

Electromagnetics Theory

Transmission Line (Part 1)

Outline:

- 7.1 General Consideration (W13)
- 7.2 Lumped-Element model
- 7.3 Transmission line (TL) equation (W14)
- 7.4 Wave propagation on TL
- 7.5 Lossless TL (W15)
- 7.6 Input impedance of lossless line
- 7.7 Special cases of lossless line

7.1 General Consideration

- TL consist of structure & media which serve to transfer energy /information between 2 points.

Ex: - nerve fiber in human body
 - acoustic wave in fluids
 - mechanical pressure wave in solids.

- In this chapter, focus on TL used for guiding EM signals.

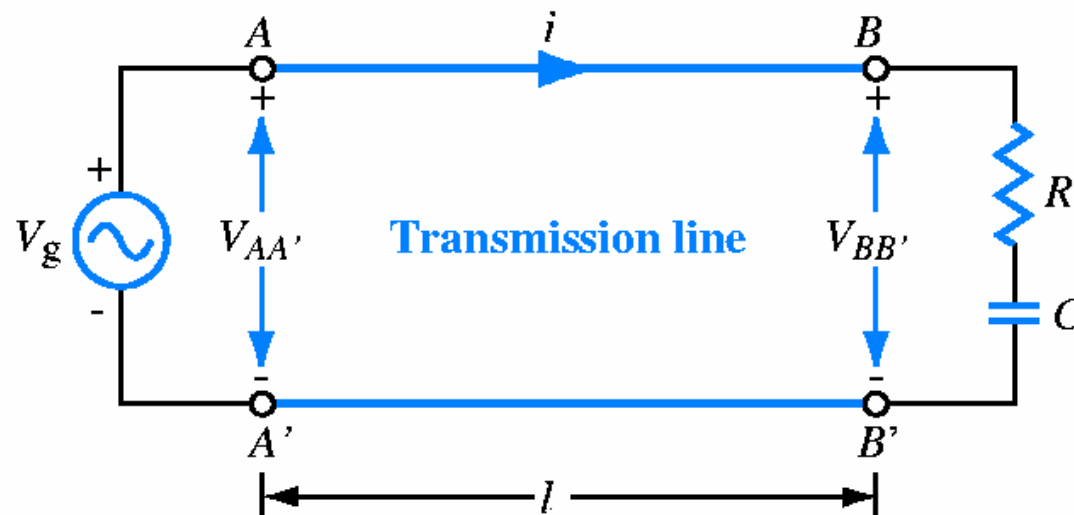
Ex: - telephone wires
 - coaxial cable
 - optical fiber carrying light wave

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- Fundamentally TL is a two-port network, each port consist of two terminal



7.1.1 Role of wavelength

- In low freq electrical circuit , wires are used to connect the elements of the circuit in desired config.
- Figure below shows the generator connected to a simple RC load via a pair of wires (TL) of length l .



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- If generator voltage is cosinusoidal in time, then the voltage across input terminal AA' is

$$V_{AA'} = V_g(t) = V_o \cos \omega t \quad (V)$$

Where

angular freq, $\omega = 2\pi f$

current flow thru wire with speed of light, c

$$c = 3 \times 10^8 \quad m/s$$

Thus

$$V_{BB'} = V_{AA'}(t - l/c)$$

Travel delay time

l/c

$$V_{BB'} = V_o \cos[\omega(t - l/c)](V)$$

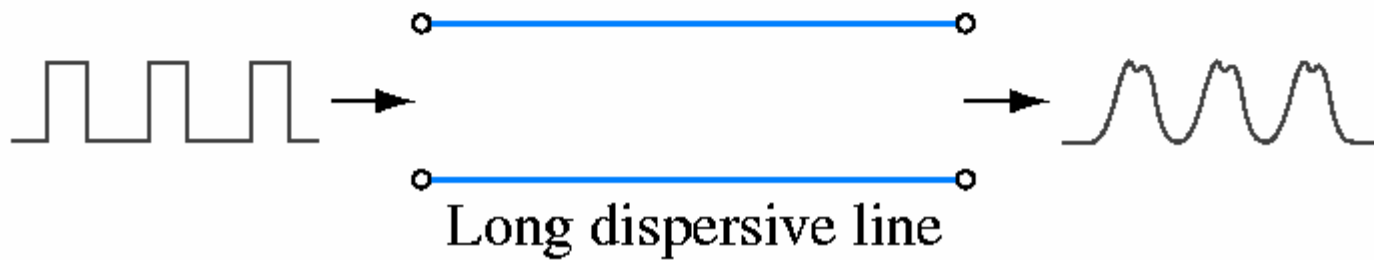
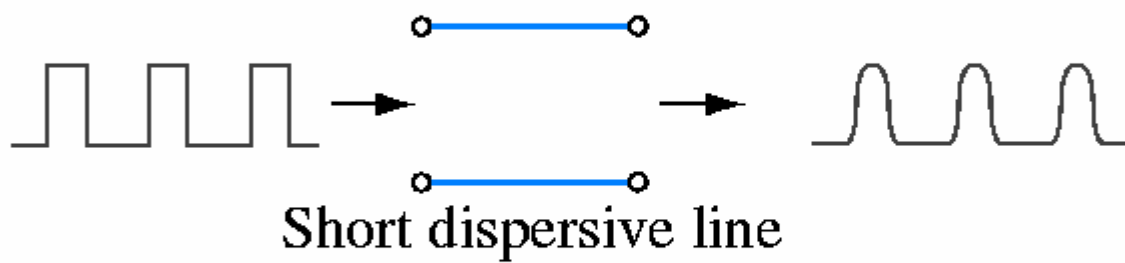
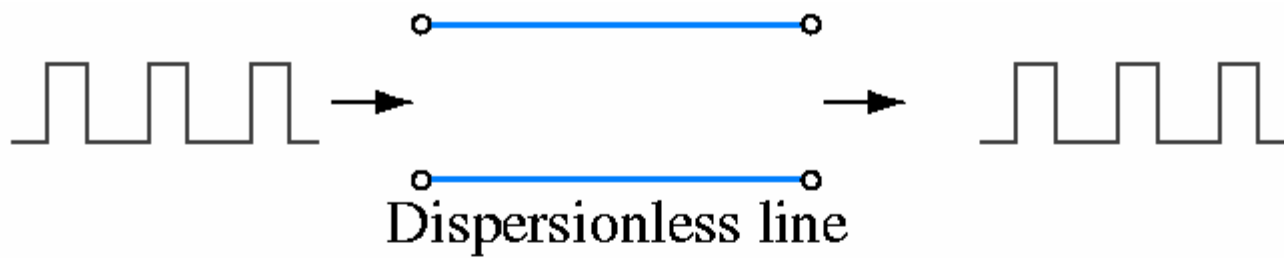
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- velocity of propagation u_p of a traveling wave is related to the oscillation freq, f and the wavelength λ by

$$u_p = f\lambda \quad (m/s)$$

- In present case, $u_p = c$. Hence, the phase factor

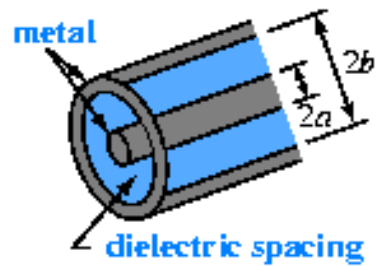
$$\frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \text{ radians}$$

- When $\frac{l}{\lambda}$ is very small, TL effects may be ignored.
- But $\frac{l}{\lambda} \geq 0.01$, present of reflected signal, power loss & dispersive effects need to be considered.

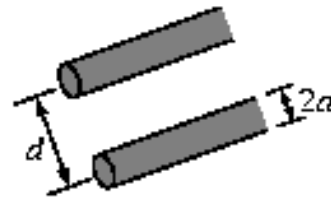


7.1.2 Propagation Modes

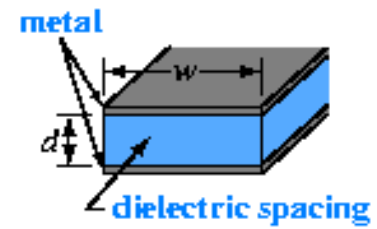
- TL can be classified into 2 basic types:
 - **Transverse electromagnetic (TEM) transmission lines**
 - Characterized by electric & magnetic fields that are entirely transverse to the propagation – **TEM mode**.
 - Ex: coaxial line, two-wire line & parallel-plate line.
 - **Higher-order transmission line**
 - Wave propagating along this line have at least one significant field component in direction of propagation.



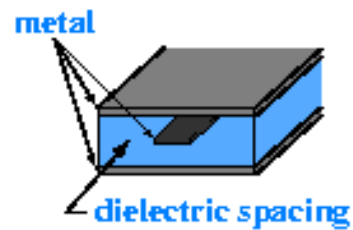
(a) Coaxial line



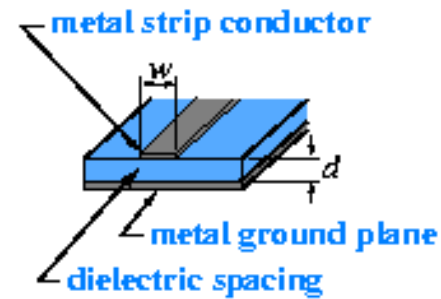
(b) Two-wire line



(c) Parallel-plate line

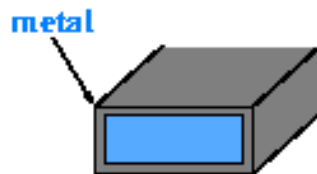


(d) Strip line

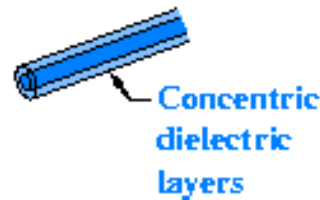


(e) Microstrip line

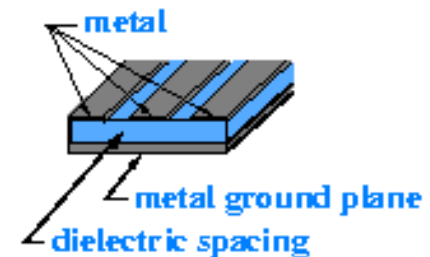
TEM Transmission Lines



(f) Rectangular waveguide



(g) Optical fiber



(h) Coplanar waveguide

Higher Order Transmission Lines

Figure 7-4

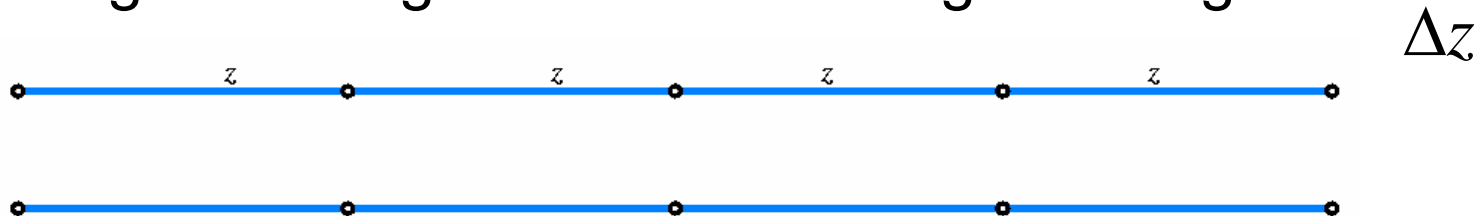
7.2 Lumped-Element model

- represent a TL by a parallel-wire configuration, regardless of the specific shape of the line under consideration.
- May represent a coaxial line, two-wire line or other TEM line.



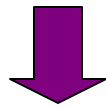
(a) Parallel-wire representation

- Orienting line along z-dir & subdividing into length of Δz

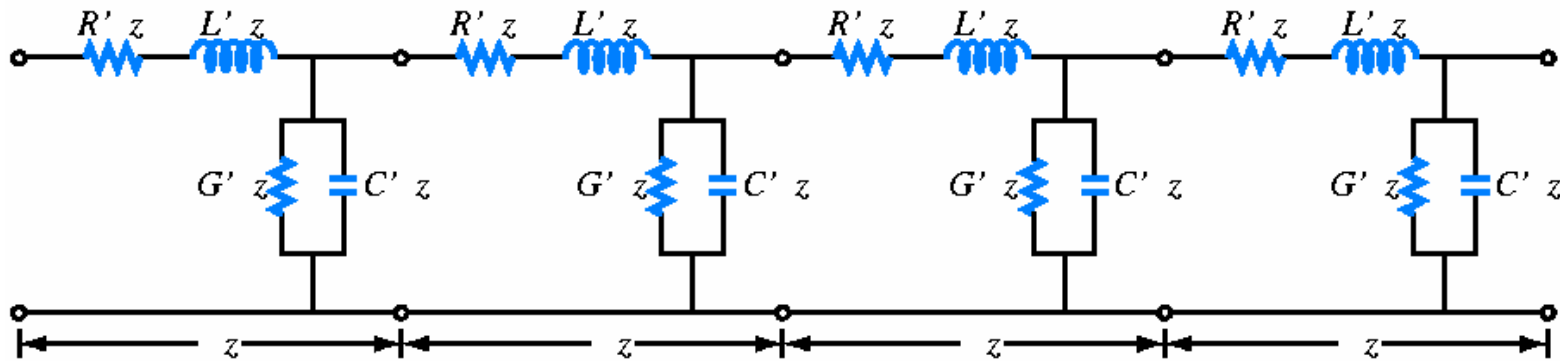


(b) Differential sections each z long

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- subdividing into length of Δz & representing each section by equivalent circuit



lumped element circuit
(consist of 4 basic element)



(c) Each section is represented by an equivalent circuit

TL parameters

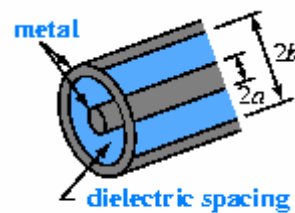
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- Basic elements:

R' : combined resistor of both conductors per unit length Ω / m

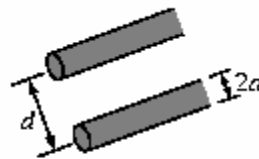
L' : combined inductor of both conductors per unit length H / m

G' : combined conductance of the insulation medium per unit length S / m

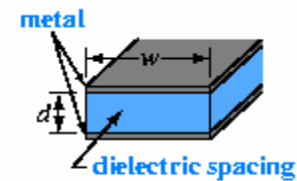
C' : combined capacitance of both conductors per unit length F / m



(a) Coaxial line



(b) Two-wire line



(c) Parallel-plate line

Table 7-1: Transmission-line parameters R' , L' , G' , and C' for three types of lines.

Parameter	Coaxial	Two Wire	Parallel Plate	Unit
R'	$\frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi} \ln(b/a)$	$\frac{\mu}{\pi} \ln \left[(d/2a) + \sqrt{(d/2a)^2 - 1} \right]$	$\frac{\mu d}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln \left[(d/2a) + \sqrt{(d/2a)^2 - 1} \right]}$	$\frac{\sigma w}{d}$	S/m
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln \left[(d/2a) + \sqrt{(d/2a)^2 - 1} \right]}$	$\frac{\epsilon w}{d}$	F/m

Notes: (1) Refer to Fig. 7-4 for definitions of dimensions. (2) μ , ϵ , and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c / \sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(d/2a)^2 \gg 1$, then $\ln \left[(d/2a) + \sqrt{(d/2a)^2 - 1} \right] \simeq \ln(d/a)$.

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- all TEM TL share the following useful relation:

$$L' C' = \mu \varepsilon \quad \frac{G'}{C'} = \frac{\sigma}{\varepsilon}$$

- Perfect conductor $\Rightarrow \sigma_c = \infty, R' \approx 0$

- Perfect dielectric $\Rightarrow \sigma = 0, G' = 0$

- If insulating medium between conductor is air, TL is called *Air Line*, ex: coaxial cable or two wire air line.

$\hookrightarrow \varepsilon = \varepsilon_0 = 8.854 \times 10^{-12} \text{ F / m}, \mu = \mu_0 = 4\pi \times 10^{-7} \text{ H / m}, \sigma = 0, G' = 0$

Exercise 1

Use table 7.1 to compute the line parameter of a two wire air line whose wires are separated by a distance of 2cm, and each is 1mm in radius. The wire may be treated as perfect conductors with $\sigma_c = \infty$.

Exercise 2

Calculate the transmission line parameters at 1MHz for a rigid coaxial air line with an inner conductor diameter of 0.6cm and an outer conductor diameter of 1.2cm. The conductors are made of copper.

Given $\mu_r = 0.999999 H / m \approx 1, \sigma = 5.8 \times 10^7 S / m$ for copper.

To be continued ...