

Appendix A

Informed Consent: Student Subjects

You are invited to participate in a doctoral research project entitled “Elementary Preservice Teachers’ Conceptions of Variation”, being conducted by Daniel Canada from the Department of Mathematical Sciences at Portland State University. The researcher hopes to develop a characterization of the knowledge held by elementary preservice teachers about this important statistical concept. You were selected as a possible participant by virtue of your enrollment in the Math 212 class.

By giving your consent to take part in this study, you are agreeing to three distinct aspects of data gathering. First of all, comments made by you in class which the researcher deems pertinent can be video or audio recorded, transcribed, and used as data. Secondly, homework which is relevant to the project can be photocopied and used as data. Thirdly, you agree to participate in at least one interview which takes place outside of the normal class hours. The interview will be scheduled at a mutually convenient time and place; it will be video recorded, and will last approximately one hour. The transcripts from this interview can also be used as data.

You as a prospective teacher will gain a direct benefit from a deeper exploration of your own ideas about this key statistical concept; this exploration allows you to extend your own learning about variation in the non-evaluative environment of the research project. Moreover, the practice in articulating your thinking is especially helpful as you make the transition to your own classroom, and invoke similar practices with your own students. Potential risks include the possibility that an unauthorized person may view the data, or that your actual name may inadvertently become associated with the data. To minimize this risk, all written responses, notes, audio or video tapes, and transcriptions will be kept confidential, and will be kept locked up in the researcher’s office in the Department of Mathematical Science at PSU. After three years, these records will be destroyed. In writing any results for the study, pseudonyms will be used so that your identity cannot be matched with the responses you have provided.

Your participation in this study is voluntary and you are completely free to withdraw from the study at any time. Your decision to participate or not will not affect your relationship with the researcher or with any academic program at PSU in any way. If you have concerns about your participation in this study or your rights as a research subject, please contact the Human Subjects Research Review Committee, Office of Research and Sponsored Projects, 111 Cramer Hall, Portland State University, (503) 725-8182. If you have any questions about the study itself, please contact Daniel Canada, at the Department of Mathematical Sciences, 334 Neuberger Hall, Portland State University, (503) 725-3621.

Your signature indicates that you have read and understand the above information and agree to take part in this study. Please remember that you may withdraw your consent at any time without penalty. Also, by signing, you are not waiving any legal claims, rights or remedies. The researcher has provided you with a copy of this form for your records.

Signature of Participant

Date

Daniel Canada, Researcher Date
Department of Mathematical Sciences
Portland State University
(503) 725-3621

Appendix B

Class Activities: Instructor's Notes

Looking at Data Sets

Materials Needed:

Poster paper

Markers

Measuring tape (cm)

Meter sticks

Scratch paper

Watches with second hands.

Data from previous classes

Procedure

1. Every student needs 9 half-sheets of paper for recording data. Have everyone find and record on a half-sheet his or her:

Head circumference	}	(To nearest half-cm)
Height		
Armspan		
Pulse rate per minute	}	(Count for a whole minute)

Copy this data down so we have 9 half-sheets, each with the same data.

2. Before passing out all the data, discuss in small groups:
 - a. What do you think will be the range for the class data in each category ?
 - b. What do you think will be a reasonable interval of heights to capture the middle 50% of the class data in each category ?
3. Make sure one half-sheet goes on each table, and one to the instructor. Now, each table and the instructor has a stack of class data. Here are the graphs each table must produce on poster paper:

Table #	Data	Type of Graph
1	Arm	Boxplot
	Height	Histogram (Use a 2cm interval width)
2	Arm	Histogram
	Head	Boxplot
3	Arm	Boxplot
	Pulse	Histogram
4	Height	Boxplot
	Head	Histogram
5	Height	Boxplot
	Pulse	Histogram (Use a 2-beat interval width)
6	Head	Histogram
	Pulse	Boxplot
7	Arm	Boxplot
	Height	Histogram (Use a 4cm interval width)
8	Head	Boxplot
	Pulse	Histogram (Use a 4-beat interval width)

They also must provide mean, median, and mode for each set of data. Meanwhile, instructor types in at least Pulse and Height data into TI-83.

4. Put the completed graphs up around the room, so that the categories are near each other (ie, put Armspan graphs together). Ask students to discuss in their groups the following:

- A. Look at the graphs for the data in each category.
What features of the graphs do you notice?
What questions do you wonder about?
- B. Find the median height for the class. Now look at the graphs for the height data.
Would you say the median height is typical for a member of the class?
- C. Compare the boxplot and the histogram for the head circumference data.
Which graph better shows how the data are spread out from the median?
- D. With the height data, compare two histograms of different interval widths.
Which one does a better job of disguising the differences among the data?
- E. How alike or unlike would you say our head circumferences are?
- F. Does it look like there is more variation in one set of data versus another set?
- G. Why aren't the armspan data closer to one another?
- H. Let's suppose another person joins the class.
How confident are you that the person's head circumference is within 2cm of the class mean height?

5. After it seems that most tables have had a chance to discuss all these questions, it is a good time to take a break. Then, discuss as a class what they notice and wonder about the graphs and the data. Use A – H above as a starting point. Themes to aim for in discussion include:

- *How well graphs do or do not display variation in the data*
- *Typical values and extreme values.*
- *What it means for (mean, median, mode) to represent an entire set of data. How the spread of the data also matters.*
- *How to describe variation.*
- *Reasons for the variation.*
- *Using both centers and spread to make predictions.*

Bring up some graphs on the TI-83 if that is needed to highlight different types of graphs.

6. The last part of this activity moves the class discussion into comparing different sets of data. Hand out graphs of data from previous classes, and ask the students to compare their own class with the previous class. Sample questions to get started include:
- a. On average, which class had faster pulse rates?
 - b. In which class were the heights more spread out?

The M&M Investigation

Materials Needed:

- 16 Regular-size (1.69 ounce) bags of M&Ms
- Stack of 2cm grid paper
- Markers

Procedure:

1. Each table receives two bags. Before opening the bags, have students make conjectures about what is inside their bag:
 - a. About how many pieces of candy do you think are inside?
 - b. Which color do you think is predominant?
 - c. What percentage of the bag is represented by each color?
2. Now open the bags, and have students write down the total number of pieces, and the frequency for each color. For each bag, construct a bar chart which shows the relative frequency of each color type (red, orange, yellow, blue, green, and brown). Before putting the graphs up in front of the class, discuss in small groups these questions:
 - a. What do you think will be the range of class data for the percentage of reds in each bag? How about the range of class data for number of pieces in each bag ?
 - b. What do you think will be a reasonable interval for number of pieces in a bag to capture the middle 50% of the class data?
3. Put all 16 individual bag charts across the top of the chalkboard. Have students discuss in small-groups:
 - a. What features of the graphs do you notice?
 - b. What questions do you wonder about?
 - c. How alike or unlike would you say our individual bag data are?
4. Have each table come up with a boxplot and a barchart which uses only the data of the percentage of reds in each of the 16 individual bags. Have students discuss in small-groups:
 - a. Do you think that the median percentage of reds is typical for an individual bag ?
 - b. Compare the boxplot and the bar chart for the data on percentage of red. Which graph better shows how the data are spread out from the median?
 - c. Why aren't the data for percentage red in an individual bag closer to one another?
 - d. Let's suppose we open up one more individual bag. How confident are you that the percentage of red in the bag will be within 2% of the class mean?

5. Now have students aggregate the data. Different teams can form and concentrate on making graphs for the following levels of aggregation:

- Combining two bags at a time (produces 8 graphs)
- Combining four bags at a time (produces 4 graphs)
- Combining eight bags (produces 2 graphs)
- Combining all bags (produces 1 graph)

Get these graphs up on the chalkboard so that five rows are formed, a row of 16 graphs (the individual bags), a row of 8 graphs, 4 graphs, etc.

6. In small groups, have students discuss the following:

- a. How alike or unlike are the data for the aggregate bags ?
- b. Does it look like there is more variation in the individual bag data or in the aggregate bag data ?
- c. What do you think is the company's distribution for the colors now that you've seen the data?

7. After it seems that most tables have had a chance to discuss all these questions, it is a good time to take a break. Then, discuss as a class what they notice and wonder about the graphs and the data. Use the questions above as a starting point. Themes to aim for in discussion include:

- *How to describe variation.*
- *Reasons for the variation.*
- *Using both centers and spread to make predictions*
- *How sample size affects variation between samples*

Then reveal what the company's distributions are. Talk about

- *Whether individual bags or aggregate bags are closer to the company distribution.*
- *How sample size affects the extent to which the sample proportions match the population proportions*

The River Crossing Game

Materials Needed:

Copies and overheads from *The River Crossing Game* (Unit VIII , Visual Encounters with Chance, Activity 3. By M. Shaughnessy & M. Arcidiacono. (1993). *Math and the Mind's Eye*. The Math Learning Center. P.O. Box 3226, Salem, OR 97302. Tel (503) 370-8130. Fax (503) 370-7961).

Dice

Colored chips

Procedures:

1. Pass out the Game Board, Arrangement Sheets, and Line Plot Grids. Explain the game by demonstrating on the overhead. Before passing out dice and chips, have students make a few conjectures about winning arrangements, and record these arrangements.
2. Pass out the dice and chips and have students play the game for awhile. Looking around the room, and after most teams have played about four games, then it is about time to stop playing. As a whole class, discuss
 - Thoughts about winning arrangements. Is there a “best” arrangement or does it depend on what arrangement you are playing against?
 - Which numbers came up the most or the least frequently.
 - How to find experimental and theoretical probabilities.
3. Have the teams (should be two teams per table) figure out their experimental probabilities for getting a “2” through “12” , and also compute the corresponding theoretical probabilities. As a table, small groups should discuss:
 - a. What do you think will be the class range for the experimental probabilities of getting a “7”?
 - b. What do you think will be a reasonable interval to capture the middle 50% of the class data for the experimental probabilities of getting a “7”
4. Put all team line plots across the top of the chalkboard. Have students discuss in small groups:
 - a. What features of the graphs do you notice?
 - b. What questions do you wonder about?
 - c. How alike or unlike would you say the team line plots are?

5. Have each table come up with a boxplot and a barchart which uses only the data for the experimental probability of getting a “7” in each of the team line plots. Have students discuss in small groups:
 - a. Would you say that a probability of $1/6$ is a typical experimental probability to obtain?
 - b. Compare the boxplot and the bar chart for the class data on experimental probabilities of getting a “7” . Which graph better shows how the data are spread out from the median?
 - c. Why aren't the data for the experimental probabilities of getting a “7” closer to one another?
 - d. Let's suppose we roll the dice 60 times. How confident are you that the number of “7”s will be between 8 and 12 ?

6. There are only about 16 values used in the graphs of Step 5 above. That is, every team's line plot data was used to find experimental probabilities of getting a “7”. Have tables figure out what the experimental probabilities of getting a “7” would be if they aggregate the team line plots to simulate a larger number of trials. Produce a graph for the class if two or three team line plots combine their trials and outcomes. Have the tables discuss among themselves:
 - a. Does it look like there is more variation for the graphs depicting experimental probabilities which are based on a larger or smaller number of total trials?
 - b. Let's suppose we roll the dice 600 times. How confident are you that the number of “7”s will be between 80 and 120 ?

7. After it seems that most tables have had a chance to discuss all these questions, it is a good time to take a break. Then, discuss as a class what they notice and wonder about the graphs and the data. Use the questions above as a starting point. Themes to aim for in discussion include:
 - *How to describe variation.*
 - *Reasons for the variation.*
 - *Using both centers and spread to make predictions*
 - *How the number of trials affects the extent to which the experimental probabilities match the theoretical probabilities.*

Conclude with any last thoughts on what an ideal winning arrangement should look like, or even if one exists.

Appendix C
Pre-Activity Documents

Informational Survey

Name :

Age:

1. Where & when did you take Math 211?

2. What other math courses have you taken in which probability and/or statistics was taught?
 - a. If there were such courses, then when & where were they taken ?

 - b. How did you feel about the probability and/or statistics at that time?

3. How comfortable do you feel about learning probability and/or statistics now ?

4. Of what use to your life do you imagine there might be in learning about...
 - a. Probability ?

 - b. Statistics ?

Pretest on Variation

1. a) What does the word “sample” mean to you?

(i) What are some characteristics of a “sample” ?

(ii) What is the use of a “sample” ?

b) What does “random” mean to you ?

(i) Give an example of something that happens in a “random” way.

c) What does “variation” mean to you ?

(i) Use the word “variation” in a sentence.

(ii) Give an example of something that “varies”.

2. Consider how long a person might wait at a transit station for MAX, Portland’s light rail system. Suppose that the following eight intervals of time between arrivals of the westbound train are measured and reported (in minutes): 7.0, 7.0, 7.0, 8.0, 10.5, 13.0, 13.0, 14.5. For the eastbound train, these eight measurements are reported (in minutes): 8.5, 9.0, 9.0, 9.5, 10.5, 11, 11, 11.5. The data are displayed graphically below (Fig. 1) :

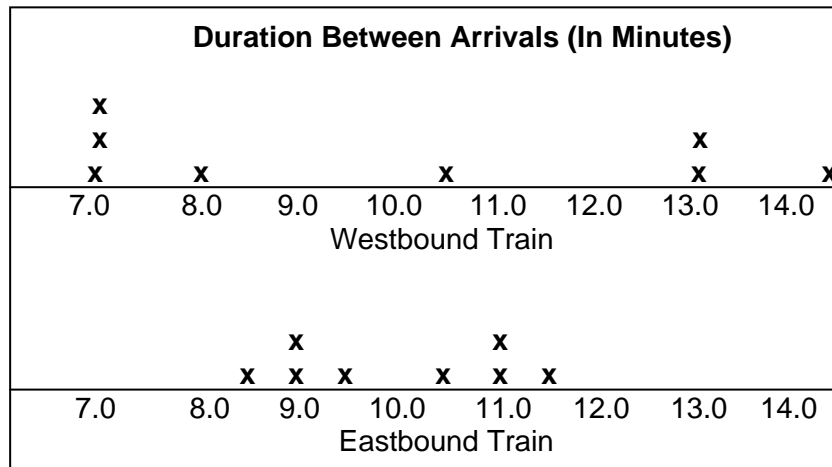


Fig. 1

a. Which of the two trains do you judge to be more consistent?

b. Which of the two sets of data do you think has more variability ?

3. A town has two hospitals: In the Large Hospital, an average of 60 babies are born each day. In the Small Hospital, an average of 20 babies are born each day. Boy babies and Girl babies each make 50% of the total baby population in town.

One year, each Hospital kept a record of how many days in the year the Daily Birth Rate of Boys was over 70%. At the end of the year, the two Hospitals looked at each other's records.

- a. Do you think that the Large Hospital had more of those days (in which Daily Birth Rate of Boys was over 70%), or that the Small Hospital had more of those days, or that the two recorded about the same number of those days?
- b. Explain how or why you chose your answer.

4. a. Do you think you'd be more surprised to see one or the other of these sequences as the result of rolling a die 18 times?

i) 2 1 1 3 4 1 5 5 6 2 2 4 3 4 6 5 1 3

ii) 1 1 1 1 1 1 1 1 1 1 1 2 2 1 1 1 1 1

iii) 1 3 4 2 5 6 6 5 2 3 1 4 2 3 1 6 4 5

- b. Why or why not ?

- c. Complete this sentence: "I would be very surprised if I rolled a die 100 times, and got LESS THAN _____ 6s or MORE THAN _____ 6s ." Why did you choose those numbers?

Appendix D

Post-Activity Documents

Homework #1. Context: Variation in Data Sets
(Follows activity *Looking at Data Sets*)

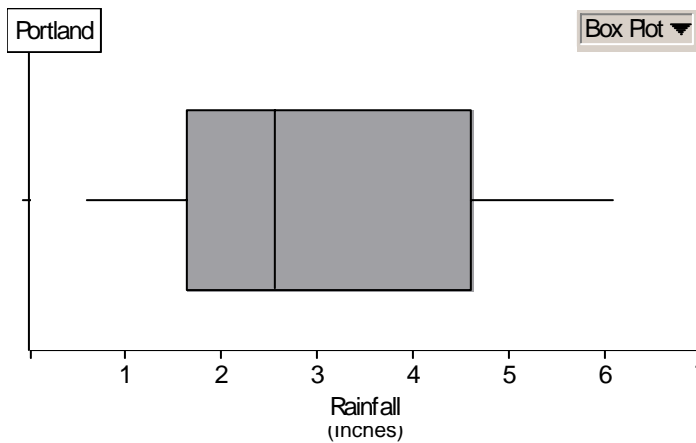
Reflections on Data Displays

WEATHER

You can use the spaces provided to write down your responses

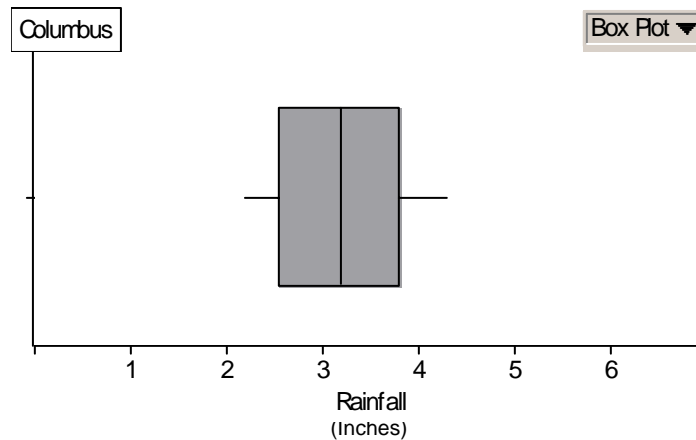
A] The following data was taken from a National Weather Service. They kept records of the rainfall in Portland, for example, to see how much rain fell in each month. After 30 years, they averaged the amounts of rainfall in each month: This is called the, average, or Normal Rainfall for the 30-year period. Look at the following data comparing Portland OR to Columbus OH:

PORTLAND AVERAGES (1961-1990)



Month of Year	Normal Rainfall (In.)
J	5.4
F	3.9
M	3.6
A	2.4
M	2.1
J	1.5
J	0.6
A	1.1
S	1.8
O	2.7
N	5.3
D	6.1

COLUMBUS AVERAGES (1961-1990)



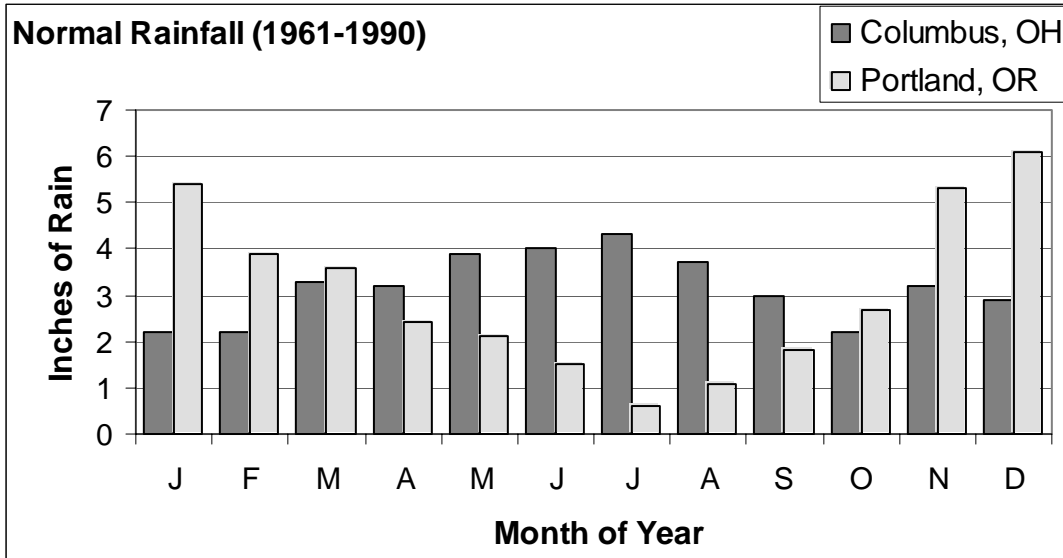
Month of Year	Normal Rainfall (In.)
J	2.2
F	2.2
M	3.3
A	3.2
M	3.9
J	4
J	4.3
A	3.7
S	3
O	2.2
N	3.2
D	2.9

i) What do the above graphs tell you about the variation in rainfall throughout the year in the two cities?

ii) Matt hates rainy weather, but he has to choose between living in Columbus or Portland. Based only on the data, which city do you advise him to avoid: That is, which city would you characterize as the rainier city? Why?

iii) Adam and Zain are two Math 212 students who answered the previous question. Adam said Portland was rainier because sometimes it got over 6 inches of rain a month. Zain said Columbus was rainier because the average monthly rainfall was higher than Portland. What do you think of Adam and Zain's reasoning?

B] Now look at the same data: This time it is displayed differently.



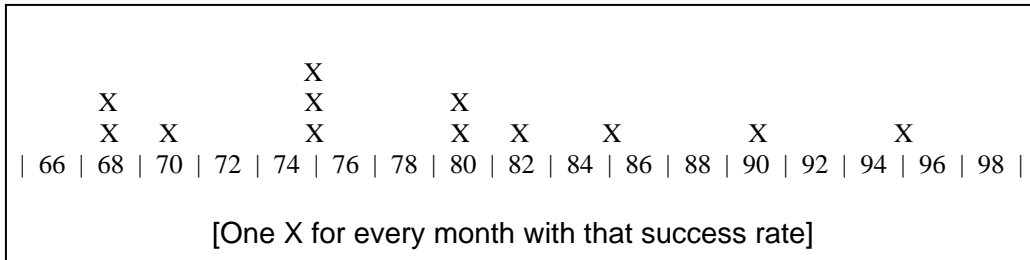
Steve isn't as concerned about which city is rainier, but he likes his rainfall to be consistent: That is, from one month to the next, he likes the amount of rainfall to be as similar as possible to the other months of the year. Based only on the data, which city do you advise for Steve, and why?

C] One of Portland's TV Meteorologists posts on the web a track record of success rates in predicting the weather. For example, if he makes a prediction for tomorrow's high temperature, he counts it as a success if he ends up being within 3 degrees of the actual high temperature. Then, at the end of the month he calculates (Number of Successes) / (Total Predictions for the Month), which he then lists as a percentage of the time he was successful.

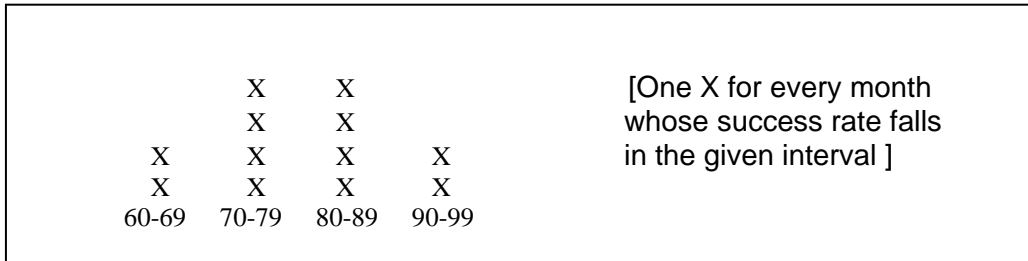
Rick tried his own skill at predicting the weather for a year; using the method described above, he then listed his own success rates for each month of the year. In a sense, these are his "Grades" showing how good a predictor he is or is not.

Rick's data : J F M A M J J A S O N D
 68 68 70 75 75 75 80 80 82 85 90 95

Rick is trying to decide which of the following two graphs to use to display his record:



Graph #1



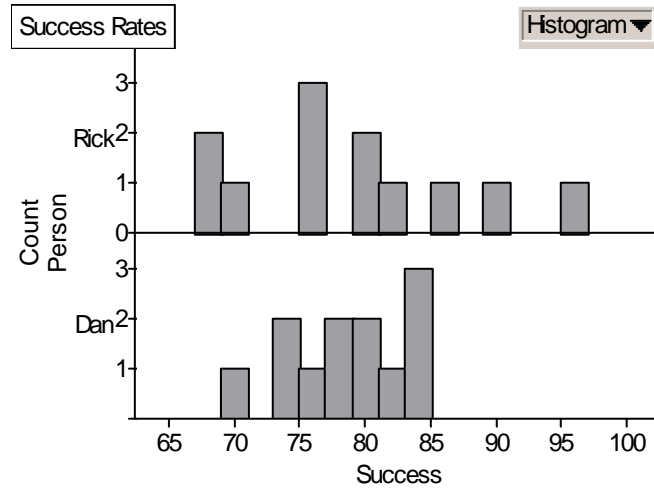
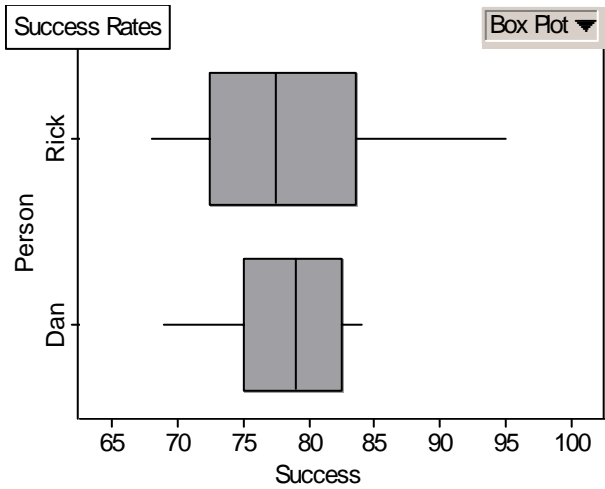
Graph #2

Which of the two graphs shown do you feel does a better job of telling the story of how good Rick is at predicting the weather, and why?

D] Dan also tried his skill at predicting the weather, and listed these success rates for the same twelve months of the same year as Rick:

Dan's data : J F M A M J J A S O N D
 69 74 74 76 78 78 80 80 82 83 84 84

The following graphs summarize Rick's data and also Dan's data:



i) Which person is more consistent in their success rates ? Why?

ii) Which person do you think is the better predictor of the weather, and why?

Homework #2. Context: Variation in Sampling
(Follows activity *The M&M Investigation*)

Reflections on Sampling

M&Ms

Use the spaces provided to write down your thoughts

- A] Regular-size bags only seem to hold about 57 pieces or so of candy. Looking at the individual bag charts, some of the percentages for the color Red looked like, for example, 8%, 15%, 18%, 25%...etc.

Why do you think there were so few (or none) of our individual bags showing a percentage of 20% Red, which company suggests is the “true percentage” ?

- B] Let's say we looked inside of 1000 individual regular-sized bags of M&Ms.

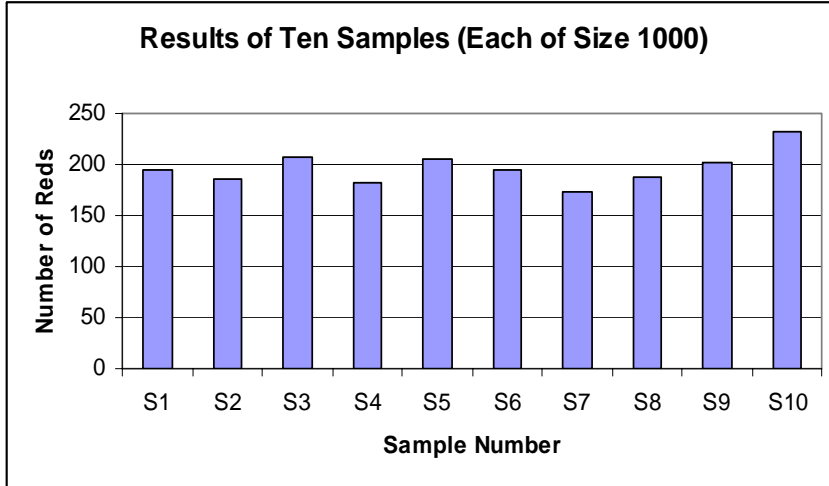
i) About how many of these 1000 bags would you think had between 16% to 24% Red ? _____

ii) Why did you pick the answer you did in part i) above?

iii) Would you be surprised if none of the 1000 bags had more than 16% red? Why or why not?

C] When I computed a grand total for both afternoon and evening classes, there were about 1000 pieces of candy. And, our final overall percentage of Red was not exactly 20% (200 Red Pieces) !

A simulation of getting 10 samples of M&Ms is conducted, where each sample contains 1000 pieces of candy. Using the company's claim of a 20% Red mixture, that would be 200 Red pieces expected in every sample of 1000. But in the simulation here are the numbers of Red in each of the 10 samples (S1, S2 ... S10) of 1000:



Sample Number	Number of Reds
S1	195
S2	185
S3	207
S4	183
S5	206
S6	194
S7	174
S8	188
S9	201
S10	233

- i) How many of these ten samples had "Percentage Red" between 16% and 24% ? _____
- ii) How many of these ten samples had "Percentage Red" between 19% and 21% ? _____
- iii) If you took 100 samples, each of size 1000, how many of your 100 samples do you think would have between 19% and 21% Red ? Why?

D] Steve and Kate each had a sample of M&Ms. Steve's sample contains 50 pieces of candy and Kate's contains 175 pieces.

One of the samples has a "Percentage Red" of less than 10% ! Do you think that this sample is

- ...more likely to belong to Steve, or
- ...more likely to belong to Kate, or
- ...equally likely to be Kate or Steve's sample. Explain why you think so.

E] Matt used regular-sized bags in his class. There are about 55 pieces of candy in each bag. Matt's class claims to have opened 15 regular-size bags of M&Ms, and they say that the following list gives the "Percentage Red" in each of the 15 bags:

18.2%	19.6%	21.3%	20.9%	19.1%
20.5%	18.3%	20.6%	19.2%	22.1%
21.8%	18.6%	19.8%	21.5%	17.9%

Do you think it is likelier that

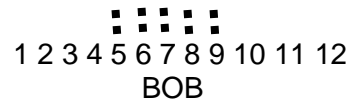
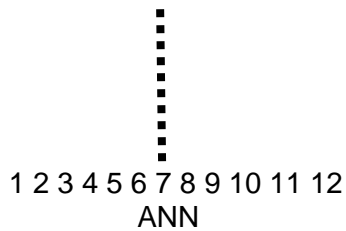
- ...Matt's class just made up these results, or
- ...those are in fact the actual results, or
- ...no one can have much confidence in saying if the results are made up or not.

Explain your reasoning:

Homework #3. Context: Variation in Chance Situations
(Follows activity *The River Crossing Game*)

Use the spaces provided to write down your thoughts

- A] Ann and Bob played the River Crossing Game using ten chips each. Their starting arrangements are shown below. (Ann put all ten chips on 7, while Bob put two chips each on 5, 6, 7, 8, and 9).



They played one hundred games, each using exactly the same arrangement. Write down how many of the games you think each player won:

ANN: I think Ann won _____ out of the hundred games

BOB: I think Bob won _____ out of the hundred games

- i) Why did you answer the way you did? _____

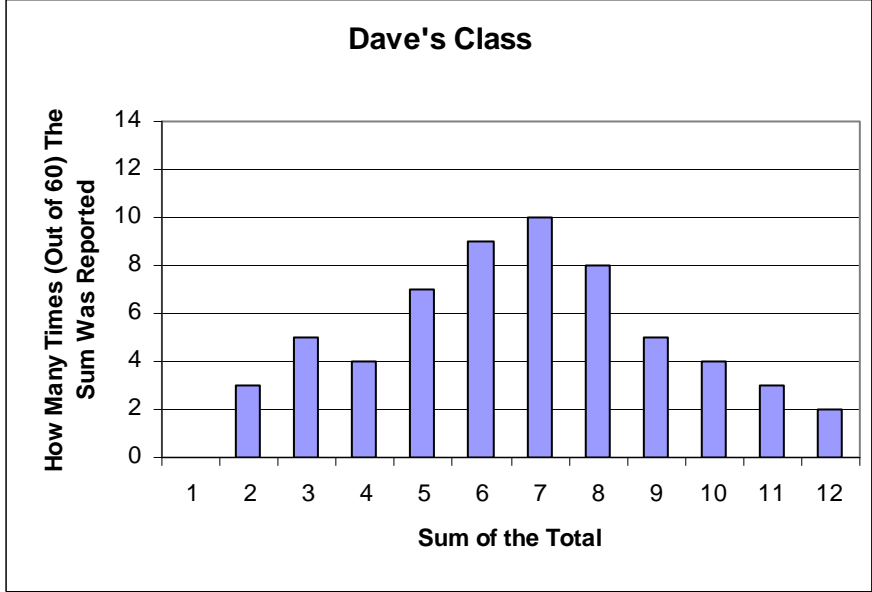
- ii) If you were playing against Bob's arrangement, then what arrangement would you choose to play :

1 2 3 4 5 6 7 8 9 10 11 12

- iii) If you were playing against Ann's arrangement, then what arrangement would you choose to play:

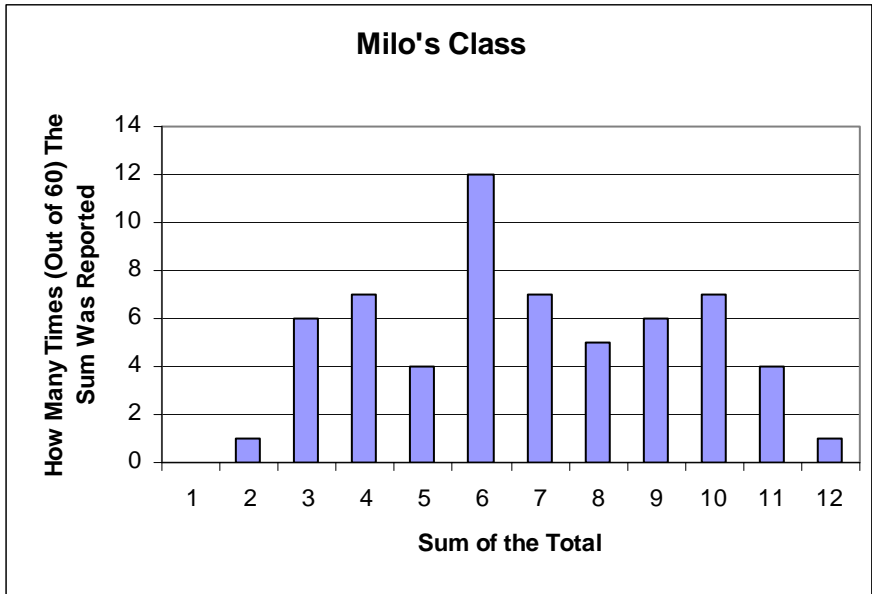
1 2 3 4 5 6 7 8 9 10 11 12

Dave went for a snack, leaving his students to do the work. When he returned he saw this chart:



Sum	How Many
2	3
3	5
4	4
5	7
6	9
7	10
8	8
9	5
10	4
11	3
12	2
Total:	60

Milo also taught a Mth212 class of 30 students. He did exactly the same as Dave in his class, and when he returned from break, he saw the following chart:



Sum	How Many
2	1
3	6
4	7
5	4
6	12
7	7
8	5
9	6
10	7
11	4
12	1
Total:	60

In reality, however, only one class actually did the 60 tosses. The other class decided to not bother doing any tosses, and instead they just made up some results that they thought would be about right.

Which class so you think made up the data, and explain your reasons: _____

- E] Later that year, Dave and Milo attended the National Conference for Mth212 Teachers. They gave a joint presentation, and shared how Dave's class reported 10 out of 60 tosses having a sum of seven, and how Milo's class only reported 7 out of 60 having a sum of seven.

At the Conference, it turns out that 100 other instructors had similar stories: they miraculously all asked their classes to generate 60 tosses and count how many sums were seven.

So, of the 100 classes, each of which did 60 tosses, which of the following seems more reasonable, and why?

- i) 80 % or more of the classes had exactly 10 (out of 60) sums of seven.
- ii) 80% or more of the classes had from 8 to 12 (out of 60) sums of seven.
- iii) Some other result, such as:

Explain: _____

Appendix E

Interview Protocol

- 1] a) In the questions you answered about the Weather (*Variation in Data Sets*), can you explain any more to me about what you put down in response to Questions A through D?

Probes: (*These are informed by their written responses*)

- b) What do you notice about the graphs in each of Questions A through D ?

Probes: (i) What can you tell about the spread of the data?

- c) What influences your choice of which city is rainier, or which weatherperson is more accurate?

2. a) In the questions you answered about the M&Ms (*Variation in Sampling*), can you explain any more to me about what you put down in response to Questions A through E?

Probes: (*These are informed by their written responses*)

- b) If I had about 100 individual bags, and I plotted a histogram showing how many bags had which "Percentage Red", what do you think my graph might look like?

Probes: (i) Can you draw it for me?
(ii) Why do you think it would look like that?

- c) Let's say you grab an individual bag. How sure are you that the bag you have has between 15% and 25% Red?

Probes: (i) Why aren't you more sure?
(ii) How about between 18% and 22% Red?

3. a) In the questions you answered about the River Crossing Game (*Variation in Chance Situations*), can you explain any more to me about what you put down in response to Questions A through E?

Probes: (*These are informed by their written responses*)

- b) When we looked at this in class, we discovered that the "Theoretical Probability" of getting a "7" from the roll of a pair of dice was $\frac{1}{6}$. What does this mean?

- c) Let's say I roll a pair of dice six times, and I don't get any "7"s. Should I be surprised?

Probes: (i) How many rolls would it take of getting no "7"s for you to be surprised?