Electromagnetics Theory

Transmission Line (Part 2)

Outline:

7.3 Transmission line (TL) equation

• TL usually connects a source on one end and a load on the other end.

•Develop eq. which describe V & I across TL

function of

time, t

spatial position, z

Time-domain form of TL eq

•Telegrapher's eq. in phasor form:

$$-\frac{d\tilde{V}(z)}{dz} = (R' + jwL')\tilde{I}(z) \quad \ldots\ldots \quad (1)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + jwC')\tilde{V}(z) \quad \ldots\ldots \quad (2)$$

7.4 Wave propagation on TL

- By differentiating both side of eq (1) & (2) wrt z will give Wave eq. for $\tilde{V}(z)$ and $\tilde{I}(z)$, are given by

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

- γ is called the *complex propagation constant* of TL.

$$\gamma = \sqrt{(R' + jwL')(G' + jwC')}$$

- γ consists of a real part α and imaginary part β .

$$\gamma = \alpha + j\beta$$

Attenuation constant (Np/m) — α

Phase constant (rad/m) — β

•With

$$\alpha = \Re e(\gamma)$$
$$= \Re e\sqrt{(R' + jwL')(G' + jwC')} \qquad (\text{Np/m})$$

$$\beta = \Im m(\gamma)$$
$$= \Im m\sqrt{(R' + jwL')(G' + jwC')} \qquad (\text{Rad/m})$$

Characteristic impedance (Z_0)of line:

is equal to the ratio of the voltage amplitude to the current amplitude for each of the traveling wave individually, but it is not equal to the ratio of total voltage $\tilde{V}(z)$ *to the total current* $\tilde{I}(z)$ *, unless one of the two wave is absent.*

$$Z_0 = \frac{(R' + jwL')}{\gamma} = \sqrt{\frac{(R' + jwL')}{(G' + jwC')}}$$

- $\alpha = 0$ ⇨ *lossless line*
- $\alpha \neq 0$ ⇨ *lossy transmission lines*

Exercise 3

An air line is a TL for which air is the dielectric material present between the two conductors, which renders G'=0. In addition, the conductors are made of a material with high conductivity so that R'=0. For an air line with characteristic impedance listed below, find the inductance per meter and the capacitance per meter of the line.

$$Z_0 = 50\Omega$$

$$\beta = 20 rad / m$$

$$f = 700 MHz$$

7.5 Lossless TL

• So far we have known two fundamental properties of transmission line

- Propagation constant, γ
- Characteristic impedance, Z_0

• Both are specified by the angular frequency ω and the line parameters R', L', G' and C'

• To minimizes ohmic losses, we need to select conductors with very high conductivities ($R' \ll \omega L'$) and dielectric materials with negligible conductivities ($G' \ll \omega C'$), therefore allow us to set $R' = G' = 0$

- For lossless TL , $\alpha = 0$

- Where we know, $\gamma = \alpha + j\beta$

thus

$$\gamma = j\beta = jw\sqrt{L'C'}$$

$$\alpha = 0$$

$$\beta = w\sqrt{L'C'}$$

$$Z0 = \sqrt{\frac{L'}{C'}}$$

- relations for wavelength λ and phase velocity Up with β

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{w\sqrt{L'C'}}$$

$$U_p = \frac{w}{\beta} = \frac{1}{\sqrt{L'C'}}$$

or

$$\beta = w\sqrt{\mu\varepsilon} \qquad (\text{rad/m})$$

$$U_p = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (\text{m/s})$$

• relation between λ and U_p is given by:

$$\lambda = \frac{u_p}{f} = \frac{c}{f}\frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$\lambda_0 = \frac{c}{f}$$

Wavelength in air corresponding to f

• In the case of TEM lossless medium, the phase velocity is independent of frequency, therefore its is a nondispersive medium.

• A nondispersive medium creates no distortion in transmitting data/ signal, due to the fact that all the phase velocity is the same for all the frequency components

Table 7-2: Characteristic parameters of transmission lines.

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(d/2a) + \sqrt{(d/2a)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(d/a)$, if $d \gg a$
Lossless parallel plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})\,(d/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, a = wire radius and d = separation between wire centers. (4) For parallel-plate line, w = width of plate and d = separation between the plates.

7.5.1 Voltage reflection coefficient

• Consider a transmission line of length *l* connected on one end to a generator circuit and on the other end to a load ZL. The load is located at *z* =0 and the generator terminals are at *z* =-*l*

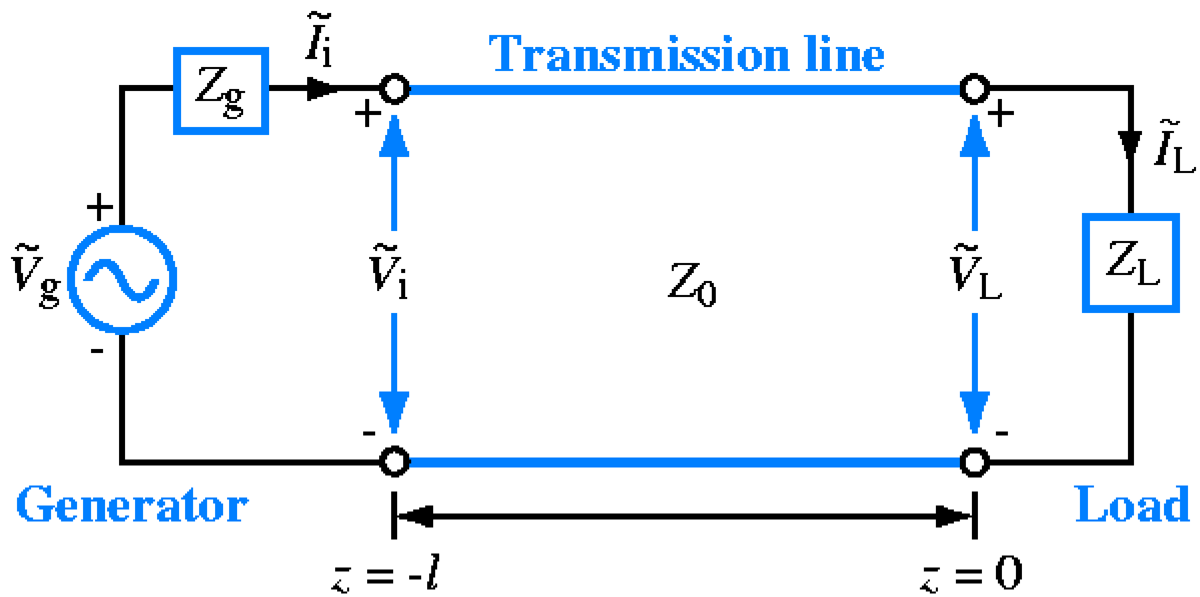


Figure 7-9

• ‘ the ratio of the amplitude of the reflected voltage wave to the amplitude of the incident wave at the load is known as the *voltage reflection coefficient* Γ ‘

Voltage amplitude of incident wave

$$\Gamma = \frac{V_0^{\;-}}{V_0^{\;+}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L / Z_0 - 1}{Z_L / Z_0 + 1} \left(\text{dim}\,ensionless\right)$$

Voltage amplitude of refected wave

• the ratio of the current amplitudes is

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma$$

•Note that Γ is governed by a single parameter, the load impedance Z_L, which is normalized to the characteristic impedance of the line Z_0 .

•Because Z_L is in general a complex quantity, hence Γ may also be complex:

$$\Gamma = |\Gamma| e^{j\vartheta r}$$

Mag. Of $|\Gamma| \leq 1$

Phase angle

- *a load is said to be matched to the line*

if $Z_L = Z_0$ because then there will be no reflection by the load ($\Gamma = 0$ and $V_0^- = 0$)'

- *when load is an open circuit* $(Z_L = \infty)$

$\Gamma = 1$ and $V_0^- = V_0^+$

- *when load is a short circuit* $(Z_L = 0)$

$\Gamma = -1$ and $V_0^- = -V_0^+$

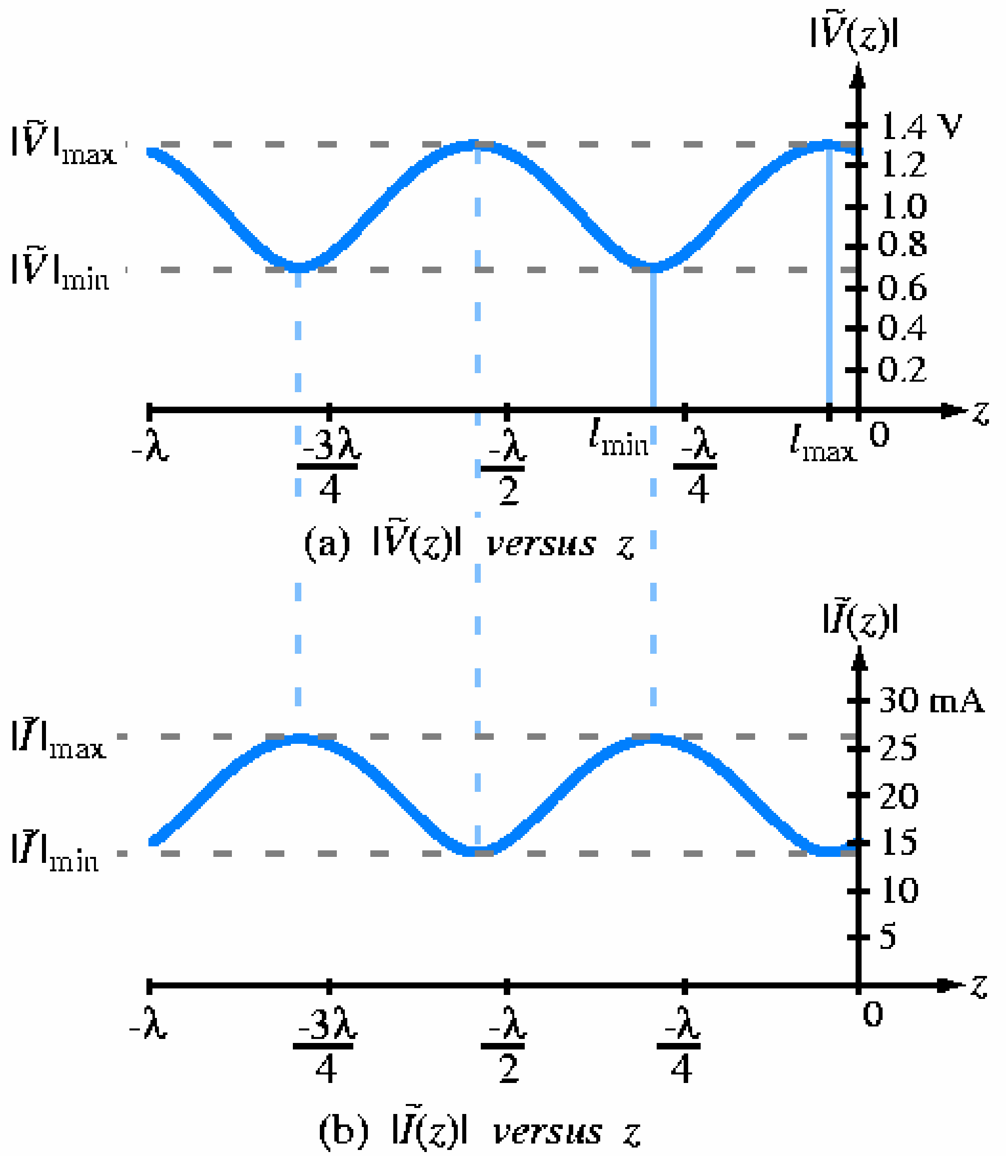
Exercise 4

A 100Ω transmission line is connected to a load consisting of a 50Ω resistor in series with a 10pF capacitor. Find the reflected coefficient at the load for a 100MHz signal.

7.5.2 Standing Waves

- shows the standing-wave pattern for (a) |*V(z)*| and (b) |*I(z)*| for a lossless transmission line of characteristic impedance Z_0=50 , terminated in a load with a reflection coefficient $\Gamma = 0.3e^{j30^0}$ .The magnitude of the incident wave $|V_0^+|$ = 1 V. The standing-wave ratio is

$$S = |\tilde{V}|max/|\tilde{V}|min$$
$$= 1.3/0.7 = 1.86$$



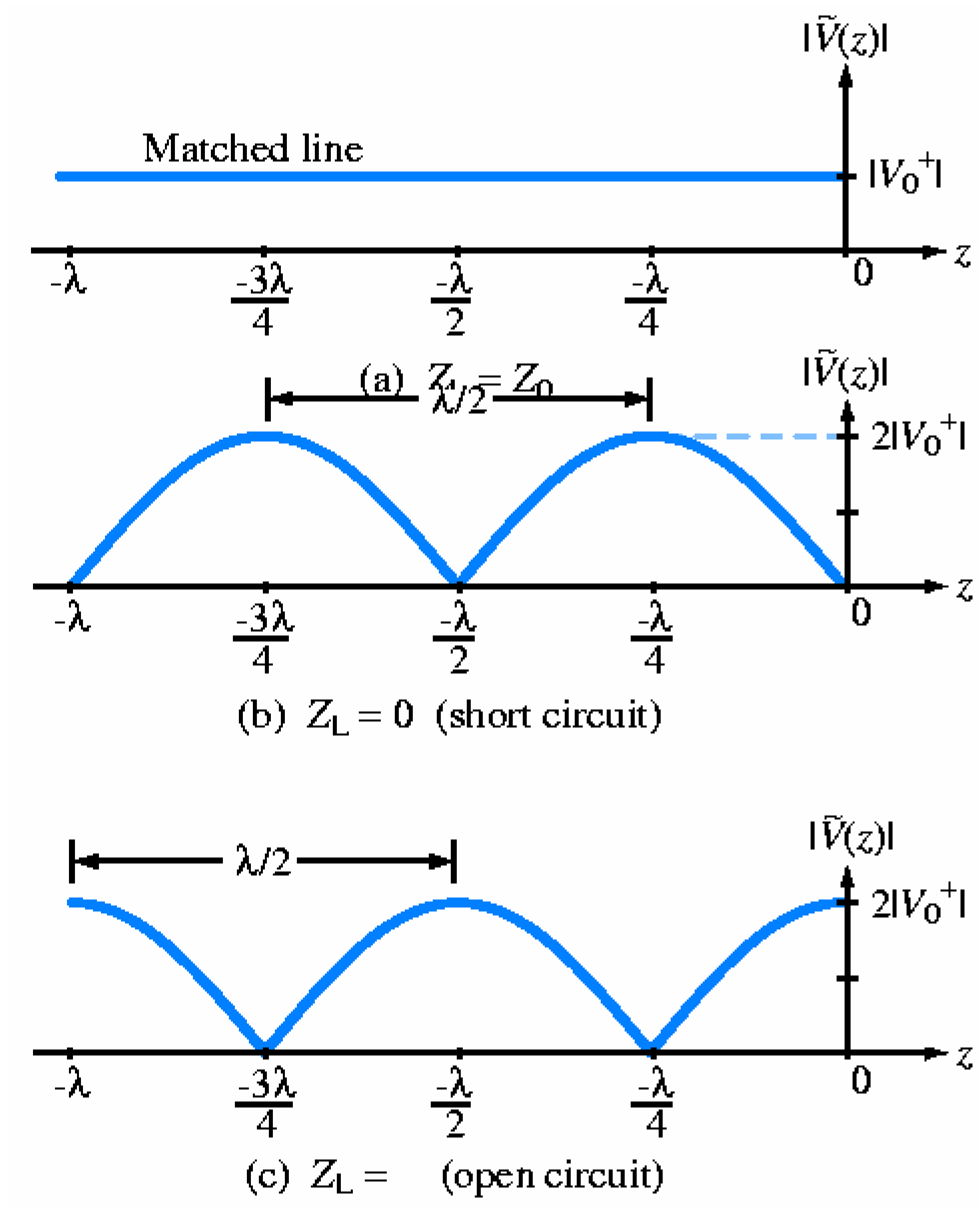
• sinusoidal pattern - standing wave caused by the interference of the two waves

•Matched line, Z_L = Z_0 with $|\Gamma|$ = 0 and |V(z)| = $|V_0^+|$ for all values of z → with no reflected wave present, there will be no interference and no standing wave

•$|\Gamma|$ = 1 and max of |V(z)| = 2|V0+| and min of |V(z)| = 0

➢Short circuit, where Γ =-1

➢Open circuit, where Γ = 1



- How to find the *max* & *min* position value of the voltage magnitude??

First voltage maximum

$$l\max = -z = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2},$$

$$\begin{cases} n = 1,2,\ldots & \text{if } \theta_r < 0 \\ n = 0,1,2,\ldots & \text{if } \theta_r \geq 0 \end{cases}$$

First voltage minimun

$$l\min = \begin{cases} l\max + \lambda/4, & \text{if } l\max < \lambda/4 \\ l\max - \lambda/4, & \text{if } l\max \geq \lambda/4 \end{cases}$$

- The ratio of $|\tilde{V}|\max$ to $|\tilde{V}|\min$ is called *the voltage standing-wave ratio, S or VSWR* and is given by

$$S = \frac{|\tilde{V}|\max}{|\tilde{V}|\min} = \frac{1+|\Gamma|}{1-|\Gamma|} (\text{dimensionless})$$

- For a matched load : $\Gamma = 0, \therefore S = 1$

- For a line : $|\Gamma| = 1, \therefore S = \infty$

Exercise 5

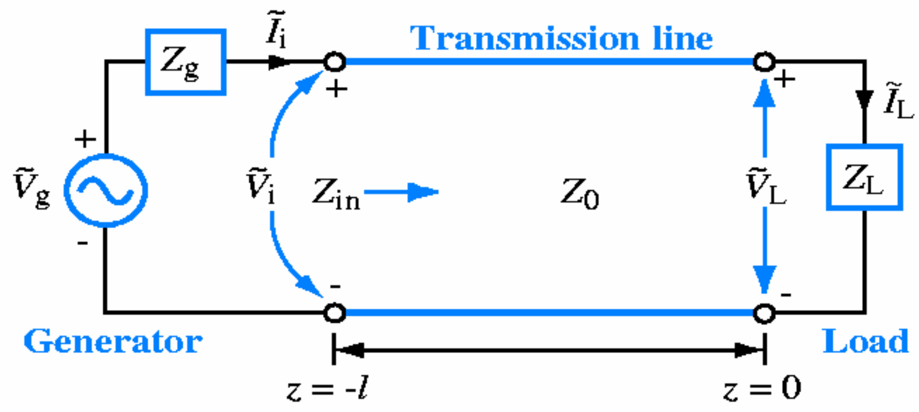
A 50Ω transmission line is terminated in a load with $Z_L = (100 + j50)\Omega$. Find the voltage reflection coefficient and the voltage standing wave ratio (VSWR).

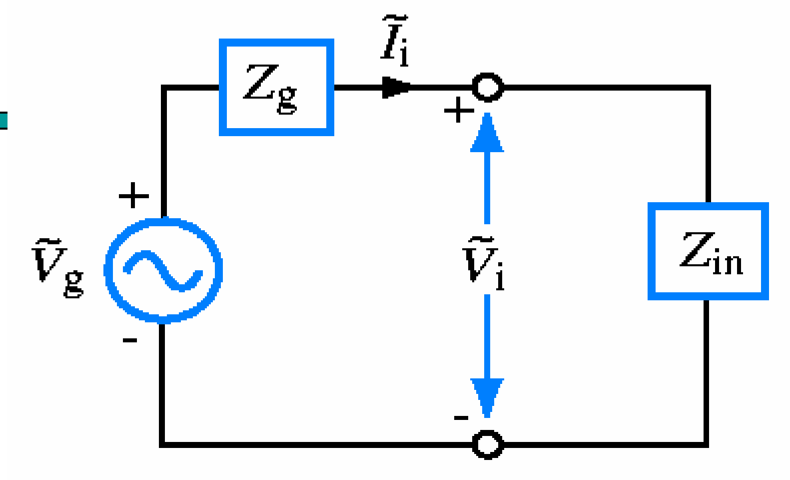
7.6 Input impedance of lossless line

- the standing wave pattern indicate that for a mismatched line the voltage and current magnitudes are oscillatory with position on the line and in phase opposition with each other.

- Hence, the voltage to current ratio Z_{in}, must vary with position

- Z_{in} is define as '*the ratio of the total voltage (incident & reflected wave) to the total current at any point z on the line.*'





$$Z_{in}(-l) = Z_0\left(\frac{Z_L\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_L\sin\beta l}\right)$$

$$= Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$$

Voltage amp of incident wave

$$V_0^+ = \left(\frac{V_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right)$$

Exercise 6

A 1.05 GHz generator circuit with series impedance Zg =10ohm and voltage source given by

$$v_g(t) = 10\sin\left(wt + 30^0\right)$$

Is connected to a load $Z_L = (100 + j50)\Omega$ through a 50Ω , 67cm long lossless transmission line. The phase velocity of the line is 0.7c , c is the velocity of light in a vacuum. Find

a) λ

b) β

c) Γ

d) Z_{in}

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All the best !!

Thank you