

ELEMENTARY PRESERVICE TEACHERS'
CONCEPTIONS OF VARIATION

by
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DISSERTATION APPROVAL

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ABSTRACT

This purpose of my study was to research elementary preservice teachers' (EPSTs') conceptions of variation. Given that variation is an essential characteristic of both probability and statistics, there have been calls for more research on the teaching and learning of variation. The research previously conducted on variation has been aimed mostly at school-age students. Very little has been known about how preservice teachers think about this concept. This project not only builds upon previous studies, but extends the work in stochastic teaching and learning to an area where new research has been needed.

My two main research questions were: One, what conceptions of variation are held by EPSTs in the three contexts of data sets, sampling, and chance situations? Two, how can the conceptions of variation held by EPSTs in these three contexts be characterized? To gather data for my study, I gave written surveys to a class of 30 students who were taking a course in mathematics for prospective elementary teachers. They completed surveys both before and after a series of classroom interventions designed to explore and discuss variation. Also, I interviewed six of the students, two times each: Once before the classroom interventions and once afterwards.

Using the survey and interview data, I developed an Emergent Framework which describes three aspects that characterize thinking about variation: Expecting, displaying, and interpreting variation. Within each

aspect, I identified dimensions which I then described in terms of different themes. The majority of my analysis was given to adding depth to the different aspects, dimensions, and themes which make up the Emergent Framework. The Framework, grounded in the survey and interview data, addresses the research questions by providing a richly detailed structure for characterizing EPSTs' conceptions of variation.

For further research, I suggest describing interactions within the Emergent Framework, and recommend studies that focus on the teaching of variation within university courses for prospective teachers. I also suggest a comparative look at both the conceptions of children and their prospective teachers.

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