Electromagnetics Theory

Plane-Wave Propagation

Outline:

1. **Review of Waves and Phasors**
2. **Time-Harmonic Fields**
3. **Plane-Wave Propagation in Lossless Media**

Introduction

$\mathbf{E}(t) \Leftrightarrow \mathbf{H}(t)$

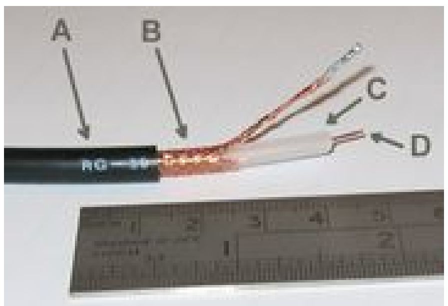
↓

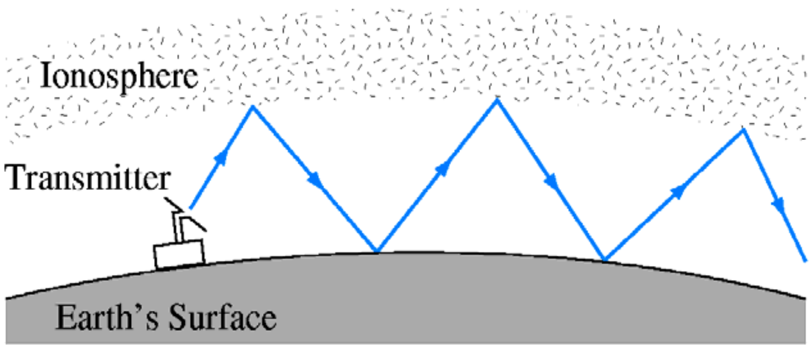
Generates EM wave

↓

Propagate

↓ Guided ↓ Wireless



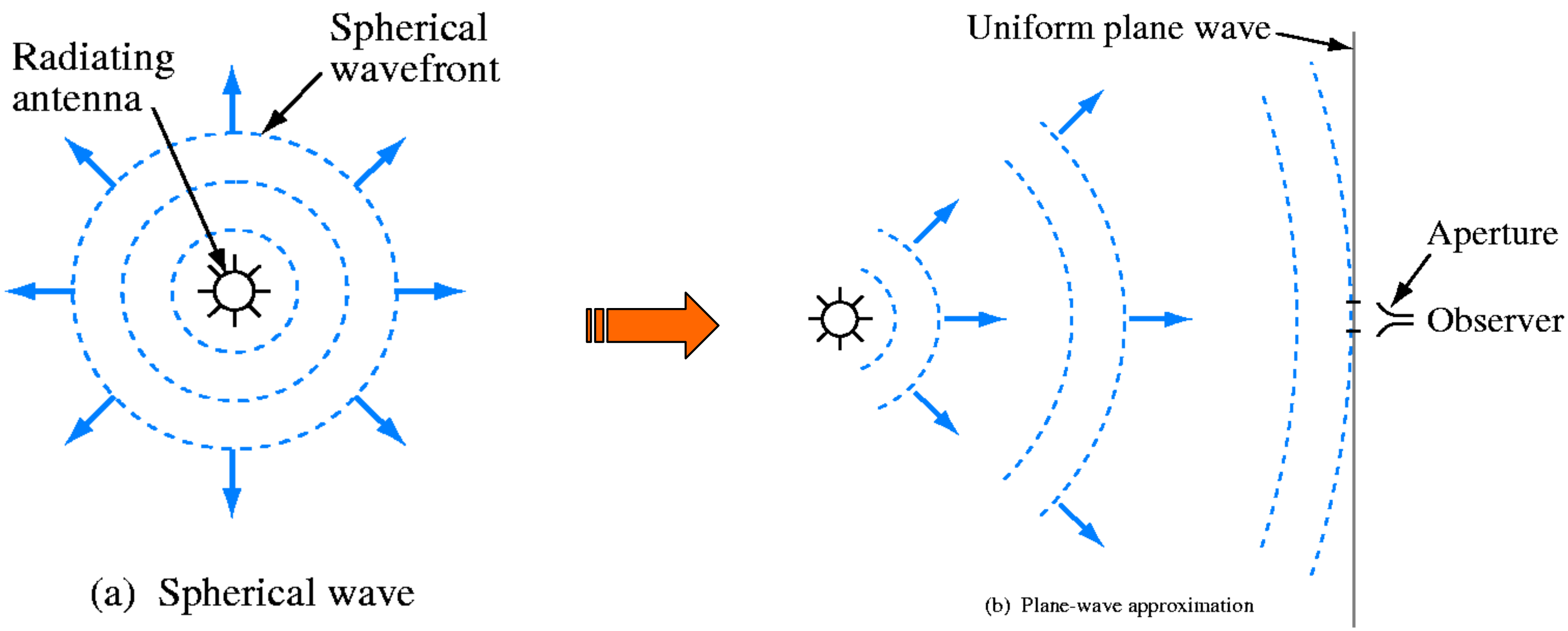


Uniform plane wave

To describe the propagation of a spherical wave

⇩

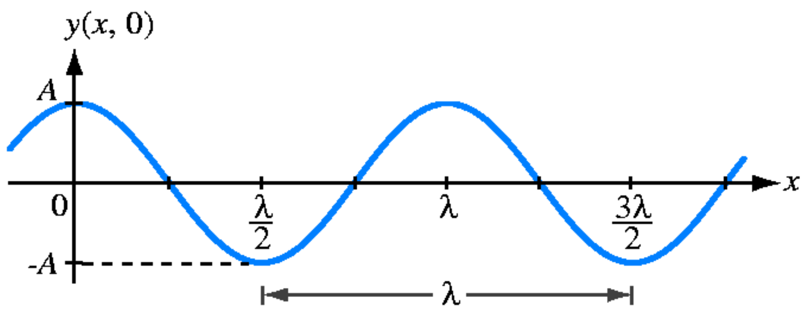
Cartesian Coordinate System



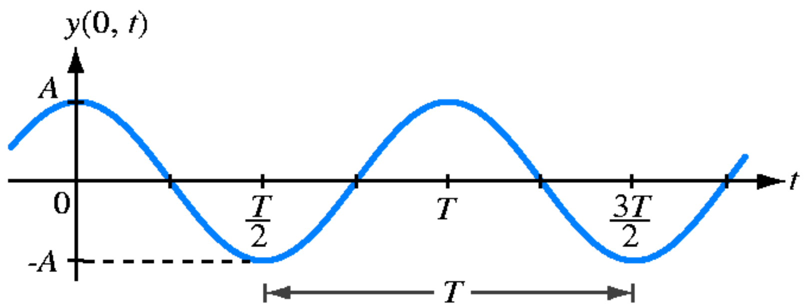
Review of Waves and Phasors

Sinusoidal Wave in a Lossless Medium

A medium is said to be ***lossless*** if it does not attenuate the amplitude of the wave traveling within it or on its surface.



(a) $y(x, t)$ versus x at $t = 0$



(b) $y(x, t)$ versus t at $x = 0$

$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)$$

Amplitude

Period

wavelength

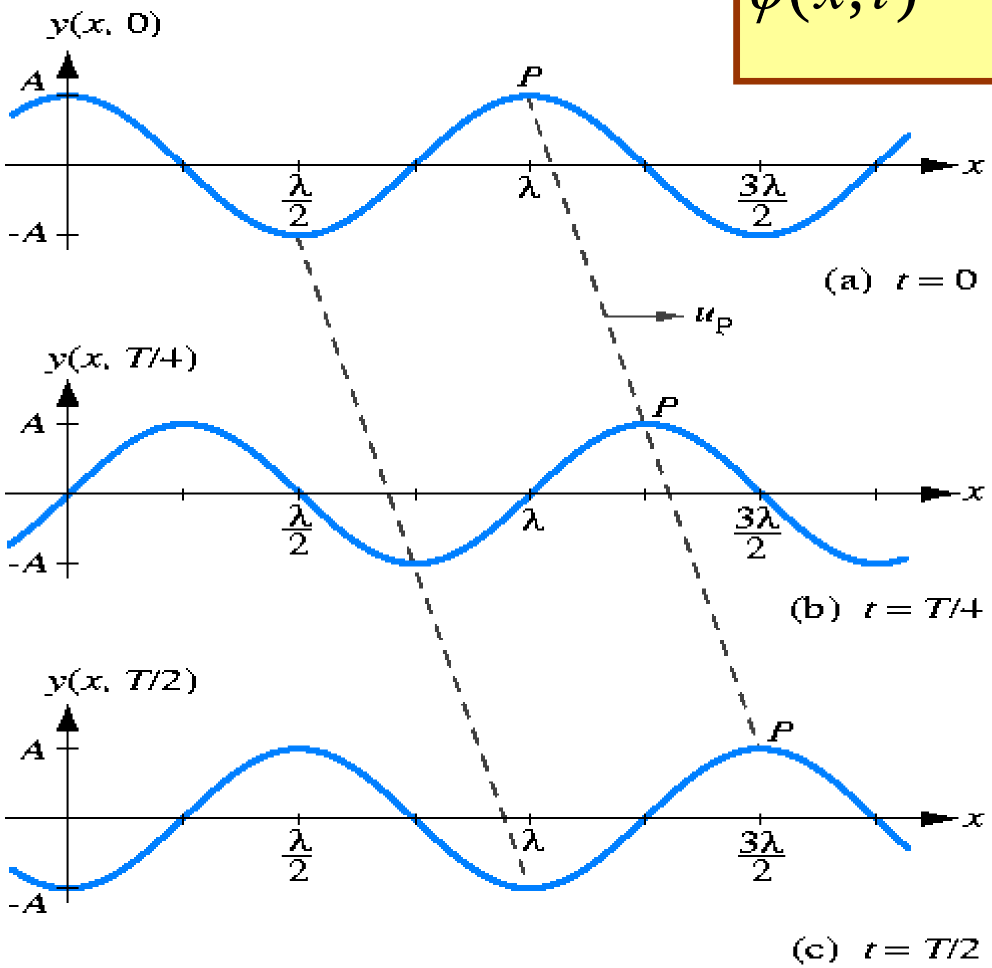
Reference phase

$$\boxed{\phi(x,t) = \left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)}$$

Phase

Phase velocity

Phase velocity (u_p)



$$\phi(x,t) = \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} = 2n\pi, \qquad n = 0,1,2\ldots$$

$$\frac{d}{dt}\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right) = 0$$

u_p

$$\frac{2\pi}{T} - \frac{2\pi}{\lambda}\frac{dx}{dt} = 0$$

$$u_p = \frac{\lambda}{T} \quad \text{(m/s)}$$

Phase velocity (cont'ed)

Phase velocity is the velocity of the wave pattern.

Direction of propagation

If one of the signs is +ve and the other is –ve → *wave in + x-direction*

If both signs are +ve or both are –ve → *wave in – x-direction*

ϕ_0 → *has no influence on either the speed or the direction of the wave propagation*

$$\boxed{\phi(x,t) = \left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right)}$$

Phase velocity, phase constant

$$y(x,t) = A\cos(2\pi f t - \frac{2\pi}{\lambda}x)$$

$$= A\cos(\omega t - \beta x)$$

$\omega = 2\pi f$ (rad/s)

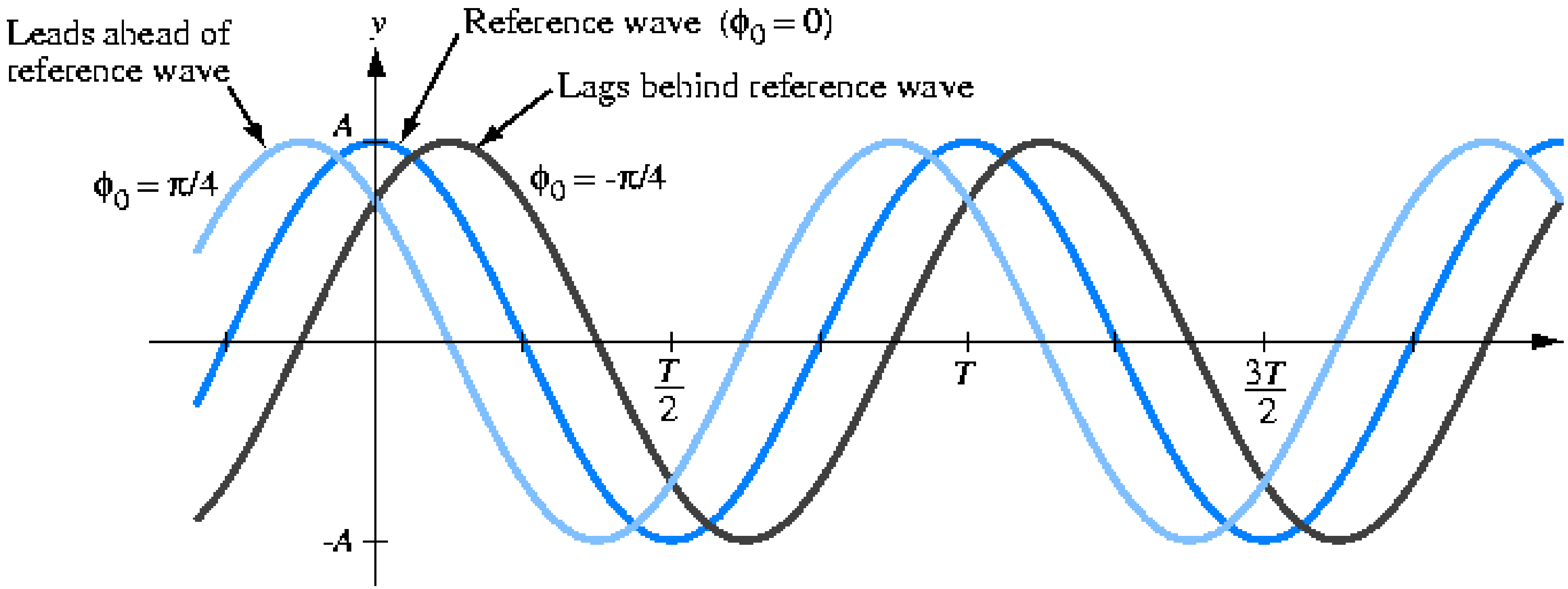
Angular velocity

Phase constant/ wavenumber

$\beta = \dfrac{2\pi}{\lambda}$ (rad/m)

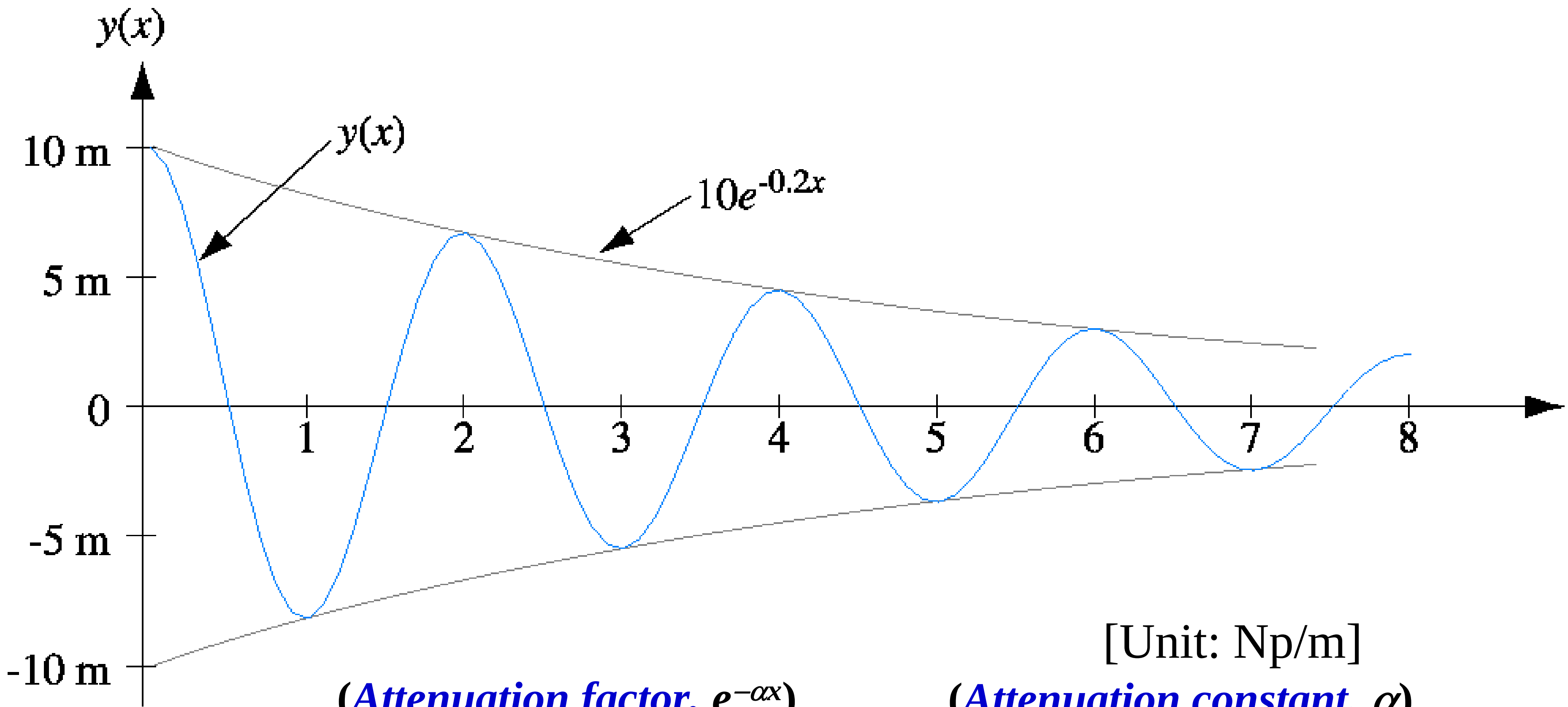
u_p in terms of ω and β ➡ $u_p = \dfrac{\omega}{\beta} = f\lambda$

The role of reference phase, ϕ_0



$$y(x,t) = A\cos(\omega t - \beta x + \phi_0)$$

Sinusoidal Wave in a Lossy Medium



[Unit: Np/m]

(***Attenuation factor***, $e^{-\alpha x}$) (***Attenuation constant***, α)

$$y(x,t) = Ae^{-\alpha x}\cos(\omega t - \beta x + \phi_0)$$

Example 6-1: Sound wave in water

An acoustic wave traveling in the x-direction in a fluid is characterized by a differential pressure $p(x,t)$. The unit for pressure is N/m^2. Find the expression for $p(x,t)$ for a sinusoidal sound wave traveling in the positive x-direction in water. Given:

$f = 1$ kHz
$u_p = 1.5$ km/s
Amplitude = 20 N/m^2
$p(x,t)$ max at $t = 0$, $x = 0.25$ m.

Treat water as a lossless medium

$$p(x,t) = 20\cos(2\pi \times 10^3 t - \frac{4\pi}{3} x + \frac{\pi}{3})$$

Example 6-2: Power Loss

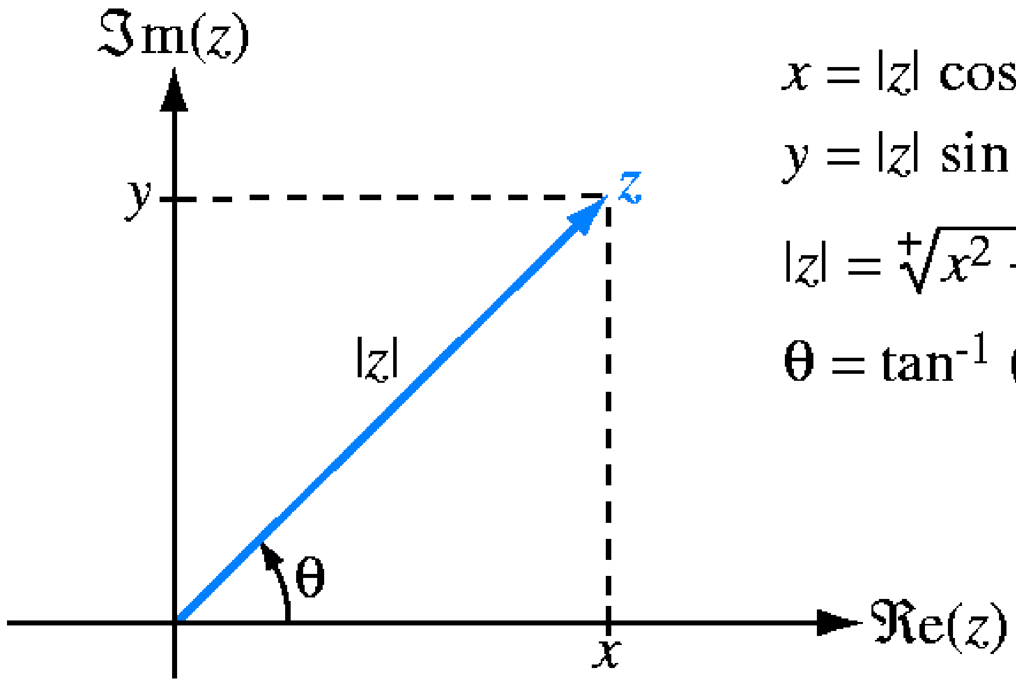
A laser beam of light propagating through the atmosphere is characterized by an electric field intensity given by:

where *x* is the distance from the source in meters. The attenuation is due to absorption by atmospheric gases. Determine:

a) The direction of wave travel
b) The wave velocity
c) The wave amplitude at a distance of 300m

Review of Complex Numbers

Complex number



$$x = |z| \cos \theta$$

$$y = |z| \sin \theta$$

$$|z| = \sqrt[+]{x^2 + y^2}$$

$$\theta = \tan^{-1} (y/x)$$

Real part

Imaginary part

$$z = x + jy$$

$$z = |z|\, e^{j\theta} = |z| \angle \theta$$

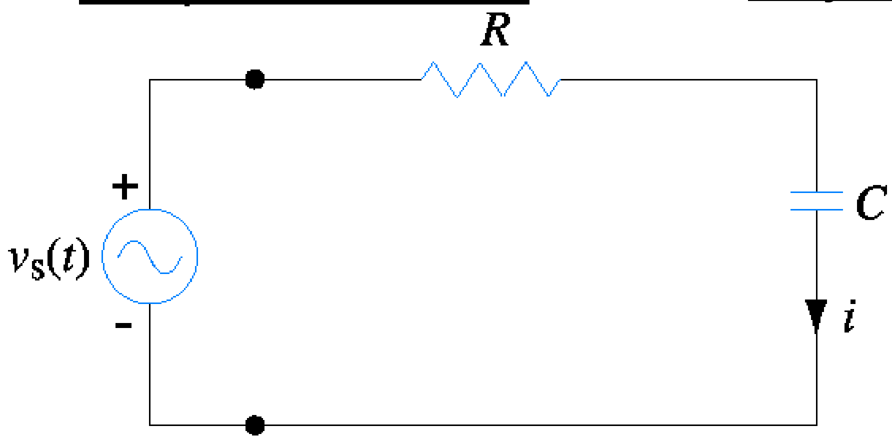
$$e^{j\theta} = \cos\,\theta + j\,\sin\,\theta$$

Review of Phasors

What is ***phasor analysis*** **?**

- Is a mathematical tools
  - Solving linear systems with periodic time function

Simple RC circuit



Objective: Obtain an expression for current $i(t)$

$$v_s(t) = V_o \sin(\omega t + \phi_0)$$

$$\boxed{i(t) = ??}$$

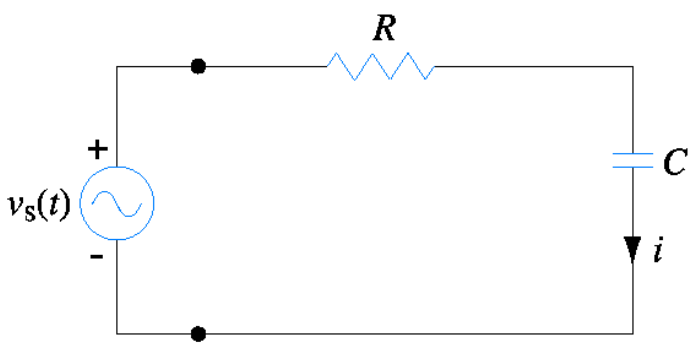
Review of Phasors (cont'ed)

Step 1: Adopt a cosine reference

$$v_s(t) = V_o \sin(\omega t + \phi_0)$$

$$\Downarrow$$

$$v_s(t) = V_0 \cos(\omega t + \phi_0 - \frac{\pi}{2})$$



Apply KVL

$$\boxed{Ri(t) + \frac{1}{C}\int i(t)dt = v_s(t) \quad \text{(time domain)}}$$

Review of Phasors (cont'ed)

Step 2: Express time-dependent variables as phasors

$$z(t) = \mathrm{Re}[\tilde{Z} e^{j\omega t}]$$

$$v_s(t) = V_0 \cos(\omega t + \phi_0 - \frac{\pi}{2})$$

Time-independent
phasors

$$v_s(t) = \mathrm{Re}[V_0 e^{(j\omega t + \phi_0 - \frac{\pi}{2})}]$$

$$= \mathrm{Re}[V_0 e^{(\phi_0 - \frac{\pi}{2})} e^{j\omega t}]$$

$$= \mathrm{Re}[\tilde{V}_s e^{j\omega t}]$$

Recall:

$$Ri(t) + \frac{1}{C}\int i(t)dt = v_s(t) \quad \text{(time domain)}$$

$v_s(t)$ is now represented with time-independent ***phasors***

What next?

Review of Phasors (cont'ed)

i (*t*) $\Rightarrow$ $$i(t) = \mathrm{Re}(\tilde{I}e^{j\omega t})$$

Recall:

$$Ri(t) + \frac{1}{C}\int i(t)dt = v_s(t)$$ (time domain)

Solution:

differentiation

$$\frac{di}{dt} = \mathrm{Re}[j\omega\tilde{I}e^{j\omega t}]$$

integration

$$\int i\ dt = \mathrm{Re}[\frac{\tilde{I}}{j\omega}e^{j\omega t}]$$

Review of Phasors (cont'ed)

Step 3: Recast the differential/integral form equation is phasor form

$$Ri(t) + \frac{1}{C}\int i(t)dt = v_s(t) \quad \text{(time domain)}$$

$$R(\mathrm{Re}[\tilde{I}e^{j\omega t}]) + \frac{1}{C}(\mathrm{Re}[\frac{\tilde{I}}{j\omega}e^{j\omega t}]) = \mathrm{Re}[\tilde{V}_s e^{j\omega t}]$$

$$\tilde{I}(R + \frac{1}{j\omega C}) = \tilde{V}_s \quad \text{(phasor domain)}$$

Review of Phasors (cont'ed)

Step 4: Solve the phasor-domain equation

$$\tilde{I} = \frac{\tilde{V}_s}{R + 1/(j\omega c)} \quad \Rightarrow \quad \tilde{I} = \frac{V_0 \omega C}{\sqrt[+]{1 + \omega^2 R^2 C^2}} e^{j(\phi_0 - \phi_1)}$$

Step 5: Find the instantaneous value

<u>Objective:</u> Obtain $i(t)$

$$i(t) = \mathrm{Re}[\tilde{I} e^{j\omega t}]$$

$$= \mathrm{Re}[\frac{V_o \omega C}{\sqrt[+]{1 + \omega^2 R^2 C^2}} e^{j(\phi_0 - \phi_1)} e^{j\omega t}]$$

$$= \frac{V_o \omega C}{\sqrt[+]{1 + \omega^2 R^2 C^2}} \cos(\omega t + \phi_o - \phi_1)$$

Time-domain vs phasor-domain

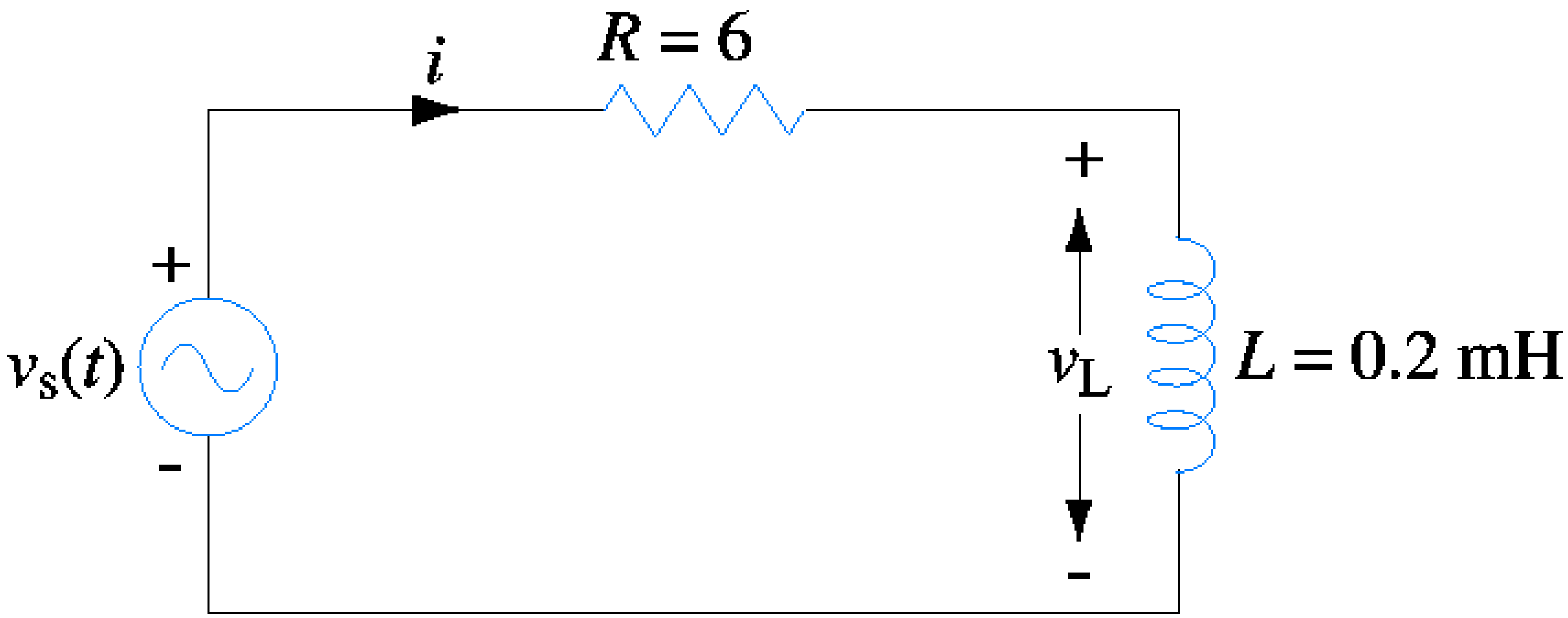
$z(t)$		$\tilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t+\phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t+\beta x+\phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t+\beta x+\phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t+\phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z_1(t))$	$\leftrightarrow$	$j\omega\tilde{Z}_1$
$\frac{d}{dt}[A\cos(\omega t+\phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z_1(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\tilde{Z}_1$
$\int A\sin(\omega t+\phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

Example 6-4: *RL* Circuit

The voltage source of the circuit shown below is given by:

$$v_s(t) = 5\sin(4\times10^4 t - 30^0)$$

Obtain an expression for the voltage across the inductor.



Time Harmonic Fields

In the *time-varying case*, the electric and magnetic fields (**E**, **D**, **B** and **H**) and source (ρ_v , **J**) are each, in general, a function of the spatial coordinates (x, y, z) and the time variable t.

$$\mathbf{E}(x, y, z; t) = \mathrm{Re}[\tilde{\mathbf{E}}(x, y, z)e^{j\omega t}]$$

Maxwell's equations in phasor domain ⇒

$$\nabla \cdot \tilde{\mathbf{E}} = \tilde{\rho}_v / \varepsilon$$

$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}}$$

$$\nabla \cdot \tilde{\mathbf{H}} = 0$$

$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\varepsilon\tilde{\mathbf{E}}$$

Complex Permittivity

$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\varepsilon\tilde{\mathbf{E}} \qquad ; \mathbf{J} = \sigma\mathbf{E}$$

$$= (\sigma + j\omega\varepsilon)\tilde{\mathbf{E}}$$

$$= j\omega\left(\varepsilon - j\frac{\sigma}{\omega}\right)\tilde{\mathbf{E}}$$

Complex permittivity, ε_c

For a lossless medium, σ = 0, $\varepsilon_c = \varepsilon$

Wave equations for a charge-free medium

Charge free $\equiv \rho_v = 0$

Maxwell's equations for charge a free medium $\Rightarrow$

$$\nabla \cdot \tilde{\mathbf{E}} = 0$$
$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}}$$
$$\nabla \cdot \tilde{\mathbf{H}} = 0$$
$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\varepsilon_c\tilde{\mathbf{E}}$$

$\Downarrow$

Homogenous wave equation for Ĕ $\Rightarrow$

$$\nabla^2\tilde{\mathbf{E}} + \omega^2\mu\varepsilon_c\tilde{\mathbf{E}} = 0$$

Propagation constant : $\gamma^2 = -\omega^2\mu\varepsilon_c$

Wave equations for **E**, **H**

Wave equations for E

$$\boxed{\nabla^2 \tilde{\mathbf{E}} + \omega^2 \mu \varepsilon_c \tilde{\mathbf{E}} = 0} \Rightarrow \boxed{\nabla^2 \tilde{\mathbf{E}} - \gamma^2 \tilde{\mathbf{E}} = 0}$$

Wave equations for H

$$\boxed{\nabla^2 \tilde{\mathbf{H}} + \omega^2 \mu \varepsilon_c \tilde{\mathbf{H}} = 0} \Rightarrow \boxed{\nabla^2 \tilde{\mathbf{H}} - \gamma^2 \tilde{\mathbf{H}} = 0}$$

Plane-Wave Propagation in Lossless Media

Lossless media ≡ *nonconducting* medium (σ=0)

$$\boxed{\varepsilon_c = \varepsilon \cancel{- j\frac{\sigma}{\omega}}}$$

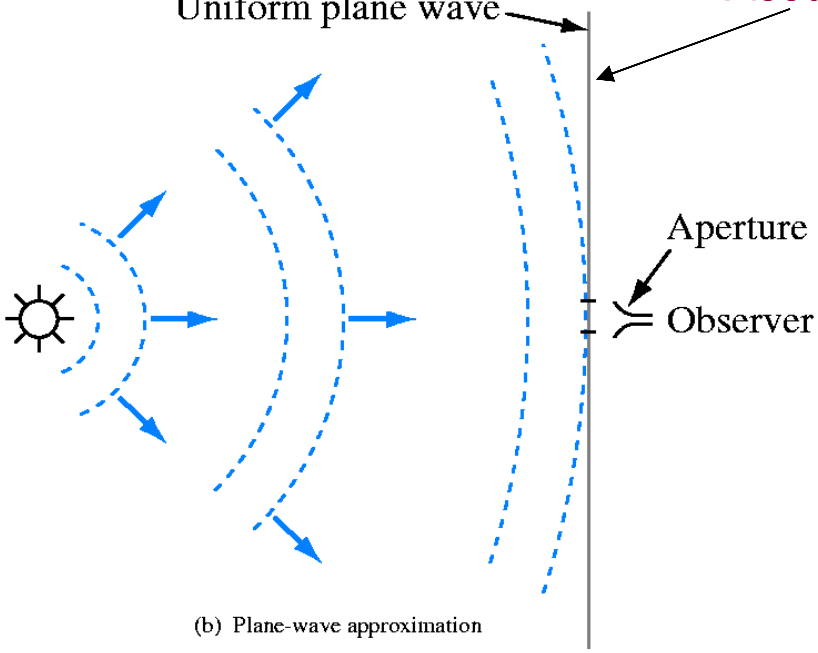
$$\boxed{\gamma^2 = -\omega^2 \mu\varepsilon}$$

Wavenumber k (lossless)

$$\boxed{k = \omega\sqrt{\mu\varepsilon}}$$

Uniform Plane Waves

A *uniform plane wave* is characterized by *electric* and *magnetic* fields that have *uniform* properties at all points across an infinite plane.



Assume x-y plane

$$\frac{\partial \tilde{E}_x}{\partial x} = \frac{\partial \tilde{E}_y}{\partial y} = 0$$

$$\boxed{\tilde{E}_z = 0}$$

$why = 0$

Uniform Plane Waves (cont'ed)

why=0

$$\boxed{\nabla \times \tilde{\mathbf{H}} = j\omega\varepsilon\tilde{\mathbf{E}}}$$

Taking the *Curl* on the *z*-component

$$\Downarrow$$

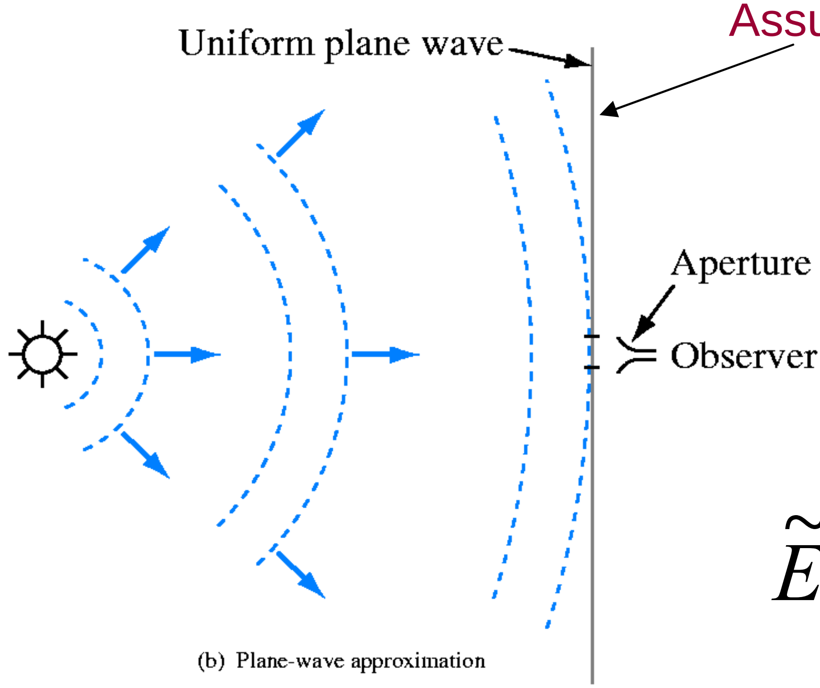
$$\boxed{\hat{\mathbf{z}}\left(\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y}\right) = \hat{\mathbf{z}} j\omega\varepsilon\tilde{E}_z}$$

$$\Downarrow$$

$$\boxed{\frac{\partial \tilde{H}_y}{\partial x} = \frac{\partial \tilde{H}_x}{\partial y} = 0, \qquad \therefore \tilde{E}_z = 0}$$

A plane wave has no electric- or magnetic-field components along its direction of propagation

Uniform Plane Waves (cont'd)



(b) Plane-wave approximation

Assume x-y plane

z-direction

$$\tilde{E}_x(z) = \tilde{E}_x^{+}(z) + \tilde{E}_x^{-}(z)$$
$$= E_{x0}^{+} e^{-jkz} + E_{x0}^{-} e^{jkz}$$

$+z$*-direction* $\qquad$ $-z$*-direction*

Uniform Plane Waves (cont'ed)

Objective: To get the relationship of **E** *and* **H** *in plane wave propagation*

$V_0^+ = Z_0 I_0^+$

Assume **E** has only a component along *x-direction, $E_y=0$*

$$\nabla \times \mathbf{E} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \tilde{E}_x^+(z) & 0 & 0 \end{vmatrix} = -j\omega\mu(\hat{\mathbf{x}}\tilde{H}_x + \hat{\mathbf{y}}\tilde{H}_y + \hat{\mathbf{z}}\tilde{H}_z)$$

$$\Rightarrow$$

$$\tilde{H}_x = \tilde{H}_z = 0$$

$$\tilde{H}_y = \frac{1}{-j\omega\mu}\frac{\partial \tilde{E}_x^+(z)}{\partial z}$$

$$\Downarrow$$

$$\tilde{H}_y(z) = \frac{k}{\omega\mu} E_{x0}^+ e^{-jkz} = H_{y0}^+ e^{-jkz}$$

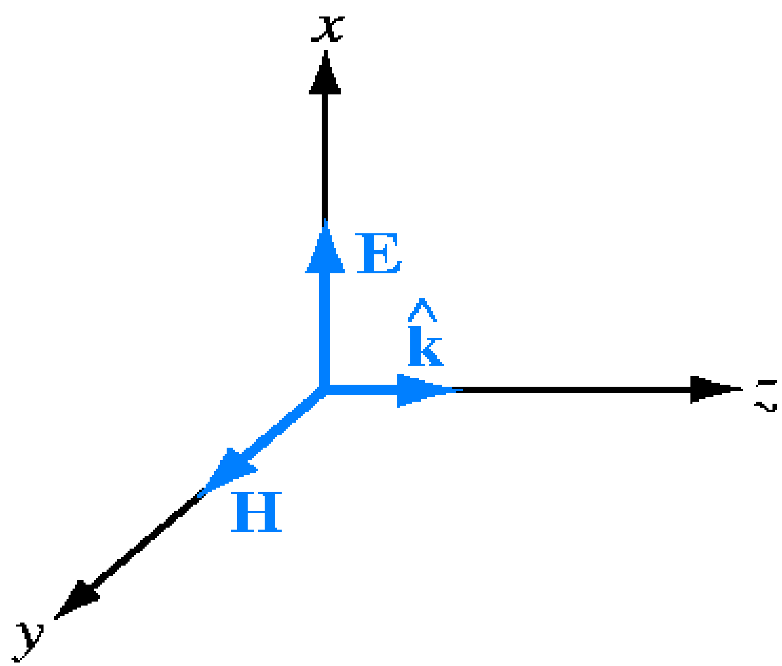
$$\Leftarrow$$

$$H_{y0}^+ = \frac{k}{\omega\mu} E_{x0}^+$$

Intrinsic impedance

$$\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}}$$

Transverse electromagnetic (TEM) wave



$$\boxed{\begin{aligned}\tilde{\mathbf{E}}(z) &= \hat{\mathbf{x}}\tilde{E}_x^+(z) \\ \tilde{\mathbf{H}}(z) &= \hat{\mathbf{y}}\,\frac{\tilde{E}_x^+(z)}{\eta}\end{aligned}}$$

complex

$$\tilde{E}_x^+(z) = E_{x0}^+ e^{-jkz} = \left|E_{x0}^+\right| e^{j\phi^+} e^{-jkz}$$

Transverse electromagnetic (TEM) wave…

The instantaneous electric and magnetic fields are given by:

$$\mathbf{E}(z,t) = \mathrm{Re}[\widetilde{\mathbf{E}}(z)e^{j\omega t}]$$
$$= \hat{\mathbf{x}}\left|E_{x0}^{+}\right|\cos(\omega t - kz + \phi^{+}) \quad \text{(V/m)}$$
$$\mathbf{H}(z,t) = \mathrm{Re}[\widetilde{\mathbf{H}}(z)e^{j\omega t}]$$
$$= \hat{\mathbf{y}}\left|\frac{E_{x0}^{+}}{\eta}\right|\cos(\omega t - kz + \phi^{+}) \quad \text{(A/m)}$$

They are ***in phase***

Phase velocity for Plane Wave Propagation

Recall:

$$y(x,t) = A\cos(2\pi ft - \frac{2\pi}{\lambda}x)$$
$$= A\cos(\omega t - \beta x)$$

$$u_p = \frac{\omega}{\beta} = f\lambda$$

$$\beta = \frac{2\pi}{\lambda} \quad \text{(rad/m)}$$

Deduce from:

$$\mathbf{E}(z,t) = \mathrm{Re}[\tilde{\mathbf{E}}(z)e^{j\omega t}]$$
$$= \hat{\mathbf{x}}\left|E_{x0}^{+}\right|\cos(\omega t - kz + \phi^{+}) \quad \text{(V/m)}$$

$$u_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{(m/s)}$$

$$\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \quad \text{(m)}$$

If the medium is vacuum, u_p = ? η =?

Example 6-5: EM Plane Wave in Air

The electric field of a 1-MHz plane wave traveling in the +*z*-direction in air points along the *x*-direction. If the peak value of *E* is 1.2π (mV/m) and *E* is max. at *t* =0 and *z* = 50 m, obtain expressions for **E**(*z*,*t*) and **H**(*z*,*t*), then plot these variations as a function of *z* at *t*=0.

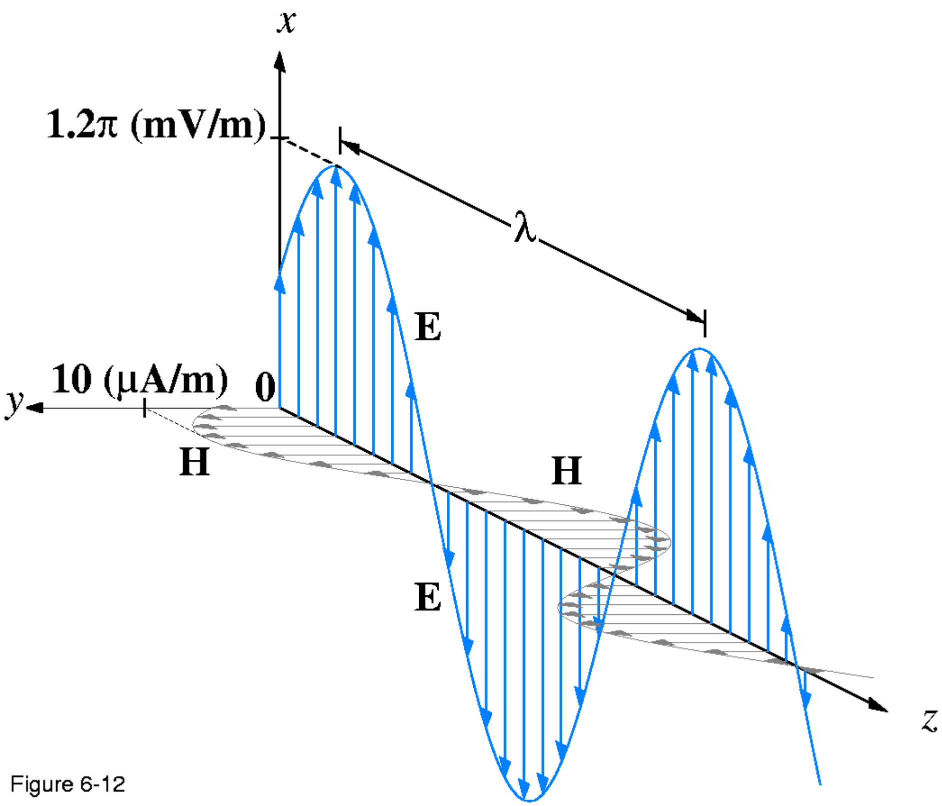
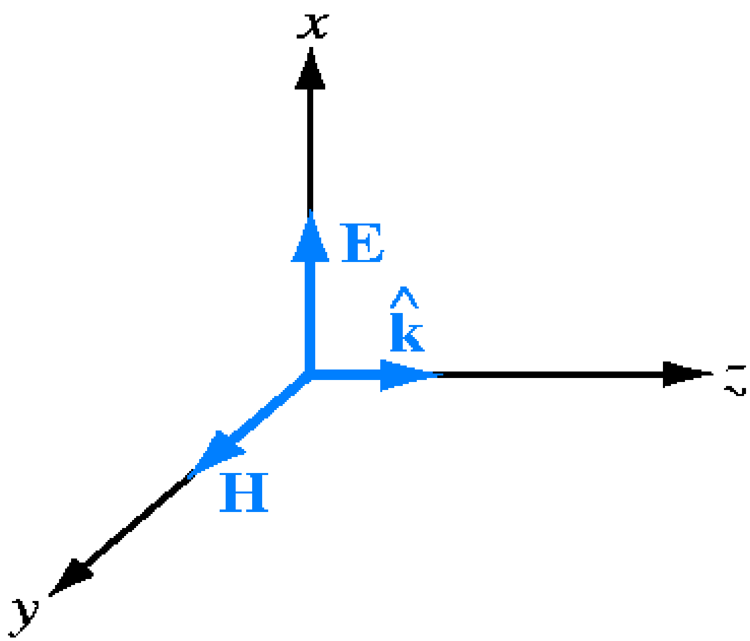


Figure 6-12

General Relation between E and H



$$\tilde{\mathbf{H}} = \frac{1}{\eta}\hat{\mathbf{k}} \times \tilde{\mathbf{E}}$$

$$\tilde{\mathbf{E}} = -\eta\hat{\mathbf{k}} \times \tilde{\mathbf{H}}$$

When we rotate the four fingers of the right hand from the direction of **E** *toward the direction of* **H**, *the <u>thumb</u> will point in the direction of wave travel,* **k**-hat

Exercise

A 10-MHz uniform plane wave is traveling in a nonmagnetic medium with $\mu = \mu_0$ and $\varepsilon_r = 9$. Find

(a) The phase velocity

(b) The wavenumber

(c) The wavelength in the medium

(d) The intrinsic impedance of the medium

Thank You