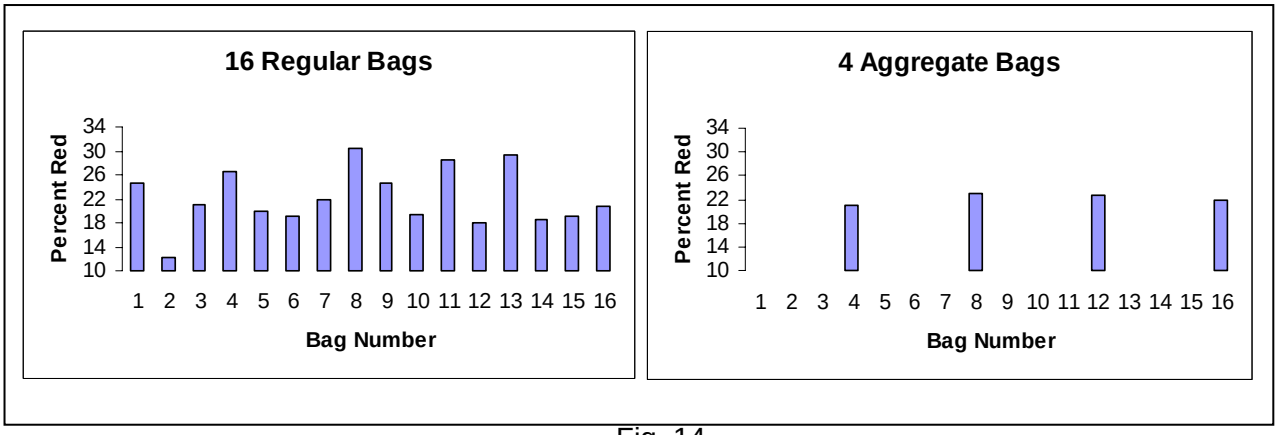

Fig. 14

To simulate a larger sample, students aggregate their sixteen regular-sized bags four at a time and recalculate the totals and color percentages. In the pilot, the larger, aggregated "bags" (each representing four regular bags) had about 228 candies in each. In the right side of Figure 13 above, aggregate bag #4 represents the contents of regular bags #1 – 4, aggregate bag #8 corresponds to regular bags #5 – 8, and so on. One question in the class discussion is how many of the sixteen regular bags were farther than expected from having 20% red M&Ms, and whether most of the aggregate bags were closer to 20% than expected. Another question is how confident students are of getting a sample of M&Ms with between 18% to 22% red. We also talk about why the bags aren't all identical. Table 3 shows a set of questions for *The M&M Investigation* which are used in small-group and class-wide discussion, as well the aspects of understanding variation that are addressed by the questions.

The M&M Investigation Discussion Guide	
Aspect	Sample Questions
1.Expecting	a. What do you think will be the range for the percentage of reds in each bag? What about for the number of pieces in each bag? b. What do you think will be a reasonable interval for number of pieces in a bag to capture the middle 50% of the class data?
2.Noticing	a. Look at the graphs for the individual bag data. What features of the graphs do you notice ? b. Find the median percentage of reds for the individual bags. Would you say that percentage is typical for an individual bag ?
3.Displaying	a. Compare the boxplot and the bar chart for the data on percentage of red. Which graph better shows how the data are spread out from the median?
4.Describing	a. How alike or unalike would you say our individual bag data are? What about for our aggregate bag data? b. Does it look like there is more variation in the individual bag data or in the aggregate bag data ? Explain.
5.Interpreting	a. Why aren't the data for percentage red in an individual bag closer to one another? b. Let's suppose we open up one more individual bag. How confident are you that the percentage of red in the bag will be within 2% of the class mean ?

Table 3

The River Crossing Game Credit for this activity belongs to the *Math and the Mind's Eye* curriculum (Shaughnessy & Arcidiacono, 1993). Students play this game in pairs. Each player gets ten chips to arrange on place markers which are numbered from one to twelve. Two dice are rolled, and if either player has a chip on a marker whose number corresponds to the sum of the dice, the player may remove a chip. A player wins when all their chips are removed. For example, consider Player A and Player B each having the following beginning arrangement (see Fig. 15):

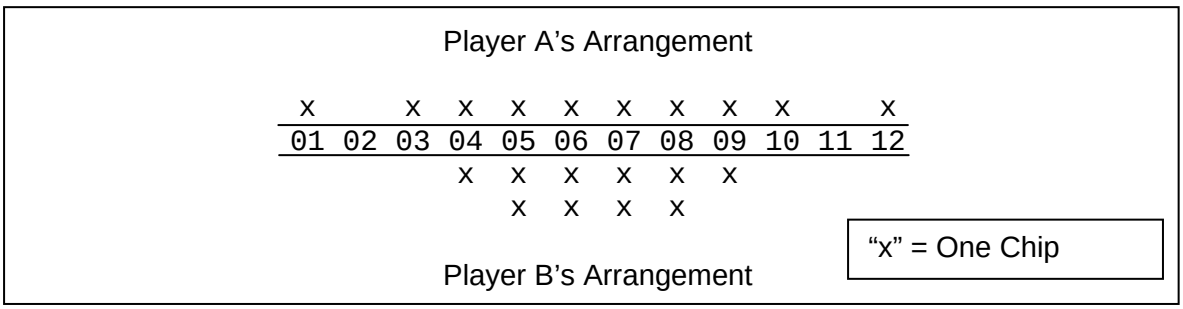

Fig. 15

Before playing the first time, I have students make conjectures about what they think a good arrangement might look like. Most students are quick to observe that Player A above will never get a sum of two dice equaling "1". Beyond that observation, the suggestions for optimal arrangements prior to actually playing the game are quite varied. Some students want a uniform distribution, some want to approximate a normal distribution, and some choose their favorite numbers. Students play several games, changing their arrangements from game to game as they wish. Also, each pair of students keeps a cumulative line plot showing the sum of the dice for every toss made for all games. Figure 16 shows an example of the line plots from two different pairs of students.

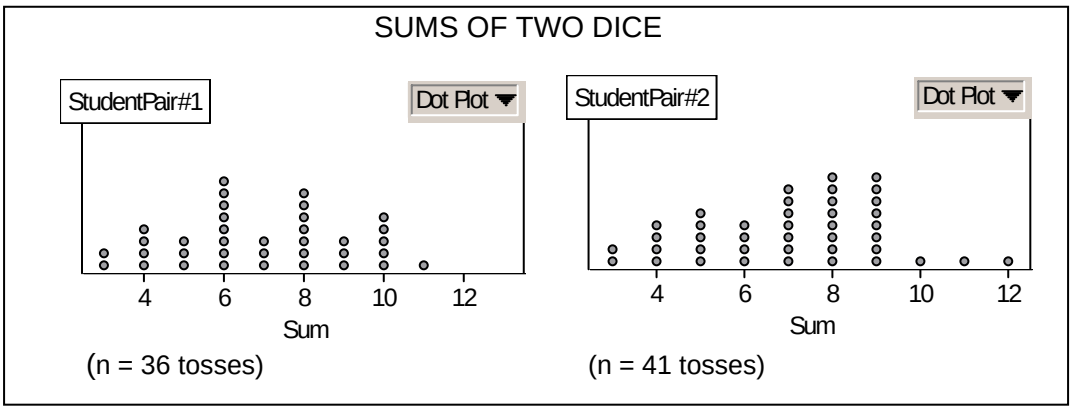

Fig. 16

The total number of tosses can be different in each graph because the number of tosses needed to win each game varies. Students obtain the experimental probability of obtaining a "7" (that is, a sum equaling seven) by counting how many sevens they got and dividing by their total tosses. Then the students calculate the theoretical probability of obtaining a "2" through "12" by listing the sample space and considering possible outcomes. Included in class discussion is the way that the theoretical probability of getting a "7" is 1/6, yet usually none of the experimental probabilities are 1/6. In fact, "7" is theoretically the most likely outcome, but many of the line plots do not even show "7" to be the most frequent sum. Table 4 shows a set of questions for

The River Crossing Game which are used in small-group and class-wide discussion, as well the aspects of understanding variation that are addressed by the questions.

The River Crossing Game Discussion Guide	
Aspect	Sample Questions
1.Expecting	a. What do you think will be the range for the experimental probabilities of getting a "7"? b. What do you think will be a reasonable interval to capture the middle 50% of the class data for the experimental probabilities of getting a "7" ?
2.Noticing	a. Look at the line plots for all pairs of students. What features of the graphs do you notice ? b. Look at the graph showing all the experimental probabilities of getting a "7" obtained by pairs of students. Would you say that a probability of 1/6 is a typical experimental probability to obtain?
3.Displaying	a. Compare the boxplot and the bar chart for the class data on experimental probabilities of getting a "7" . Which graph better shows how the data are spread out from the median?
4.Describing	a. How alike or unalike would you say our data on experimental probabilities of getting a "7" are? b. Does it look like there is more variation for the graphs depicting experimental probabilities which are based on a larger or smaller number of total trials?
5.Interpreting	a. Why aren't the data for the experimental probabilities of getting a "7" closer to one another? b. Let's suppose we roll the dice 60 times. How confident are you that the number of "7"s will be between 8 and 12 ?

Table 4

All three activities described above provide opportunities for students to respond to questions about variation, either in small groups or in class discussion. Class and small-group discussions vary, but some specific questions have been listed for each activity which are related to the five aspects of understanding variation. The activities give students an opportunity to gather data, display data, and discuss the variation represented in the graphs of the data. The activities lay the groundwork for the homework and interviews, in which students answer more questions which are embedded in the context of data sets, sampling, and chance situations.

Observations

The purpose of using observations as a data collection method is to record the overall class contexts in which the activities occurred, and to capture the contributions of my eight

cases. I want to find out what they say to one another in their groups, and what they share with the class, as they engage in the activities and discussion.

Observations are a common data collection technique in case studies (Stake, 1995; Merriam, 1998). Best and Kahn (1998) write that "when observation is used in qualitative research, it usually consists of *detailed notation* of behaviors, events, and the contexts surrounding the events and behaviors" (p. 253, italics in original). Patton (2001) includes the following three dimensions of concern when conducting observations: the role of the observer, the disclosure of observation, and the recording procedures. In my research, I will have roles as both participant and observer. I will be a participant by virtue of being the instructor for the Math 212 class, and an observer by virtue of having my class activities recorded. Patton claims that

> "the participant observer employs multiple and overlapping data collection strategies, being fully engaged in experiencing the setting (participation) while at the same time observing and talking with other participants about whatever is happening" (pp. 265, 266).

Regarding disclosure, the participant observation is what Fraenkel and Wallen (2000) called *overt*, because the researcher will be identified and the cases will know they are being observed. For recording procedures, I will have the three class activities videotaped. To "minimize the errors resulting from faulty memory," caution Best and Kahn (1998), "simultaneous recording of observations is recommended" (p. 295). Videorecording of the classroom during the activities will show the context of the learning environment, the overall flow of the class, and the specific contributions of my cases to the class discussion. Also, the tables where my cases sit will be audiorecorded, so that I can capture what they are saying to each other in small-group discussions during the activities.

Written Documents

The use of written documents as a method for collecting data in a case study is well regarded (Stake, 1995; Merriam, 1998; Patton, 2001). Written documents can supplement

observational data, and "quite often, documents serve as substitutes for records of activity that the researcher could not observe directly" (Stake, 1995 p. 68). In my study, there are two types of documents I will collect from my cases: pre-activity documents and post-activity documents. These documents are listed in Appendices C and D.

Pre-Activity Documents The pre-activity documents are completed by the all students at the beginning of the Math 212 course. One of the documents is a description of students' prior experience with stochastics, including a self-reporting by the students of classes that they remember having taken in the past. The purpose of the description is so that I can differentiate subjects according to their experience. The other document is a pretest consisting of four questions similar to those that have been used by other researchers. The pretest illuminates subjects' existing intuitions and ways of reasoning about variation. The first question asks the subjects for a description of what the following terms mean to them: sample, random, and variation. The question is similar to one used by Watson, Kelly, Callingham, and Shaughnessy (2002). The second question presents two different sets of data in a line plot, and asks the subjects direct questions about which set has more variability in the data. The context for the question is variation in data sets, and the question itself is akin to one used by Loosen, Lioen, and Lacante (1985). The third question is the Hospital Problem, and the context is variation in sampling. Finally, the fourth question uses a probability context. Students are asked what outcomes would be considered surprising in an experiment involving repeated rolls of a die. The question is motivated by tasks used is research reported by Shaughnessy and Ciancetta (2001).

The reason for administering a pretest is so that I can see how my subjects respond to questions on variation before any class activities have been conducted. My research is not a teaching experiment, but instruction and classroom interactions about variation can still affect the way that subjects write and talk about the topic. As I write up the case studies and describe the participants, I want to profile their stochastic backgrounds and give an example of how they responded to some questions about variation at the beginning of the course.

Post-Activity Documents The post-activity documents comprise three homework assignments closely related to the three class activities. After each of the activities, students will take home the assignments and answer the questions. Taken together, the questions in the assignments address the five aspects of understanding variation which are a part of the conceptual framework. As Marshall and Rossman (1995) note, "the decision to gather and analyze documents…should be linked to the research questions developed in the conceptual framework of the study" (p. 85). For my study, gaining a written record of how the participants respond to questions on variation is essential. The homework on variation in data sets, following the class activity *Looking at Data Sets*, places more emphasis on the aspects of noticing, displaying, and describing variation. The two assignments on variation in sampling and variation in probability situations follow *The M&M Investigation* and *The River Crossing Game*. Both assignments highlight the aspects of expecting, noticing, describing, and interpreting.

In addition to providing a written record of explanation and justification in response to questions on variation, the preliminary analysis of the written documents provides insight into questions to ask in the subsequent interviews. Patton (2001) says that "documents prove valuable not only because of what can be learned directly from them but also as stimulus for paths of inquiry that can be pursued only through direct observation and interviewing" (p. 294).

Interviews

Interviewing is a common and powerful method of trying to understand other people (Fontana & Frey, 1994). Best and Kahn (1995) note that "interviews are used to gather information regarding an individual's experience and knowledge" (p. 255), and Patton (2001) says that the purpose of interviewing "is to allow us to enter into the other person's perspective. Qualitative interviewing begins with the assumption that the perspective of others is meaningful, knowable, and able to be made explicit" (p. 341). While interview types run the gamut from being highly structured to completely unstructured, the protocol for this research is best described as semistructured (Fontana & Frey, 1994). Merriam (1998) maintains that

"interviewing in qualitative investigations is more open-ended and less structured" (p. 74). The interviews for my study follow the script listed in Appendix E. The initial questions are largely open-ended, and the possibilities for the use of subsequent probes depend on the responses of the interviewees.

Although Fraenkel and Wallen (2000) suggest that "structured and semistructured interviews are best conducted toward the end of a study" (p. 510), this shouldn't be taken as an absolute rule in qualitative research. The reason that interviews for this study will occur only after the written responses have received preliminary analysis is because one function of the interview protocol is to solicit further thoughts on the tasks presented earlier.

The interviews are thus task-based, since "the subjects' interactions are not merely with the interviewers, but with the task environments" (Goldin, 2000, p. 519). Once the participants have gone through the class sessions in which the activities on variation were completed, and have provided written responses to questions based on those activities, the tasks during the interview should appear familiar. That is, the contexts for the tasks are either the same as those used before or very similar.

Goldin (2000) goes on to mention the value in task-based interviews, noting that the tasks can be adjusted in wording and content according to the results of previous research. He adds:

> Interview contingencies can be decided explicitly and modified when appropriate. In comparison with paper-and-pencil test-based methods, task-based interviews make it possible to focus research attention more directly on the subjects' processes of addressing mathematical tasks" (p. 520).

Thus, a semistructured, task-based interview format is the third method used for gathering data.

The interview script does contain questions directed at specific tasks and in specific contexts, but the questions are open-ended and include the possibility of extensive probing in an effort to determine conceptions of variation held by the participants. The interviews will be videotaped so that the subjects' explanations can be recorded. In the interviews, there are

opportunities for interviewees to provide some written work, but this work will likely by accompanied by verbal communication about the thought processes behind the work. Any written work, as well as observations I make during the interviews, will become a part of the overall data for the study, along with the transcriptions of the interviews.

Sources of data for my study therefore include observations of the classroom environment as well as during the interview setting. Also, a description of prior stochastical experience, a pretest, three homework assignments, and any work provided during the interview all contribute to the written records collected for the study. Lastly, semistructured task-based interviews will be conducted and transcribed. Together these methods allow for a degree of triangulation, and all provide data contributing to the answering of the study's research questions. To use the data, however, requires analysis, a process discussed in the subsequent section.

Procedures for Data Analysis

This section first presents the role of grounded theory in the data analysis process, both in general terms and also as it applies to my study. Subsequently, the role of computer software in facilitating the process of data analysis in my study is discussed.

Role of Grounded Theory

The general principles for the data analysis are derived from grounded theory, applied both within cases and across cases. The aim in the analysis is to describe and categorize the data, resulting in a tentative theory of what are the key features of the subjects' conceptions of variation. Strauss and Corbin (1994) said that "grounded theory is a *general methodology* for developing theory that is grounded in data systematically gathered and analyzed" (p. 273, italics in original). The techniques prescribed fit the aim of the analysis needed for this study.

The ultimate aim of most qualitative studies, claim Huberman and Miles (1994) is "to describe and explain (at some level) a pattern of relationships, which can be done only with a set of conceptually specified analytic categories" (p. 431). Grounded theory begins by

describing and building categories, the dimensions of which are defined by their conceptual properties. Emergent tentative hypotheses suggest links between categories and properties (Patton, 2001; Merriam, 1998).

The development of these categories, properties, and hypotheses occurs through a method of constant data comparison, "a process whereby the data gradually evolve into a core of emerging theory" (Merriam, 1998, p. 191). More specifically, data analysis begins even while the data is being collected, possibly in the form of memos regarding the researcher's conjecture for possible categories. Then, the data is iteratively compared to the emerging categories, while the categories are refined in light of reviewing the data (Strauss & Corbin, 1994). "This process of taking information from data collection and comparing it to emerging categories is called the *constant comparative* method of data analysis" (Creswell, 1998, p. 57, italics in original). Patton (2001) calls this comparative analysis "a central feature of grounded theory development" (p. 490).

Because my study is concerned with EPSTs conceptions, the techniques of grounded theory are of particular appeal because "concepts are the basic units of analysis in the grounded theory method" (Strauss & Corbin, 1990, p. 63). The authors continue by stating that "conceptualizing our data becomes the first step in analysis" (p. 63). Open coding is described as the process of identifying the concepts and discovering their properties. A detailed line-by-line analysis, while labor intensive, is extremely generative and especially useful at the outset of a study. The establishing of categories has much in common with the clustering technique of Miles and Huberman (1994), in which the aim was to "understand a phenomenon better by *grouping* and then *conceptualizing* objects that have similar patterns or characteristics" (p. 249). An example of the use of this technique in the study of understanding variation is provided by Torok and Watson (2000). Adding to the power of open coding is axial coding, defined as "the process of relating categories to their subcategories, termed 'axial' because coding occurs around the axis of the category, linking categories of the level of properties and dimensions"

(Strauss & Corbin, 1998, p. 123). Microanalysis is the combined approach of open and axial coding, using a line-by-line analysis, to "generate initial categories (with their properties and dimensions) and to suggest relationships among categories" (p. 57).

In analyzing my data, both within-case and cross-case analyses will be done. Within-case analysis "means that the researcher identifies themes within a single case" (Creswell, 1998, p. 252), and provides a detailed description for that case (Huberman & Miles, 1994). For my study, since variation is looked at in several distinct contexts, it is important to look for themes occurring within each individual case before comparing across cases. For within-case analysis, each participant is considered an individual case and analyzed on that basis (Merriam, 1998; Creswell, 1998). Cross-case analysis "involves examining themes across cases to discern themes that are common to all cases" (Creswell, 1998, p. 250), and it typically follows within-case analysis when multiple cases are studies. Some authors warn of the complexities of cross-case analysis (Huberman & Miles, 1994; Miles & Huberman, 1994), but Merriam (1998) says that "ultimately, cross-case analysis differs little from analysis in a single qualitative case study" (p. 195). The result of the analysis is a comprehensive set of "categories, themes, or typologies that conceptualize the data from all the cases" (p. 195).

While discussing multiple cases, Creswell (1998) brings up the topic of how many cases is enough, cautioning against a rationale for improved generalizability, "a term that holds little meaning for most qualitative researchers" (p. 63). Rather, a more fitting rationale for the tenets of grounded theory is a desire for saturation, meaning that point at which new cases are not adding any new information to the establishment of categories (Creswell, 1998).

Role of Computer Software

Grounded theory techniques allow for the inclusion of a wide scope of data, apropos of this study, such as the written responses, transcribed interviews, and observational notes. Also, memos suggesting the continual conjecturing and refinement of categories and concepts also become a part of the data, as the process of theory development moves through cycles of

constant comparison. As such, data management becomes a crucial issue in using grounded theory. Richards (1994) boldly states that “all researchers working in the qualitative mode will clearly be helped by some computer software” (p. 105). The use of qualitative data analysis software facilitates not only the management of data, but “it can offer leaps in productivity for those adept at it” (Patton, 2001, p. 447).

The software used in this study will be NUD•IST (Non-numerical Unstructured Data Indexing, Searching, and Theorizing), a theory-building program that aids in data storage, coding, retrieval, and category comparisons and linking (Richards and Richards, 1994; Richards, 1994; Patton, 2001). NUD•IST is well-suited for the analysis techniques of grounded theory, although it cannot be emphasized enough that software only assists in the process – software does *not* analyse data for the researcher (Patton, 2001; Creswell, 1998). My own previous use of NUD•IST in a grounded theory study on parent perceptions of mathematics provided experience in the various features of the software. Well over one thousand responses - whose length ranged from a few words to whole paragraphs - were coded and indexed. The flexibility in coding, categorizing, and revising made the development of theory a dynamic and reflexive process. The single biggest obstacle was in learning what NUD•IST could do for the analysis, and how to get the program to do what I wanted. Once the various ways of categorizing, indexing, and coding were learned – the logistics of the program as well as the potential – I found that I was able to fully immerse myself in finding out what the data was trying to say.

Some researchers have used NUD•IST to do automatic coding. Watson and Moritz (2000a) mention how, in their research, “command files automatically assigned responses to categories, which were manually checked for accuracy” (p. 11). However, in that instance the researchers were intent on assigning SOLO modes and levels to the emergent categories. In my study, as mentioned earlier, the more laborious yet potentially illuminative task of line-by-line coding will be used at the outset, and the creative yet systematic process of theory building is

unencumbered by a rigid framework like the SOLO taxonomy. It is expected that NUD•IST will, as it was in my previous experience, be an enormous asset in the analysis process.

In summary, the data from this study will receive both within-case as well as cross-case analyses, and the techniques are derived from grounded theory. The aim of the analysis is to inform the research questions by providing a rich description and an overall characterization of EPSTs conceptions.

Conclusion

Subjects will be chosen from the Math 212 class, an environment in which activities having to do with variation are enacted. The style of pedagogy in the class encourages students to communicate their thinking, both verbally in writing. Therefore, it is expected that the subjects will be familiar with and able to provide the kind of expression of thinking needed for this study. Within the array of possible methods in the qualitative research paradigm, a case study design is chosen as suitable for answering my research questions.

An informational survey will be given to subjects at the outset of the Math 212 course, as will a pretest focusing on variation. Three specific class activities will next be conducted, and these activities correspond to the three different contexts for understanding variation which are a part of my conceptual framework. The class activities will be observed, and the discussion of the class and among my cases will be recorded. Next, homework related to the class activities will be assigned and subsequently collected and analyzed. Finally, individual interviews will be conducted. Collectively, the data collection methods address all three different contexts chosen for exploring the topic of variation, and all five aspects of understanding variation which are also a part of my conceptual framework. It is expected that the data will provide the kind of description of conceptions held by the participants needed to achieve the goals of my study.

The use of grounded theory techniques allow for the characterization of the subjects' understanding to naturally emerge in the shape of distinct but linked categories. The process of data analysis will be facilitated by the use of the NUD•IST computer software. The final result

will be a rich description and tentative theory of the conceptions of variation held by elementary preservice teachers.