ELECTROMAGNETICS THEORY
BEKP3553
1-2006/2007

Tutorial 2:

Vector Calculus

1. Find the gradient of the following scalar function:

   (a) $T = 2 / (x^2+z^2)$

   (b) $U = z\cos\phi / (1+r^2)$

   (c) $W = e^{-R}\sin\theta$

2. For a scalar function $V = xy - z^2$, determine its directional derivative along the direction of vector $\mathbf{A} = (\hat{\mathbf{x}} - \hat{\mathbf{y}}z)$ and then evaluate it at $P(1,-1,2)$.

3. For the vector field $\mathbf{E} = \hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy$, verify the divergence theorem by computing:

   (a) the total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each and parallel to the Cartesian axes

   (b) the integral of $\nabla \cdot \mathbf{E}$ over the cube's volume.

4. For the vector field $\mathbf{E} = \hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)$, calculate:

   (a) $\oint_C \mathbf{E} \cdot d\mathbf{l}$ around the triangular contour shown in Figure 1.

   (b) $\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s}$ over the area of the triangle.

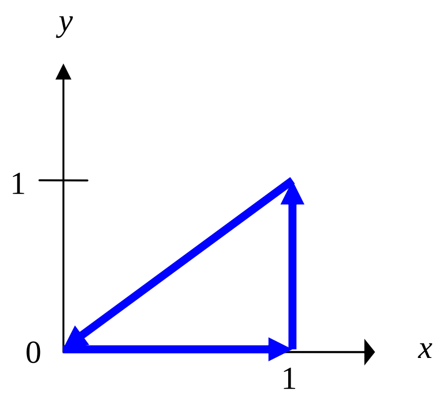

Figure 1: Contour for problem 4 (a)

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5. Verify Stoke's theorem for the vector field

$$\mathbf{B} = (\hat{\mathbf{r}} r \cos\phi + \hat{\phi} \sin\phi)$$

by evaluating the following:

(a) $\oint_C \mathbf{B} \cdot d\mathbf{l}$ over the semicircular contour shown in Figure 2.

(b) $\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s}$ over the surface of the semicircle.

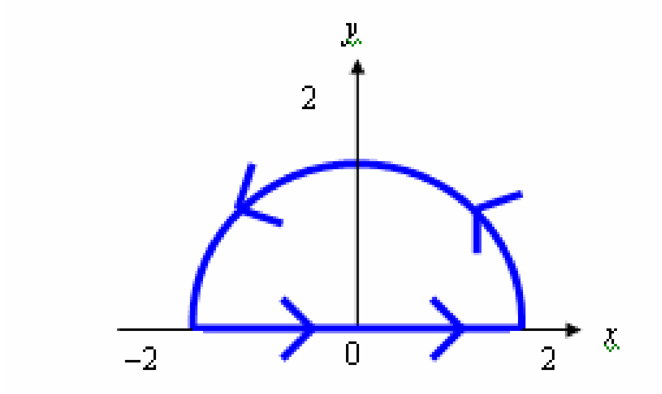

Figure 2: Contour path for problem 5(a)

6. Find the Laplacian of the following scalar functions:

(a) $V = xy^2z^3$
(b) $V = xy + yz + zx$